

2
$1-4$


Since the completion of the bridge it has had to sustain large and irregular-moving loads. Being a work of great local interest, it has on several occasions been crowded with people. It is evident, however, that the most severe strain which it will have to resist is that resulting from heavy gales of wind. South-westerly gales blow nearly in the direction of the length of the bridge, and the high ground on either side diminishes their effect upon the structure. But gales from the north-west or south-east, being nearly in the direction of the deep gorge of the River Avon at the place where the bridge is constructed, impinge upon the work with great violence, so much so that it is difficult at times to stand upon the roadway. On these occasions three effects are observed:

First, there is a small horizontal deflection, which is just sufficient to be perceptible to the eye when placed in range with the suspension-rods. Secondly, there is an undulation from end to end of the bridge. It is a slow and steady movement of the structure, which manifests itself by a rising

Table of Experiments on Cast-iron Beams, showing the Weight on the Centre Span at the
Time of Fracture.


Table showing the Deflections with 100 lbs . uniformly Distributed over Centre Span.

| $\begin{gathered} \text { Condition } \\ \text { of } \\ \text { Loading. } \end{gathered}$ | No. 1. <br> Uniform Continu- ous Beam. | No. 2. <br> Continuous Beam, increased three times in width over the Piers the weight of creased 50 per cent. | No. 3. <br> Continuous Beam increased $\begin{aligned} & \text { to } \\ & \text { double the depth }\end{aligned}$ over the Piers, the weight of creased 25 per cent. | Detached Beam loa <br> Length of bearing eq of Beams, N | $\begin{aligned} & \text { led uni } \\ & \text { al to } \\ & \text { s. } 1,2, \end{aligned}$ | mly. re span |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Loaded only on centre span, the ends being fastened down. | $\begin{array}{cc} \text { Exp. } & \text { incl. } \\ \text { No. } 1 . & \cdot 1703 \end{array}$ | Exp.  inch. <br> No. $\cdot$ $\cdot 1308$ <br> , 2 $\cdot$ -1274 <br> $" 3$ $\cdot$ 1381 <br>    <br> Mean -1321  | Exp.  <br> No. inch.  <br> No $\cdot 1114$ <br> $" 2$ $\cdot 1090$ <br> $" 3$ $\cdot 1066$ <br> $", 4$ $\cdot$ <br> Mean $\cdot$ <br> 1187  |  |  |  |
|  | 2 . 1572 |  |  |  |  |  |
|  | , 3 . 1501 |  |  |  |  |  |
|  | 4 - 1669 |  |  | Mea |  |  |
|  | Mean . 1611 |  |  | Summary showing the Deflection as compared with a detached Beam. |  |  |
| The load a foot on centre span being twice that on side spans. | $\begin{array}{cc} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & \cdot 1428 \end{array}$ | $\begin{array}{cc} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & \cdot 0979 \end{array}$ | $\begin{array}{ll} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & \cdot 0795 \end{array}$ | Condition of Loading. | $\begin{gathered} \text { No. } \\ \text { of } \\ \text { Beam. } \end{gathered}$ | Mean relative Deflections, that detached Beam being100 . |
|  | , 2 . 1317 | , 2 . 0935 | " 2 . 0788 |  |  |  |
|  | $3 \cdot \cdot 1317$ | 3 . 0916 | , 3 . 0723 |  |  |  |
|  | ,, 4 . 1246 | , 4 . 0955 | , 4 . 0723 | Loaded only on centre span, the ends being fastened down . | $\} \begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | $\begin{aligned} & 46 \cdot 04 \\ & 37 \cdot 75 \\ & 31 \cdot 83 \end{aligned}$ |
|  | Mean . 1327 | Mean . 0946 | Mean . 0757 |  |  |  |
| Beam loaded uniformly over its whole length. | $\begin{array}{cc} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & \cdot 0843 \end{array}$ | $\begin{array}{cc} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & 0496 \end{array}$ | $\begin{array}{cc} \text { Exp. } & \text { inch. } \\ \text { No. } 1 . & 0228 \end{array}$ | The load per foot |  | 37.92 |
|  | 823 | , 2.0477 | $, 2.0250$ | on centre span <br> being twice that on side spans | , $\begin{aligned} & 2 \\ & 3\end{aligned}$ | 27.0421.63 |
|  | -0799 | , 3 . 0472 | , 2.0250 |  |  |  |
|  | 799 | ,, 4 . 0497 | 4 . 0283 | Beam loaded uniformly over its whole length | $\left\{\begin{array}{l}1 \\ 2 \\ 3\end{array}\right.$ | $\begin{array}{r} 23 \cdot 31 \\ 13 \cdot 86 \\ 7 \cdot 14 \end{array}$ |
|  |  | Mean . 0485 | Mean. -02:5 |  |  |  |

and falling os the roadway about half-way between the centre and the abutments, alternating from the north-eastern to the south-western portion of the bridge. The third effect produced by wind is the motion imparted to the land chains. It will be observed that there are no suspension-rods between the piers and the land-saldles, so that the chains hang without anything to restrain their motion. Violent gusts of wind are capable of deflecting these chains literally, notwithstanding their weight, the longitudinal strain upon them, and the comparatively small surface expos ${ }^{~}$ d.
Two features of this bridge appear worthy of remark. One is the facility with which the work was executed; the other is the comparatively inexpensive nature of the scaffolding or temporary staging required for erecting the chains, considering the distance between the points of support. Buth these results are due to the principle of suspension. Already spans crossed by bridges on the principle of suspension far exceed those of any form of girder.

In relation to this subject it may be right to mention a construction of continuous girder which approaches, in regard to economy of materials employed, to a stiffened suspension bridge. In a continuous girder, if, instead of using an equal depth throughout, a greater depth and a greater sectional area be given over the piers, it lias been found by experiments on solid bars, as well as from a theoretical investigation of the case as applied to lattice girders, that an increase of strength is obtained in a much higher ratio than that of the increased weiglit of metal employed. The experiments on the bars referred to were made by W. H. Barlow, in 1858 , and the material employed was cast-irnn, which was selected because the relative strengths of the different forms were indicated in a distinct and decided manner, by the actual rupture of the beam. The experiments extended to a comparison of strengths and stiffness of four descriptions of bars;-

1st. Detached bars, parallel throughout. 2nd. Continuous, bars, of equal section throughout. 3rd. Parallel continuous bars, in which the sectional area was increased over the piers, without increasing the depths. 4th. Continuous bars, in which both the sectional area and the depth were increased over the piers. The continuous bars were tried under three separate conditions of loading; -1 st. The centre span alone was loaded. 2nd. The load on the centre span per unit of length was made double the load in the side spans. 3rd. The load was evenly distributed throughout the whole length of the bar. The detached bar was 6 ft. between the b carings; the continuous bars were 15 ft . long, having a centre opening of 6 ft ., and two side openings, each 4 ft .6 in. The results are given in the p. 764 tables.

In the last form, where the area and the depth are each made double over the pier, while the bar is increased in weight only 25 per cent. in the centre opening, the strength and stiffness are increased in much higher proportion.

The mean relative strength of this bar, in the three conditions of loadiug, that of the detached bar being taken as 1 , were as follows;-when loaded only in the centre span, $2 \cdot 60$; when the load a foot on the centre span was twice that of the side span, $\dot{3} \cdot 93$; when the load was equally distributed throughout the bar, $4 \cdot 42$. The stiffness was increased in a still higher proportion, for, taking the deflection of a detached beam at 100 , the deflections of the continuous beams under the three conditions of loading, were 31,21 , and 7 rtspectively .

The introduction of a material of twice the tensile strength of iron would enable a bridge of twice the span of the Clifton Bridge to be made with the same sectional area in the chains, the ratio between the versed sine of the curve and the length of the chains being similar, and the weight of the roadway and load a foot the same in each case; whereas to make a bridge of double that span in wrought-iron would require nearly four times the section in the chains. The importance of so great a reduction in weight in crossing openings of large span can hardly be overrated. It operates not only in diminishing the actual quantity of material in the structure itself, and the consequent strains on the abutments or anchorages, but it diminishes very greatly the cost of the temporary staging and scaffolding required to construct the work.

Comparative merits of different Systems of Iron Bridge Building accurately examined. Taken from Jules Gaudard's excellent work, 'Étude Comparative de Divers Systèmes de Ponts en Fer.'

## General Symbols.

$M=M$ ment of rupture, or moment of resistance which is equal to it ;
$\mathbf{F}=$ Stress or transverse strain;
$\mathrm{R}=$ Resistance the square mètre ( 6000000 for tension or compression, 4000000 for shearing strain) (see No. 18);
$p=$ Permanent continuous load, the lineal mètre of girder ;
$p^{\prime}=$ Moving load, the lineal metre of girder ;
$q=$ Proportion $\frac{p^{\prime}}{p+p^{\prime}}$ of the moving to the whole load;
$\mathbf{P}=$ Sometimes a weight applied to a certain point of a girder, sometimes the weight of the girder itself in its length of bearing;
$l=$ Bearing of the girder ;
$\mathrm{L}=$ Length of the girder;
$\frac{\mathrm{P}}{l}=$ Mean weight of girder, the lineal mètre (see No. 37);
$\frac{\mathrm{PL}}{l}=$ Weight of the girder ;
${ }_{d}=$ Void between the abutments;

$\frac{P L}{l d}=$ Weight the lineal mètre of girder spanning the void;
$\delta=$ Interval between two successive points to which the load is applied, in girders loaded in a discontinuous manner ;
$\mathrm{N}=\frac{\mathrm{L}}{\delta} ;$
$m=\frac{l}{2 \delta}$ or $\frac{l-\delta}{2 \delta}$, according as $\delta$ enters an even or an odd number of times into $l$;
$n=$ Number indicating the order of a section of flange or a lattice-bar, the numbering beginning at the middle of the bearing;
$\alpha=$ Angle of inclination of the struts;
$\beta=$ Angle of inclination of the ties;
$h=$ Height of the girder:
$t=$ Weight the lineal mètre of a prism capable of supporting a strain of 1 kilogramme, or proportion of the weight of a cubic mètre to the resistance per square mètre. For iron $t=7800 \mathrm{k}-600000 \mathrm{O}=0.0013 ;$
$t=.00 / 3$ éloys $x$
$\mathrm{U}=$ Coefficient applied to the flanges, the actual weight of which is always greater than the theoretical weight (see Nos. 63 and following);
$\mathrm{V}=$ Mean coefficient of stiffness applied to compressed bars (see Nos. 68 and following);
$\Omega=$ Supplementary term taking into account the weight of various accessories independent of $h$.
The dimensions of an iron plate are denoted by the symbol $a / b$, the letters $a$ and $b$ being replaced by numbers indicating respectively the breadth and the thickness of the section.

For an angle-iron, we write $a / b / c$ or $\frac{a / b}{c}, \alpha$ and $b$ being the breadth of the arms, and $c$ the mean thickness; for a simple $T, \frac{a / b}{c / d}, a$ being the total breadth of the double arm and $c$ its thickness, $b$ the length of the single arm (including the thickness $c$ ), and $d$ its thickness.

For a double $T$ rolled, $\frac{a / b}{c / d}$, $a$ being the total height, $c$ the thickness of the web, $b$ the breadth of the flanges, and $d$ their thickness; and, generally, for a double $T$, consisting of plate and angle iron, we indicate successively the dimensions of the web, one of the four angle-irons, and one of the flanges.

When angle-iron with arms of unequal length is used, the lesser arm is applied to the plate and bears the bolts, and the greater forms the projecting mouldings or flanges.

1. Moments of Rupture and Transverse Strain for Girders resting frcely upon Tirn Sunnorts.-Let us consider, Figs. 1573, 1574, a portion ABCD of a girder included between any section CD and a support B, and subjected to various loads or vertical forces, as $P$. The reaction of the abutment is a force $\mathbf{R}$ also vertical. The reactions exerted at CD may be reduced to a force F and a couple. The other forees being vertical, F must be vertical also: it is in equilibrio with the vertical transverse strain in the section considered, the value of which is $F=R-\Sigma P$, the sign $\Sigma$ indicating a sum. The condition of equilibrium relative to the moments of the forces requires that the moment of the couple, or moment of resistance of the section CD, should be equal to the moment of the other
 forces with respect to this same section. This latter moment, called moment of rupture, has the value $\mathrm{M}=\mathrm{R} x-\Sigma \mathrm{P}\left(x-x_{1}\right)$. Its differential $\frac{d \mathrm{M}}{d x}=\mathrm{R}-\Sigma \mathrm{P}=$ transverse strain.

The vertical rib, or web of the girder, is besides subjected to a sliding strain upon its longitudinal fibres or horizontal transverse strain.' Indeed, if we conceive the fragment COO'C limited by two sections infinitely near $\mathrm{CO}, \mathrm{C}^{\prime} \mathrm{O}^{\prime}$, and by a horizontal fibre $\mathrm{O}^{\prime} \mathrm{O}^{\prime}$, this fragment is subjected, upon CO , to a certain pressure T , and upon $\mathrm{C}^{\prime} \mathrm{O}^{\prime}$ to $\mathrm{T}+d \mathrm{~T}$; the resultant $d \mathrm{~T}$ ought to be held in equilibrio by a force of adhesion $\mathrm{R}_{1} e d x$ upon $\mathrm{OO}^{\prime}\left(e=\right.$ thickness of the web, $\mathrm{R}_{1}=$ resistance to shearing strain.

The maximum of this action occurs when $\mathrm{OO}^{\prime}$ is situate upon the neutral axis; for then T is resisted by the force of all the compressed fibres situate above this axis. As the flange has a section usually greater than the rib, and, besides this, contains the fibres which are most acted upon, the point of application of the resultant $T$ will be very near the flange, and its value will be approximatively equal to the moment of rupture M divided by the height $h$ of the girder. Consequently we shall have $d \mathrm{~T}=\frac{d \mathbf{M}}{h}=\mathbf{R}_{1} c d x$, whence $e=\frac{d \mathrm{M}}{d x} \times \frac{1}{\mathbf{R}_{1} h}=\frac{\mathbf{F}}{\mathbf{R}_{1} h}$, which may be explained by saying that the vertical section $e h$ of the rib should be capable of resisting a shearing force equal to the transverse strain, a condition which enables us to calculate the thickness $e$.
2. Moment of Rupture and Transverse Strain due to a Load uniformly distributed.-If the load $p$ per lineal mètre extends throughout the whole length of bearing $l$, the figure of the moments of rupture will be the parabola $\mathrm{M}=\frac{1}{2} p x(l-x)$, the abscissæ being reckoned from the abutment. This parabola has its axis vertical, and the value of its parameter is $\frac{2}{p}$; the maximum moment in the middle of the bearing is $\frac{p l^{2}}{8}$. The transverse strain is represented by the right line having

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for equation $\mathbf{F}=p\left(\frac{l}{2}-x\right)$; it is $=0$ in the middle of the bearing. Considering the beginning of the co-ordinates in the middle of the girder, we should have $\mathrm{M}=\frac{p}{2}\left(\frac{l^{2}}{4}-x^{\prime 2}\right)$ and $\mathrm{F}=-p x^{\prime}$.
3. Let us now consider a load $p^{\prime}$ per lineal mètre, uniformly distributed as $p$, but extending only over a portion of the bearing included between the fixed abscissæ $x=a$, and $x=a+b$, reckoned from the left abutment. Then, in the first interval, of length $a$, the moment of rupture due to $p^{\prime}$ will be represented by a right line $\mathrm{M}=\frac{p^{\prime} b}{2 l}(2 l-2 a-b) x$; in the second interval, of length $b$, by ${ }^{\circ}$ an arc of a parabola tangential to the preceding right line, and expressed by $\mathrm{M}=\frac{p^{\prime} b}{2 l}(2 l-2 a-b) x-\frac{1}{2} p^{\prime}(x-a)^{2}$; and in the third interval, of length $l-a-b$, by a new tangent to the parabola, namely, $\mathbf{M}=\frac{p^{\prime} b(2 a+b)}{2 l}(l-x)$. The figure of the transverse strains is a broken line, the extreme portions of which are horizontal.

For the maximum of the moment of rupture $p^{\prime}$ must extend over the whole bearing; but for the maximum of strain at the point, the abscissa of which is $x, p^{\prime}$ must lie only between this point and the most distant support. If it lies between the point in question and the right abutment, the force is positive, if we consider as positive those forces which act upwards; it is expressed by $\mathbf{F}=\frac{p^{\prime}}{2 l}(l-x)^{2}$, and is consequently represented by an arc of a parabola, the ordinate of which upon the left support is $\mathrm{A} \mathrm{C}=\frac{p^{\prime} l}{2}$, Fig. 1575, the ordinate in the middle $\mathrm{O} \mathrm{E}=\frac{p^{\prime} l}{8}$, and the ordinate at $B=0$; this latter point is the summit of the parabola. When, on the contrary, the load is placed upon the length included between A and the point the abscissa of which is $x$, we have as the figure of the straining forces, then negative, another parabola AFD, having its summit in A, and the maximum ordinate of which, absolutely expressed, is $-\mathrm{BD}=-\frac{p^{\prime} l}{2}$. Thus the load $p^{\prime}$, by changing its
 position, produces at each point of the bearing transverse strains sometimes positive, sometimes negative.
4. In brief, if we have at the same time a dead weight $p$ and a moving weight $p^{\prime}$, the maximum moment of rupture will be $\mathrm{M}=\frac{1}{2}\left(p+p^{\prime}\right) x(l-x)$, and the transverse strain in the left half bay $\mathbf{F}=\frac{1}{2}\left(p+p^{\prime}\right)(l-2 x)+\frac{p^{\prime} x^{2}}{2 l}=\frac{1}{2}\left(p+p^{\prime}\right)\left(l-2 x+\frac{x^{2}}{l} q\right), q$ denoting the proportion of the moving weight $p^{\prime}$ to the total weight $p+p^{\prime}$.

The value of the area comprised between the figure of the moments of rupture and the axis of the $x^{\prime}$ s is $\frac{1}{12}\left(p+p^{\prime}\right)^{3}$, and the area of the maximum straining forces, absolutely expressed, is $\frac{1}{4}\left(p+p^{\prime}\right) l^{2}\left(1+\frac{1}{6} q\right)$ for the whole bearing. Dividing these areas by $l$, we shall have the mean ordinates of the moments of rupture and of the maximum straining forces. Sometimes the effects do not depend exclusively upon $\mathbf{F}$ or $\mathbf{M}$, but upon a function of the form BF-A M. Now if we consider only $p^{\prime}$, this function will be greatest when $p^{\prime}$ extends between the point considered and the farthest abutment; it will then attain the value $\frac{p^{\prime}(l-x)^{2}}{2 l}(\mathrm{~B}-\mathrm{A} x)$.
5. Effect of Loads uniformly distributed at certain intervals.-When a girder supports discontinuous weights $\mathrm{P}, \mathrm{P}^{\prime} \mathrm{P}^{\prime \prime}, \ldots$ applied to points having as their abscisse $a, a+b, a+b+c$, and so on, and such that $\mathrm{P}=\frac{1}{2} p(a+b), \mathrm{P}^{\prime}=\frac{1}{2} p(b+c)$, and so on, the figure of the moments of rupture is a polygon inscribed in the parabola which would correspond with a load $p$ per lineal mètre, uniformly distributed over all the parts of the girder.

Supposing each of the extreme intervals equal to $a$, all the intermediate intervals to $\delta$, and $l=2 a+\mathrm{N} \delta$, the permanent weights applied to the points of division will be equal to $p \delta$, except at the extreme points, where they will be $\frac{1}{2} p(a+\delta)$. The load $p^{\prime}$ will furnish analogous weights, but they can be only on a certain number of consecutive points of division. The transverse strain for a given load is constant in each interval, but varies from one to the other. In order that it may reach its maximum in the interval preceding the point having as its abscissa $\alpha+\kappa \delta$, the load must be applied to this point and to all those which follow it, as far as the right abutment, the point in question being supposed on the left of the middle of the girder. We shall thus obtain for this maximum,

$$
\mathbf{F}=\frac{\delta}{2}\left(p+p^{\prime}\right)(\mathrm{N}-2 \kappa+1)\left[1+\frac{\kappa(\kappa-1) \delta^{2}+a(2 \kappa \delta-\delta+a)}{\delta(\mathrm{N} \delta+2 a)(\mathrm{N}-2 \kappa+1)} q\right]
$$

but this formula does not apply when $\kappa=0$, that is for the extreme interval; in that case we have $\mathbf{F}=\frac{1}{2}\left(p+p^{\prime}\right)(\mathrm{N} \delta+a)$.

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6. Usually all the intervals are equal to each other. Making, therefore, $a=0$, and $l=\mathrm{N} \delta$, we shall have $\mathrm{F}=\frac{\delta}{2}\left(p+p^{\prime}\right)(\mathrm{N}-2 \kappa+1)\left[1+\frac{\kappa(\kappa-1)}{\mathrm{N}(\mathrm{N}-2 \kappa+1)} q\right]$ in the interval included between the abscissæ $(\kappa-1) \delta$ and $\kappa \delta$. This formula holds good for the first interval.

To find the sum of the values of the transverse strain in all the intervals when N is an even number, take the sum of the preceding expression in which $\kappa$ assumes the successive values $1,2, \frac{\mathrm{~N}}{2}$, and double the result for the whole bearing. We thus obtain

$$
\Sigma \mathrm{F}=\left(p+p^{\prime}\right) \frac{\mathrm{N} l}{4}\left(1+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}^{2}} q\right)
$$

and multiplying by $\delta$ we shall have the area of the polygon of the maximum straining forces absolutely considered. When N is an odd number, find the sum on the supposition that $\kappa$ varies from 1 to $\frac{\mathrm{N}-1}{2}$, double the result, and add the force of the middle interval which will correspond with $\kappa=\frac{\mathbf{N}+1}{2}$. We thus obtain $\Sigma \mathbf{F}=\left(p+p^{\prime}\right) l \frac{\mathbf{N}^{2}-1}{4 \mathbf{N}}\left(1+\frac{1}{6} q\right)$

The area of the polygon of the moments of rupture may be found by taking from the area of the circumscribed parabola N small segments, the chords of which have each $\delta$ as a horizontal projection, and the surfaces of which have the constant value $\frac{1}{12}\left(p+p^{\prime}\right) \delta^{3}$. We thus obtain for the area sought $\left(p+p^{\prime}\right) l^{3} \frac{\mathrm{~N}^{2}-1}{12 \mathrm{~N}^{2}}$, a formula which applies whether N be an even or an odd number. We should have $\frac{1}{12}\left(p+p^{\prime}\right)\left(l^{3}-\mathrm{N} \delta^{3}-2 a^{3}\right)$ in the more general case in which $l=2 a+\mathrm{N} \delta$.
7. When we have to consider an expression of the form BF -AM, where M denotes the moment of rupture with respect to the point the abscissa of which is $\kappa \delta$, and $F$ the force in the interval preceding this point, we shall raise it to its maximum by applying the load to the point the abscissa of which is $\kappa \delta$ and to the following points of division as far as the right abutment, $\kappa \delta$ being supposed less than $\frac{l}{2}$. We obtain in this way the maxima value

$$
\frac{p^{\prime} \delta}{2 \mathrm{~N}}(\mathrm{~N}-\kappa)(\mathrm{N}-\kappa+1)(\mathrm{B}-\mathrm{A} \kappa \delta)
$$

the term due to the permanent load not included. If, on the contrary, F were the force beyond the point the abscissa of which is $\kappa \delta$, to obtain the maximum we should have to suppress the weight at this point, and the formula would then be $\frac{p^{\prime} \delta}{2 N}(N-\kappa)(N-\kappa-1)(B-A \kappa \delta)$.

If the expression considered were of the form $\mathrm{BF}-\mathrm{AM}-\mathrm{A}^{\prime} \mathrm{MI}^{\prime}, \mathrm{M}$ being the moment at the point the abscissa of which is $\kappa \delta$, $\mathbf{I I}^{\prime}$ that at the preceding point the abscissa of which is $(\kappa-1) \delta$, and $F$ the force between these two points, this expression would be

$$
\frac{p^{\prime} \delta}{2 \mathrm{~N}}(\mathrm{~N}-\kappa)(\mathrm{N}-\kappa+1)\left[\mathrm{B}-\mathrm{A} \kappa \delta-\mathrm{A}^{\prime}(\kappa-1) \delta\right]
$$

8. Moments of Rupture produced by the Passage of a Wheel.-A weight P applied to the point the abscissa of which is $a$, reckoning from the abutment, would produce moments of rupture reprosented by two right lines beginning at the supports and meeting at the loaded point where the ordinate reaches the value $\frac{\mathrm{P} a(l-a)}{l}$. But if this weight P be a wheel rolling from one end of the bearing to the other, the maximum moment at any given point will be produced when the wheel is passing over this point, and the maximum moments will be the parabola having the equation $\mathrm{M}=\frac{\mathrm{P} x(l-x)}{l}$ and the parameter $\frac{l}{\mathrm{P}}$

If the girder support besides a uniform load P per lineal mètre, it will be sufficient to substitate for P in the preceding equation $\mathrm{P}+\frac{p l}{2}$

But the parabola representing the moments due to $P$ is only an imaginary one, since at any given instant there exists only one of its points, the one determined by the ordinate on the right of the load; and the moments of rupture at this instant are the broken line of which we have already spoken. The force F is also not derived from the parabola, but the inclination of one or the other of the right lines composing the broken line, according as we wish to find the force on one side or the other of the section considered.
9. Moments of Rupture produced by Two Wheels. -The two wheels occupying a determinate position, the moments of rupture are a broken line composed of three right lines. But to have at a given point the maximum moment we must bring one of the wheels of the vehicle upon it, the other wheel so placing itself that their common distance $d$ remains constant. If the left wheel P be placed upon the point the abscissa of which is $x$, the moment of rupture will be $\mathbf{M}=\frac{x}{l}\left[\mathrm{Q}(l-x)-\mathrm{P}^{\prime} d\right], \mathrm{Q}$ representing the quantity $\mathrm{P}+\mathrm{P}^{\prime}+\frac{p l}{2}$, on the hypothesis that the girder supports besides a permanent load $p$ a lineal mètre.

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If $P^{\prime}$ be brought upon the section in question, the wheel $P$ removing to the left, we shall have

$$
\mathrm{M}^{\prime}=\frac{x}{l}[\mathrm{Q}(l-x)+\mathrm{P} d]-\mathrm{P} d
$$

The first equation will give a greater value than the second so long as $x$ is $<\frac{\mathrm{P} l}{\mathrm{P}+\mathrm{P}^{\prime}}$; beyond this limit we must take the second equation.

These two equations represent two parabolas A C E, B F E, Fig. 1576, of the same parameter $\frac{l}{\mathrm{Q}}$, and having their axes vertical. But if the left wheel P be heavier than the right wheel $\mathrm{P}^{\prime}$, the parabola A C E produced by bringing the wheel P on the point the abscissa of which is $x$, will descend lower than the other. Now, when the vebicle returns in the opposite direction, we shall reproduce symmetrically the two parabolas, and then it will be the curve ACE and its symmetrical which will furnish the greatest ordinates, so that we shall have to lay aside the second equation.

We must observe again that the equation of the parabolas supposes both wheels always acting, even when one of thein has passed beyond the support,
 in which case it will rest no longer upon the bearing, but upon the support. As the girder is supposed to terminate at the support when one of the wheels has passed beyond it, the curve is no longer applicable: we must in that case consider the parabola A C F B due to a single wheel towards the middle of the bearing. It is applicable only when the common distance $d$ of the wheels exceeds half the bearing; and supposing this condition fulfilled we obtain the various values numbered on the figure.

If with a certain section the girder offers an insufficient moment of resistance $\mu$ in the parts subject to the greatest strain, it will be necessary to strengthen the flanges throughout a length $\frac{1}{\mathrm{Q}}\left(\mathrm{P}^{\prime} d+\sqrt{\left.\left(\mathrm{Q} l-p^{\prime} d\right)^{2}-4 \mathrm{Q} l \mu\right)}\right.$, on the condition that this value be greater than $2 d-l$. In the contrary case, the theoretical length of the strengthening will be $\sqrt{\frac{\overline{\mathrm{P} l^{2}-4 l \mu}}{\mathrm{P}}}$.
10. When the two wheels of the vehicle are equal to each other the maximum moment occurs at the abscissa $\frac{2 \mathrm{P}(2 l-d)+p l^{2}}{2(4 \mathrm{P}+p l)}$ and its value is $\frac{\left[2 \mathrm{P}(2 l-d)+p l^{2}\right]^{2}}{8 l(4 \mathrm{P}+p l)}$.
11. Moments of Rupture due to Three Wheels.-Let $\mathrm{P}, \mathrm{P}^{\prime} \mathrm{P}^{\prime \prime}$ represent the weights of the three wheels moving from left to right, and $d, d^{\prime}$ their common distances.

There are three geometrical figures of the moments of rupture to be considered during the passage of the vehicle, and each of these figures is composed of three parabolical arcs, the equations changing when one or two of the wheels pass beyond the bearing.

1. Figure of the MIoments obtained by bringing $\mathrm{P}^{\prime \prime}$ upon the Variable Section considered.

Making $\mathrm{P}+\mathrm{P}^{\prime}+\frac{p l}{2}=\mathrm{Q}, \mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}+\frac{p l}{2}=\mathrm{Q}^{\prime}$, and $\mathrm{P}+\mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}+\frac{p l}{2}=\mathrm{Q}^{\prime \prime}$.
From $x=0$ to $x=d^{\prime}, \mathrm{P}^{\prime \prime}$ acts alone and gives $\mathrm{M}=\frac{x}{l}(l-x)\left(\mathrm{P}^{\prime \prime}+\frac{p l}{2}\right)$;
From $x=d^{\prime}$ to $x=d+d^{\prime}$ we have $\mathbf{M}=\frac{x}{l}\left\{\mathbf{Q}^{\prime}(l-x)+\mathrm{P}^{\prime} d^{\prime}\right\}-\mathrm{P}^{\prime} d^{\prime}$;
From $x=d+d^{\prime}$ to $x=l, \mathbf{M}=\frac{l-x}{l}\left\{\mathrm{Q}^{\prime \prime} x-\left(\mathrm{P}+\mathrm{P}^{\prime}\right) d^{\prime}-\mathrm{P} d\right\}$.
2. Figure of the Moments obtained by bringing $\mathrm{P}^{\prime}$ upon the Section.

From $x=0$ to $x=d$, we have $\mathbf{M}=\frac{x}{l}\left\{\mathbf{Q}^{\prime}(l-x)-\mathrm{P}^{\prime \prime} d^{\prime}\right\}$;
From $x=d$ to $x=l-d^{\prime}, \mathrm{M}=\frac{x}{l}\left\{\mathrm{Q}^{\prime \prime}(l-x)+\mathbf{P} d-\mathrm{P}^{\prime \prime} d^{\prime}\right\}-\mathbf{P} d$;
From $x=l-d^{\prime}$ to $x=l, \mathbf{M}=\frac{x}{l}\{\mathrm{Q}(l-x)+\mathrm{P} d\}-\mathrm{P} d$.

## 3. Figure of the Moments obtained by bringing P upon the Section.

From $x=0$ to $x=l-d-d^{\prime}, \mathbf{M}=\frac{x}{l}\left\{\mathrm{Q}^{\prime \prime}(l-x)-\left(\mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}\right) d-\mathrm{P}^{\prime \prime} d^{\prime}\right\} ;$
From $x=l-d-d^{\prime}$ to $x=l-d, \mathrm{M}=\frac{x}{l}\left\{\mathrm{Q}(l-x)-\mathrm{P}^{\prime} d\right\}$;
From $x=l-d$ to $x=l, \mathrm{M}=\frac{x}{l}(l-x)\left(\mathrm{P}+\frac{p l}{2}\right)$.

The parameter of each parabola is the reciprocal quantity of the mean load per lineal mètre, reckoning only the half of the uniformly distributed load $p$.

Having constructed these three geometrical figures, we have to consider each of them only in that portion in which it gives ordinates greater than the others; and as the vehicle may be brought back in the other direction, we can reproduce symmetrically the maxima curves, which will always. eliminate, if not already eliminated, the cases of a single wheel expressed by the first and last of the nine equations given above.
12. The following particular case deserves notice. If the vehicle be turned so that

$$
\mathrm{P}>\mathrm{P}^{\prime \prime}+\mathrm{P}^{\prime} \frac{d-d^{\prime}}{d+d^{\prime}}
$$

it will be sufficient to consider the first half of the girder. Again, if $l-d-d^{\prime}$ exceeds each of the quantities $d, d^{\prime}$, and $\frac{\mathbf{P}(d+d)}{\mathbf{P}^{\prime}+\mathbf{P}^{\prime \prime}}$, we must consider ouly the seventh and fifth of the nine equations, the seventh between the abscissæ O and $\frac{\mathrm{P} l}{\mathrm{P}+\mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}}$, and the fifth between the latter abscissa and the middle of the girder. A girder, the simple section of which has a moment of resistance $\mu$ inferior to the maximum moment of rupture, should be strengthened throughout a length expressed by $\frac{\mathrm{P}^{\prime \prime}\left(d+d^{\prime}\right)+\mathrm{P}^{\prime} d}{\mathrm{Q}^{\prime \prime}}+\frac{1}{\mathrm{Q}^{\prime \prime}} \sqrt{\left\{\mathrm{Q}^{\prime \prime} l-\mathrm{P}^{\prime \prime}\left(d+d^{\prime}\right)-\mathrm{P}^{\prime} d\right\}^{2}-4 \mathrm{Q} l \mu \text {, on the condition that this }}$ length exceed $\frac{\mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}-\mathrm{P}}{\mathrm{P}+\mathrm{P}^{\prime}+\mathrm{P}^{\prime \prime}} l$. In the contrary case the theoretical length of the strengthened part should be $\frac{\mathbf{P} d-\mathrm{P}^{\prime \prime} d^{\prime}}{\mathrm{Q}^{\prime \prime}}+\frac{1}{\mathrm{Q}^{\prime \prime}} \sqrt{ }\left(\mathrm{Q}^{\prime \prime} l+\mathrm{P} d-\mathrm{P}^{\prime \prime} d^{\prime}\right)^{2}-4 \mathrm{Q}^{\prime \prime} l(\mu+\mathrm{P} d)$. The value of the maximum moment is $\frac{1}{4}\left\{\mathrm{Q}^{\prime \prime} l-2 \mathrm{P} d-2 \mathrm{P}^{\prime \prime} d^{\prime}+\frac{\left(\mathrm{P} d-\mathrm{P}^{\prime \prime} d^{\prime}\right)^{2}}{\mathrm{Q}^{\prime \prime} l}\right\}$.

For example, if $l=6$ mètres, $\mathrm{P}=\mathrm{P}^{\prime}=\mathrm{P}^{\prime \prime}=6000$ kilogrammes, $r=0, d=1.40$ m tre, $d^{\prime}=1.90$ mètre, the maximum moment will be 17120 , equal to that which would be given by a uniformly distributed load of 3800 kilogrammes the lineal mètre.
13. Moments of Resistance of Solid Girders.-Girders with Equal Flanges.-For girders of inconsiderable height, the moment of resistance is calculated by the exact formula

$$
\frac{\mathrm{R}}{6 h}\left(b h^{3}-b^{\prime} h^{\prime 3}-b^{\prime \prime} h^{\prime \prime 3}-b^{\prime \prime \prime} h^{\prime \prime \prime 3}\right), \text { Fig. } 1577
$$

Tables I. and II. give the moments ready calculated of various sections. Part of these results has been taken from more extensive Tables published by M. Foy in the 'Annales de la Construction' of M. Opperman, 1863. As usual, the value of R has been taken as 6000000 ; but if it be required to give to $R$ another value, it will be easy to modify these results by a simple proportion.

When a section according to the Table would be too weak or too strong for the girder proposed, we can modify the thickness of the flanges or of the vertical rib, remembering always that every millimètre added to or taken from the latter increases or diminishes the moment of a quantity $=1000 h^{2}$; and that every millimètre in each of the two flanges causes an alteration of 6000 bh .
14. When the girder is not a very low one, the moment of resistance may be rapidly calculated by an approximative formula. With the dimensions denoted in Fig. 1577, the moment of resistance of the horizontal flanges, abstracting the rib and angle-irens, is exactly $\mathrm{R} b \frac{h^{3}-h^{\prime 3}}{6 h}=\mathbf{M}$; but if we take $\mathbf{M}^{\prime}=\mathbf{R} b h^{\prime} . \frac{h-h^{\prime}}{2}=\mathbf{R} s h^{\prime}, s$ being the section of one of the horizontal flanges, we shall have $\frac{\mathbf{M}^{\prime}}{\mathbf{M}}=\frac{3 h h^{\prime}\left(h-h^{\prime}\right)}{h^{3}-h^{\prime 3}}$.

For example, if $h^{\prime}$ is at least 0.8 of $h$, and this is always the case when the girder is of inconsiderable height, the proportion $\frac{M^{\prime}}{M}$ will be equal to $0 \cdot 983$, which is very near unity.

As $h^{\prime}$ differs little from $h$, we may consider the moment of resistance of the vertical rib of thickness $e$ as equal to $\mathrm{R} e \frac{h^{\prime 2}}{6}$ or $\frac{1}{6} \mathrm{R} s^{\prime} h^{\prime}$, $s$ being the section $e h^{\prime}$. Thus the vertical rib gives a moment of resistance equal to only $\frac{1}{3}$ of that which the same quantity of material would possess if equally distributed between the two horizontal flanges.

For the angle-irons, if we limit ourselves to $\frac{80 / 80}{10}$ and $\frac{100 / 100}{12}$, we can make use of Table III., which gives the moments of resistance ready calculated of four angle-irons (two at the top and two at the bottom), supposing that $R$ reaches the value 6000000 in the fibres situated at the extremities of the height $h^{\prime}$. If we get beyond these limits, we can reduce the values of the Table. Thus for angle-irons of $\frac{100 / 100}{15}$ we must add $\frac{1}{4}$ of the value given by the Table for angle-irons of 100/100
$\frac{12}{2}$. When the height is inconsiderable, instead of considering the angle-irons apart, we may include them in the section $s$ of the flange.

Let us take, by way of example, a girder 0.80 mètre in height, consisting of a vertical rib of $700 / 12$ millimètres, four angle-irons of $\frac{80 / 80}{.10}$, and two flanges of $300 / 50$. We shall have $h^{\prime}=0 \cdot 70, s+\frac{1}{6} s^{\prime}=0 \cdot 0164$, and consequently

The moment of resistance of the rib and flanges $=6000000 \times 0.0164 \times 0.70=68880$
The moment of resistance of the angle-irons, according to Table III. .. .. 11014

$$
\text { Total .. .. .. .. .. .. .. .. } \overline{79891}
$$

The exact calculation gives 78158. Supposing the moment of rupture to be equal to 79894, the girder would have to bear a strain of $6 \cdot 13$ kilogrammes instead of 6 , a difference which is of no importance.
15. Moments of Resistance of Unequal Sections with respect to the so-called Neutral Axis.-The strain which iron is usually reckoned to bear is 6 kilogrammes the square millimètre, both in those parts which are subject to compression and in those which are subject to tension, the compressed parts requiring only a proper form to prevent twisting. But if it be required to employ a higher coefficient for the flange subject to tension than for the one subject to compression, we must increase the strength of the latter, that is, we must adopt an unequal section with respect to the neutral axis.

These unequal sections may be unavoidable in certain cases; for example, in girders of the form given in Fig. 1588. In such a case, it is often possible to assimilate the section to an equal one having the two flanges equal to the weakest of the given section. But if the exact calculation be required, we must proceed as follows:-

First find, by the theorem of moments, Fig. 1578, the sides of a height $\delta$ and $\delta^{\prime}$ which fix the position of the centre of gravity $G$ of the section, which coincides with the neutral axis when the girder is only subject to forces normal to its length. Then, between the coefficients $R$ and $R_{1}$, which express respectively the maximum resistance of the two flanges, we shall have the proportion $\frac{\mathrm{R}}{\mathbf{R}_{1}}=\frac{\delta}{\delta^{\prime}}$. The moment of resistance may be expressed either in terms of $R$ or of $R_{1}$ which are connected by the preceding proportion; in terms of R, for example, Fig. 1579, its value will be

1579.
 $\frac{\mathrm{R}}{3 \delta}\left(b \delta^{3}+b_{1} \delta_{1}{ }^{3}-b^{\prime} \delta^{\prime 3}-b_{1}{ }^{\prime} \delta_{1}{ }^{\prime 3}\right)$.

In the particular case of a simple T, Fig. 1579, we shall have $\delta=\frac{1}{2} \cdot \frac{b h^{\prime 2}-b^{\prime} h^{\prime 2}+b^{\prime} h^{2}}{b h^{\prime}-b^{\prime} h^{\prime}+b^{\prime} h}$. Moment of inertia $\mathrm{I}=\frac{1}{3}\left\{b \delta^{3}-\left(b-b^{\prime}\right)\left(\delta-h^{\prime}\right)^{3}+b^{\prime}(h-\delta)^{3}\right\}$. Moment of resistance $=\frac{\mathrm{R}_{1} \mathrm{I}}{h-\delta}, \mathrm{R}_{1}$ referring to the fibre which is farthest from the neutial axis.
16. Solid Girders of small dimensions.-Coefficient R.-Rivetings.-Theoretically the thickness of the vertical rib should vary with the strain to which it is to be subject; but in small girders this thickness is fixed, and offers an excess of resistance, for the conditions of rigidity prohibit the reduction of the thickness of the iron plates below certain limits. We may, therefore, consider the thickness $e$ of the rib as given.

If we first suppose the section constant, and represented in detail by Figs. 1580 to 1585 , the value of the moment of resistance will be

$$
\mathbf{M}=\frac{\mathbf{R}}{6 h}\left\{b h^{3}-(b-e)(h-2 \epsilon)^{3}\right\}=\mathbf{R} \epsilon(b-e)\left(h-2 \epsilon+\frac{4 \epsilon^{2}}{3 h}\right)+\frac{\mathbf{R} e h^{2}}{6} .
$$

We may neglect, relatively to $h$, the term $\frac{4 \epsilon^{2}}{3 h}$, and take

$$
\begin{equation*}
\mathrm{M}=\mathrm{R}\left\{\epsilon(b-e)(h-2 \epsilon)+\frac{e h^{2}}{6}\right\} ; \tag{1}
\end{equation*}
$$

whence $b=\frac{\mathrm{M}}{\mathrm{R} \epsilon(h-2 \epsilon)}+e-\frac{e h^{2}}{6 \epsilon(h-2 \epsilon)}$.
The area of the section, or the volume of the lineal mètre of girder, is expressed by

$$
2 b \epsilon+(h-2 \epsilon)(e+\psi) ;
$$

$\psi$ representing the volume of the joint-plates and mouldings of the rib the square mètre, that is, the additional thickness which must be given to the rib to equal the weight of these mouldings and joint-plates.
 and it is reduced to a minimum by the following value of $h$ :

$$
\begin{equation*}
h=2 \epsilon+2 \sqrt{\frac{3 M-2 \mathrm{R} e \epsilon^{2}}{2 \mathrm{R}(2 e+3 \psi)}} \tag{2}
\end{equation*}
$$

This formula gives the height to be adopted in order to have the lightest possible girder, offering a given moment of resistance, the quantities $c, \psi$ and $\epsilon$ being also given.

## BRIDGE.

1580. 

Two-girder Bridge, of high girders, Toaded on the upper part.

1582.
Arad/st

Another kind of console.
1583.


Eliminating M between [1] and [2], and supposing $e=\epsilon$, we obtain

$$
\begin{equation*}
h-\frac{6 e(b+\psi)}{e+\psi} \tag{3}
\end{equation*}
$$

a formula which establishes between $h$ and $b$ the most advantageous relation, $e$ being always given, and, in the interests of economy, requiring to be chosen as small as the necessary rigidity will allow. In girders of rolled iron $\psi=0$, and, consequently, $h=6 b$; thus, for this kind of girder, if we choose the area $S$ of the section, the best form will be obtained by adopting for $e$ the smallest possible value, and then taking $h=\frac{3 \mathrm{~S}}{4 e}$ and $b=\frac{\mathrm{S}}{8 e}$

Formula [2] assumes the following form in the case of a girder supporting a uniformly distributed load $p$ per lineal mètre of bearing $l$.

$$
\begin{equation*}
h=2 \epsilon+\frac{1}{2} \sqrt{\frac{3 p l^{2}-16 \overline{\mathrm{Re} e \epsilon^{2}}}{\mathrm{R}(2 e+3 \psi)}} . \tag{4}
\end{equation*}
$$

17. Let us now consider the case of girders having flanges of varying thickness. As the increase of thickness is obtained by placing one plate upon another, the variation takes place by redans or gradations, from the middle to the extremities of the bearing. If $\mu$ denote the moment of resistance of one of the sections, the additional material to be added in the middle will, theoretically, be equal in length to $\sqrt{l^{2}-\frac{8 \mu}{p}}$, supposing the girder loaded with $p$ the lineal mètre. This length should, in practice, be slightly increased, in order that the added plate may form sufficient consistency with the flange by means of its extreme rivets, at those points in which the strengthening becomes necessary.

Let us suppose that Figs. 1580 to 1585 represent the section at the ends of the girder, $\epsilon$ being the least thickness it is thought proper to adopt for the flanges; then, admitting approximatively that from the extremities this thickness increases as the ordinate of a parabolical segment, so as to reach its greatest value $z$ in the middle of the bearing, the thickness $z$ will be determined by the following equation, in which the moment of resistance is expressed in the approximative manner explained in 14.

$$
\begin{equation*}
\mathrm{R} h\left(b z+\frac{e h}{6}\right)=\frac{p l^{2}}{8}, \text { whence } b z=\frac{p l^{2}}{8 \mathrm{R} h}-\frac{e h}{6} . \tag{5}
\end{equation*}
$$

Let $\kappa$ be a coefficient $1 \cdot 10$ taking into account the joint-plates of the flanges. The volume of the lineal mètre of girder will then be $l h(e+\psi)+2 \kappa l b\left\{\epsilon+\frac{2}{3}(z-e)\right\}$; or, by substituting the value of $b z, l h\left\{\psi+e\left(1-\frac{2}{9} \kappa\right)\right\}+\frac{2}{3} \kappa l b \epsilon+\frac{\kappa p l^{3}}{6 \mathrm{R} h}$.

This expression becomes a minimum if we take

$$
\begin{equation*}
h=l \sqrt{6 \mathrm{R}\left\{\psi+c\left(1-\frac{2}{9} \kappa\right)\right\}}, \tag{6}
\end{equation*}
$$

which shows that the height of girders should vary as $l \sqrt{p}$.
The height being determined, the middle section of the flanges is calculated by equation [5]. The breadth $b$ must be sufficient to ensure the rigidity of the girder, and to prevent the thickness $z$ from exceeding a certain limit, such as 0.06 or 0.07 mètre.

Examples. - Let $l=20$ mètres, $p=6000$ kilogrammes, $e=0.010$ mètre, $\psi=0.008, \mathrm{R}=$ 6000000 , and $\kappa=1 \cdot 10$. We find $h=2 \cdot 17$ mètres, and $p z=0.0194$. If, for example, we make $z=0.050$, we shall have $b=0.39$ mètre. Supposing $\epsilon=0.010$, the girder will weigh about 11 tons, or 22231 lbs.

The above formulæ apply only to cases in which we are free to realize the most advantageous forms. It frequently happens that the height allowed is limited, in which cases we must adapt the height of the girder to the circumstances imposed upon us. The thickness of the vertical rib varies in girders of great importance.
18. Coefficient R.-The practical resistance of iron both to tension and to compression is usually takeu as 6 kilogrammes to the square millimètre, and this coefficient is regarded as applicable to the solid section, that is, without any deduction for rivet-holes. In the parts subject to compression, these holes do not weaken the piece in the slightest degree, if they are exactly filled by the shank of the rivets ; but this condition is not always fulfilled. In the parts subject to tension the hold which the heads of the rivets have taken upon the plate in consequence of the shrinking of the metal, partly compensates the loss of strength occasioned by the hole; but we must always reckon upon a loss in virtue of which $R$ will exceed 6 kilogrammes.

When certain points, such as the situation of joint-plates, for example, require a greater degree of perforation than the other parts, it will always be advisable to slightly increase the section in these points.

In the case of bars fixed to gussets by several rows of rivets, the loss may in most cases be rendered trifling by a proper arrangement of the holes: for example, one rivet only may be put in the transverse section A B, Fig. 1586, two in C D, three in the following row, and so on; for it is enough if, in any section, the net resistauce retained equals the strain upon the bar diminished by the resistance of the rivets which precede the row under consideration. Or, instead of this, we may vary only the diameter of the rivets which should increase towards the end of the piece. Or, again, the two methods may be combined. In lattice-girders, in which every bar is of inconsiderable section, a single hole, even when it is of small size, may be an important
 fraction of the section, in which case it will be advisable to increase the section, at least in the bars subject to tension; in those subject to compression it is often necessary to enlarge the dimensions for the sake of rigidity, and this excess of section, needed in the middle to prevent deflection, is serviceable at the extremities in compensating the loss occasioned by the rivets.

Thus in certain cases it is necessary to take into account the weakening of the material by rivet-holes, but, generally, we may consider that the coefficient $R=6000000$ is sufficiently low to permit us to neglect it.
19. Rivetings and Bolts.-The resistance of these to sheciring force might be considered equal to their resistance to tension or compression, when it is effected under favourable circumstances. But as the joints often require a greater number of rivets, they will not resist in an equal degree. For
this reason it is usual to reckon their resistance at 4 kilogrammes the square millimètre. In
 $1 \frac{1}{2}$ ton, or 3031 lbs ; and one of $0^{\text {m }} \cdot 25,2$ tons, or 4042 lbs . The distance of the holes apart may be about $0^{\mathrm{m} \cdot 08}$.

When a small surface only of a bar rests upon its gusset, it is often necessary to double the sections of the rivets to enable them to resist the shearing strain; this is effected by placing a joint-plate on the opposite side of the gusset, extending an equal distance along the bar. The space between the bar and the joint-plate is then filled up, so that the gusset is fixed into a kind of fork, which can be torn away only by shearing the rivets through two sections. In certain cases, it will be easier and less expensive to enlarge the gusset and retain only a single section of bolt. By doubling the gussets and joint-plates, the rivets will offer a quadruple section. The thickness of the gussets should be so calculated that there may be no risk of their giving way along the perimeter enveloping the rivets of the bar.

We may observe, with respect to the joint-plates of the flanges, that it will be prudent to place considerably apart the holes of those transverse rows which are nearest to the joint and to the ends of the joint-plate, and, on the contrary, to place them closer together in the intermediate rows. This condition may be disregarded with respect to the rows nearest the joint, if we increase the thickness of the joint-plate. But even then it is as well to increase the length, and to place the rivets farther apart.

The vertical rib of a malleable iron girder has a tendency to slide between the two angle-irons which fix it to each of the horizontal flanges. We saw in 1 that this horizontal shearing strain requires a thickness of rib $e$ given by the equation $\frac{d M}{h}=\mathrm{R}, e d x$. Taking account of the rivets, we shall have $\frac{d \mathrm{M}}{h}=n \mathbf{Q} d x, n$ being the number of rivets the lineal mètre, and $\mathbf{Q}$ the strain borne by one of them upon a double section. Whence we deduce $n=\frac{1}{\mathrm{Q} h} \cdot \frac{d \mathrm{M}}{d x}=\frac{\mathrm{F}}{\mathrm{Q} h}$.

Supposing $p \kappa$ the lineal mètre of girder, the maximum of strain $\mathbf{F}$ will be $\frac{p l}{2}$, and by using rivets of $0^{\mathrm{m}} \cdot 025$ we shall have $\mathrm{Q}=4000$ kilogrammes; consequently $n=\frac{p l}{8000 h}$, that is, at the extremities of the girder, the rivets fixing the rib to the angle-irons of the flanges should be placed apart the distance of $\frac{1}{n}=\frac{8000 h}{p l}$. With a diameter of $0^{\mathrm{m}} \cdot 022$ the distance should be 6000 h

The same considerations apply to the rivets which hold together the plates of the flanges, or which fix them to the horizontal arm of the angle-irons, but in these cases the distance may be increased on account of the relatively less sliding strain.

The foregoing condition will, in general, allow us to place the rivetings wide apart throughout the greater part of the length of the flanges; but in the flange subject to compression the rivetings serve another purpose, which must be borne in mind; namely, that of preventing the exfoliation of the plates, which would inevitably occur if they were insufficiently fixed together. This condition requires that the plates should not be left free throughout a length exceeding $0 \mathrm{~m} \cdot 15$; but when there are several rows of rivets, the distance may in certain cases be doubled, provided we take care to alternate the holes.

The holes should be placed closer together in a line with the joint, in order to avoid uselessly increasing the length of the joint-plates.

When the number of plates of which the flange is composed is grea, the joints are near together; and in this case the rivetings should extend, at small intervals, throughout the whole of the length.
20. Flooring of Bridges.-Parts beside the chief Girders.-When the rails do not rest directly upon the principal girders, these are made to support cross-girders, upon which again are placed, longitudinally, other and lesser girders, and it is upon these latter that the rails are laid.

We may consider these minor supports as simply resting upon the cross-girders, and we may take as their bearing the distance of these latter from each other. If the rolling load consists of the wheels of a locomotive, the weight of which is 7 tons, and their distance apart $1^{\mathrm{m} \cdot 40}$, one of the wheels in the middle of the girder will give the maximum moment, when the bearing is less than $2^{\mathrm{m}} \cdot 40$. Above this it will be necessary to consider two wheels. The dead weight, which is of small importance relatively to the rolling load, may be reckoned at the rate of 500 kilogrammes the lineal metre. With these hypotheses, we can adopt, according to the various lengths of bearing and the height to be given to the girder, the double T sections of Table IV., in which $l$ denotes the bearing, $M$ the maximum moment of rupture, and $h$ the height of the girder. As we have done in the Table of the moments of resistance, so in this we describe a section by indicating, first the dimensions of the vertical rib, then those of one of the four angle-irons, and last, when there is occasion for it, those of one of the flanges; for these latter, we give also the length, which in secondary girders of more than 3 mètres may be considerably less than the bearing, for these horizontal plates must be regarded as simple additions made for strength. When the length is omitted, it is understood to be equal to that of the angle-irons. The letter P denotes the mean weight of the girder a lineal metre, having taken into account the reduction in the length of the strengthening portions, and the cutting away of the angle-irons at the ends of the girder, where the vertical rib alone is retained to be fixed to vertical flanges in the cross-girder. These vertical flanges are not reckoned here, because we consider them as forming a part of the cross-girder.

## BRIDGE.

In sections consisting only of a rib and angle-iron, the latter should offer a broad surface to the balks of timber carrying the rails; it will sometimes be advisable to substitute unequal-armed angle-iron for equal-armed, care being had to preserve nearly the same weight: the resistance will be increased by the change, but if this condition is already amply satisfied, we need alter only the top angle-irons. In a bridge hereafter described, we have secondary girders reaching a length of bearing equal to 6 mètres. The mean weight of each of these is about 110 kilogrammes the lineal mètre.
21. When the height allowed is very limited, hollow girders in the form of caissons are some times used. This form is less economical than that of the double $T$, but it is favourable to the ieduction of the thickness of the flooring without diminishing the necessary thickness of the timbers. Figs. 1591, 1594, 1595, give an example of this arrangement. Other sections suitable for bearings of $1^{\mathrm{m}} 50$ to $2^{\mathrm{m}:} 50$ are found in Figs. 1587 to 1590. It is useless to consider longer bearings than these, for when the height is very limited the cross-girders should not be placed far apart.
1587.

For a bearing of $1^{\mathrm{m}} \cdot 50$.
1588.1589.

Hollow Minor Girders. Scale, $0 \cdot 05$.
For a bearing of $2 \mathrm{~m} \cdot 00$.
For a bearing of $2^{\mathrm{m}} \cdot 500$.

1591.

Three-girder Bridge, loaded on the lower part.
Very low cross-girders. Distance apart, $1^{\mathrm{m}} \cdot 500$.
Weight a lineal mètre of flooring under two lines
of the cross-girders, $975^{\mathrm{k}}$
$1350^{k}$ of rails
\} of the hollow girders, $375^{\mathrm{k}}$ \}

$$
000
$$

- 

$$
\text { of the hollow girders, } 375^{\mathrm{k}}
$$


1593.

High Cross-girders. Distance apart, $4^{\mathrm{m}} \cdot 00$.
 lines of rails of the minor girders, $320^{\mathrm{k}}$ \}

1594.


Hollow girder of $93^{k}$ the mètre,

Dctails, to a scale of 0.05 .


Length of the 1 st plate of the flanges, $3^{m} \cdot 40$.



When the thickness will allow us to give a height of, at least, $0^{\mathrm{m} \cdot 350}$ to the cross-girders, we may adopt the double $T$ form for the longitudinal girders.
22. Cross-girders for Bridges of Three Girders.-Figs. 1591 to 1598 represent various kinds. When the height allowed is very limited, the distance of the cross-girders apart should be reduced to about $1 \mathrm{~m} \cdot 50$. It will be useless to go lower than this, for we should still have to consider the crossgirder as loaded, in a line with each rail, with one wheel of the locomotive. In these cases, the principal girders should be placed as near together as possible, in order to diminish the bearing of the cross-girders. Taking their distance apart equal to $4^{\mathrm{m} \cdot 20}$, as in Figs. 1591, 1594, 1595, the length of the cross-girder will be 4 metres, its actual bearing being, in consequence of the projecting of the gussets, $3^{\mathrm{m}} \cdot 60$. Under these circumstances, the height of the cross-girder may be reduced to $0^{\text {in }} \cdot 220$, as in the figure; its weight is about 730 kilogrammes, including its two gussets; the cross-girders, therefore, will weigh 975 kilogrammes the lineal mètre of flooring under two lines of rails.

For a height of $0^{\mathrm{m}} 250$, we may take a rib of $210 / 10$ millimètres, angle-irons of $110 / 70 / 11$, and flanges of $270 / 20$ composed of two plates, one extending over a length of $3^{\mathrm{m}} \cdot 50$, and the other over $2^{\mathrm{m}} \cdot 70$ only. The weight of the cross-girders will be reduced to 830 kilos.

It will be brought down to 715 kilos., if we can take a section of $0^{\mathrm{m} \cdot 300 \text { in height, composed }}$ of a rib of $260 / 12$, angle-irons of $70 / 70 / 10$, and flanges of $220 / 20$, the first plate of which should extend $3 \mathrm{~m} \cdot 50$ and the other $2^{\mathrm{m}} \cdot 70$.

Again, this same weight will be reduced to 640 kilos. for cross-girders of $0^{\mathrm{m}} \cdot 350$, the rib being $330 / 10$, the angle-irons $80 / 80 / 10$, and the flanges $250 / 10$ throughout a length of 3 metres.
23. But generally we shall follow the example given in Figs. 1592, and 1596 to 1598 , in which the chief girders are placed at a distance of $4^{\mathrm{m} \cdot 50}$ apart. In this case the space between the two lines of rails, known as the "six-feet way," is increased to 3 mètres at each end of the bridge, in
order to keep clear of the middle girder. The length of the cross-girders is $4 \mathrm{~m} \cdot 30$, and they may be regarded as simply resting upon two supports, the actual bearing being about $4^{\mathrm{m}} \cdot 10$. The rolling load acts in two distinct points, in a line with each rail, and we may suppose the dead weight to act in the same manner. The moment of rupture will be equal to $1: 30 \mathrm{P}$, the letter P denoting the maximum total weight acting in a line with each rail. This weight increases with the distance between the cross-girders; and if we consider the case of locomotives carrying 10 tons per axle, with a distance of $1^{\mathrm{m}} \cdot 10$ between the axles, or carrying 12 tons, with a distance of $1^{\mathrm{m}} \cdot 40$ and $1^{\mathrm{m}} \cdot 90$, we may allow the following values of P :-

| Distance apart of cross-girders | $\begin{gathered} \text { Mètres. } \\ 1.50 \end{gathered}$ | $\begin{aligned} & \text { Mètres. } \\ & 2 \cdot 00 \end{aligned}$ | $\begin{aligned} & \text { Mètres. } \\ & 2.50 \end{aligned}$ | $\begin{gathered} \text { Miètres. } \\ 3 \cdot 00 \end{gathered}$ | $\begin{aligned} & \text { Mètres. } \\ & 3 \cdot 50 \end{aligned}$ | $\begin{gathered} \text { Mètres. } \\ 4 \cdot 00 \end{gathered}$ | $\begin{gathered} \text { Mè̀tres. } \\ 4 \cdot 50 \end{gathered}$ | $\begin{gathered} \text { Mlètres. } \\ 5 \cdot 00 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Portion of $\mathbf{P}$ due to the dead weight .. .. .. .. .. | $\begin{aligned} & \text { Kilos. } \\ & 1700 \end{aligned}$ | Kilos. $2000$ | Kilos. $2100$ | $\begin{aligned} & \text { Kilos. } \\ & 2300 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 2500 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 2800 \end{aligned}$ | Kalos. $3100$ | Kilos. $3500$ |
| Portion of $\mathbf{P}$ due to the rolling load | 7600 | 9500 | 10600 | 11400 | 12300 | 13000 | 13600 | 14000 |
| Total value of P .. .. .. | 9300 | 11500 | 12700 | 13700 | 14800 | 15800 | 16700 | 17500 |

We are thus able to form a Table (see Table V.) in which M denotes the maximum moment of rupture. The weights include the vertical flanges by means of which the lesser girders are fixed to the cross-girders, and the triangular gussets by means of which the latter are fixed to the principal girders. On the other hand we have deducted the portions of angle-iron or horizontal plates, which are removed towards the ends of the cross-girders.
24. Cross-girders for Two-girder Bridges.-Exemplars of these are given in Figs. 1599 to 1604. The total length of the cross-girder is supposed equal to 8 metres, and we may admit $7^{\mathrm{m}} \cdot 80$ as the actual bearing. The maximum moment of rupture, between the inner rails, will be $4 \cdot 30 \mathrm{P}$, when the two lines are simultaneously loaded, whilst it will be only $2 \cdot 70 \mathrm{P}$ in the case of one line being loaded. We may reckon the mean $3 \cdot 50 \mathrm{P}$ as equal to the moment of resistance $\mathrm{R}=6000000$. By this means we shall obtain cross girders supporting geṇerally a strain of 4.5 kilos. the square millimètre, omitting the exceptional case of both lines being loaded simultaneously, when the strain may rise to $7 \cdot 5$ kilos. In a line with the exterior rails, the mean moment will be $2 \cdot 40 \mathrm{P}$.

Table VI. gives a
 certain number of sections for various distances. The weights include all accessories.
25. Vertical Trussing. - When the girders of a bridge are very high and their flanges comparatively narrow, they are liable to incline or sag from one side to the other, especially if they are loaded on the upper part. If the direction of the sagging is different at the two ends of the girder, a strain of torsion is produced, which changes the plane of the vertical rib into an oblique surface. The principal object of vertical trussing is to counteract these effects of sagging. The strain borne by these pieces is caused by the motion of the trains, which produces lateral oscillations, or by the deflections which occur in the flange subject to compression. But it is often needful to strengthen the oblique pieces of the transverse section, because they may serve as supports to the cross-girders, of which arrangement Figs. 1605 to 1611 and Figs. 1580 to 1585 are examples.

The object of vertical trussing cannot be to equalize the deflections of the several girders in cases when these girders may be unequally loaded, for in such cases the deflections must of necessity differ : it could be prevented in part only by means of very strong pieces, and there would then result a strain of torsion. Yet, when the girders are high and rigid, the deflections are so inconsiderable that the vertical trussing does offer some resistance, because, by its elasticity, it adapts itself sufficiently to the different curves which the girders assume under the rolling load. Between two girders equally loaded, such as two girders supporting directly the

## BRIDGE.

rails of one line, these strains do not exist. In four-girder bridges under rails, if the height be inconsiderable, it will be sufficient to simply tie the two lines.
26. Horizontal Trussing. - One use of horizontal trussing is to prevent lateral vibration, and in order to effect this in the most complete manner the trussing should be placed as near the plane of the rails as possible. Another use of this kind of trussing is to form, with the aid of the crossgirders, a resisting plane which shall provent the girder from twisting under the torsive strain of the compressed flange. From this point of view the trussing should, when possible, bind together the compressed flanges of the several girders. When the bridge is loaded on the upper part, a top trussing will satisfy the double condition. In tubular bridges loaded on the lower part, an upper and a lower trussing will be necessary. And in bridges of girders loaded on the lower side, a top trussing being impossible, we can have only a lower one, but care should be had to prevent the twisting of the girders by widening their top flanges and the upright stays which serve to stiffen them.

The trussings or bracings act in the same way as horizontal lattice-bars, but the strains to which they are subject are of $\Omega$ capricious nature that eludes calculation They are usually formed of bars of a uniform dimension throughout the whole length of flooring; but in important works it may be advisable to strengthen the bars near the supports, especially if the line be curred upon


## BRIDGE.



Two-girder Bridge, loaded on the upper side.
Height of girders, 3 m . Distance apart of the cross-girders and consoles, $3^{\mathrm{m}} \cdot 000$. Scale, 0.03 .


## BRIDGE.

the bridge, which may be the case in a certain degree when the principal girders are straight. The section of the iron is usually flat, because, of the two diagonals forming a Saint Andrew cross, there is always one which resists tension, whilst the cross-girders serve as compression-bars; besides, the compressed diagonal does not remain quite inactive when it is fixed in several points throughout its length, as is usually the case. When, on the contrary, the bars are left free throughout a considerable length, it is preferable to make them of T-iron, to a void yielding.

In bridges of small span - of less than 25 mètres, for example -the breadth of the flooring is a large fraction of the length, and the trussing may be omitted, because the rigidity of the flooring itself offers a sufficient resistance to transverse oscillations.

For a bridge loaded on the upper part, Figs. 1612, 1613, and Figs. 1580 to 1585 , the horizontal trussing may weigh, according to the span, from 50 to 90 kilogrammes the lineal metre of flooring under two lines of rails (bars of 160/12 to $200 / 16$ millimètres), including the gussets.


For a three-girder bridge, loaded on the lower part, we may reckon from 80 to 150 kilogrammes; for a two-girder bridge, loaded on the lower side, from 50 to 100 kilogrammes. Tubular bridges absorb the lineal mètre, from 130 (two-girder bridges) to 160 kilogrammes (three-girder bridges) and above, for their double trussing or cross-bracing, which, added to their great breadth, gives them a transverse rigidity greater than bridges of an equal span loaded on the upper part, and having corbels, could possess.

In ordinary road bridges the horizontal cross-bracing may be considerably inferior
1613.

Scale, 0.005.


Cross-bracing about $50^{\mathbf{k}}$ a mètre of flooring.
1612.

Four-girder Railuay Bridge. Height of girders, $4^{\mathrm{m}}$. Scale, $0 \cdot 03$.
1609.

Road Bridge.


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in strength to that required for railway bridges. Flat bars of $100 / 12$ are in general amply sufficient.
27. In cases when the height allowed is unlimited, the lines of rails may rest upon the upper part of the girders. When the span is inconsiderable we shall have, generally, four principal girders, supporting directly the rails, as in Figs. 1612 to 1614; for in such cases the other pieces serve only

1615.

Two-girder Railway Bridge, loaded on the upper side.
Girders of moderate height. Scale, 0.02.

to tie the girders, and may be of small dimensions. But when the span is great, the saving in the secondary pieces should be sacrificed in favour of the more important saving which may be effected by reducing the number of the principal girders. One girder costs less than two that are only half loaded, and, besides, this one girder will be subject to the full stiain only in the exceptional case of two trains crossing the bridge simultaneously. Figs. 1580 to 1585 , and Figs. $1606,1615,1616,1617$, give examples of bridges of two girders of various heights, and, consequently, applicable to bridges of small and great span. Between two-girder and four-girder
bridges are those of three girders, Figs. 1605 and 1618 to 1620 , which require strong crossgirders, but less important than those of two-girder bridges.

When the height allowed is limited, the load is placed upon the lower part of the girders; in such cases the girders form the sides or parapets of the bridge when they are not very high, and when their height is great, advantage is taken of the circumstance to tie them on the upper part; this latter arrangement constitutes the tubular system. Figs. 1591 to 1598 are examples of three-girder bridges constructed according to the first method, and Figs. 1621 to 1624 are examples respectively of three-girder and twogirder tubular bridges. In the various kinds of bridges loaded on the lower part, we distinguish again those in which the height of the crossgirders is limited, and those in which this height is great, for this is of considerable importance when we regard the weight of the minor and cross girders. For example, in Figs. 1591, 1594, 1595, in which the cross-girders have a height of 0220 metre, thís weight reaches 1350 kilogrammes the lineal mètre of flooring under two lines of rails; whilst in Fig. 1593, with a height of 0.850 mètre, it is reduced to 630 kilogrammes. A limited thickness obliges us to increase the number of cross-girders, and induces us to give the preference to three-girder rather than to two-girder bridges.

Regarded from the point of view of tying the principal girders, the distance between the cross-girders will be determined by the transverse rigidity of the former. Girders with narrow flanges may need to be tied every 2 mètres, or 2.50 mètres; whilst in large bridges, provided the cross-girders be strong and firmly fixed, they may be placed 4 mètres apart and even more.

## 1616.

Bolting of the sleepers upon the girders.


Three-girder Railway Bridge, loaded on the upper side.


28. The weight of the various parts beside the principal girders may be taken generally as follows:-
Weight of the parts beside the chief Girders and the horizontal Trussin.', the lineal mètre of flooring under two lines of rails.
(Distance between cross-girders, 2 mètres: 320 to 450 kilogrammes, Four-girder according as the height of the girders varies from $1 \cdot 50$ to $4 \cdot 00$ mètres;

1. Loaded on the upper side.

## 2. Loaded on

 thee lower side.bridges.

Distance between cross-girders, 3 mètres: 260 to 350 kilogrammes, according as the height of the girders varies from $1 \cdot 50$ to $4 \cdot 6$ mètres;
Three-girder bridges: 460 to 800 kilogrammes, according as the height varies from 2 to 5 mètres;
Two-girder bridges: 540 to 920 kilogrammes, according as the height varies from 3 to 8 mètres;
Three-girder bridges: 960 to 630 kilogrammes, according as the height of the cross-girders varies from 0.35 to 0.85 mètre;
Three-girder bridges; 1050 to 730 kilogrammes, according as the height of the cross-girders varies from $0 \cdot 60$ to $1 \cdot 10$ mètre ;
Three-girder tubular bridges: 1050 to 730 kilogrammes with cross-girders of $0 \cdot 35$ to 80 mètre;
Two-girder tubular bridges ; 900.

We omit those exceptional cases of bridges loaded on the lower part, in which the height allowed is so limited as to render necessary very expensive arrangements.
1621.

Three-girder Tubular Bridge. Scale, 0.01.


Top cross-bracing.
Bottom cross-bracing.

29. The planking of railway bridges is composed of planks 0.06 or 0.07 mètre thick. The longitudinal timbers supporting the rails are 0.13 or 0.15 thick by 0.25 or 0.30 broad. As the calculation of the timbers can be always easily made, it is useless for us to consider it here. Generally we shall have from $(0 \cdot 60)^{3}$ to $(0 \cdot 70)^{3}$ mètre, the lineal mètre of flooring under two lines of rails.
30. For ordinary road bridges we prefer wood in the construction of the flooring to corrugated iron, because with wood we may place the beams farther apart, which is a more economical arrangement. It is true that the wood will require to be sooner renewed, but that is a labour which is readily executed, even without interrupting the traffic. In Figs. 1607 to 1625 , the planking is 0.07 mètre thick, and is covered with a boarding 0.03 mètre thick which serves the double purpose of protecting the planking, and of increasing its power of resisting by preventing a single plank from yielding beneath the weight of the wheel of a vehicle. The planking rests upon joists 0.250 mètre deep by 0.125 mètre broad, placed at distances of 0.375 mètre apart.

## 1623.

Two-girder Tubular Bridge.
Cross-bracing in T-iron. Cross-girder with vertical gussets, $660^{\mathbf{k}}$. Scale 0.01 .

1624.

Upper cross-bracing. Lower cross-bracing above the cross-girders.


When corrugated iron is employed, it may be 0.003 metre thick for a distance of 1.40 mètre between the crossgirders, the ridges being 0.080 mètre in height and having a mean breadth of $0 \cdot 160$ mètre; but this iron does not resist well unless it is covered with a sufficiently thick layer of earth to prevent the wheels from concentrating their action upon a single ridge.

There yet remains for very important works the heavy but indestructible system of brick arches, springing from iron beams. But our purpose is only to study the most economical arrangements.

31. The planking of the above-mentioned figures, cubes a square mètre, or hai as its reduced thickness 0.183 mètre. With this planking, the proper distance between the cross-girders is 1.80 metre. We may consider the cross-girders as subject to a maximum weight of 1000 kilogrammes

the lineal metre (the layer of earth varying from 0.15 to 0.25 mètre), and to an accidental load of two wheels, each exerting a pressure of 3 tons, placed 1.50 mètre apart, and in the most unfavourable position, which gives the moment of rupture expressed by the formula in 10. When the bearing of the cross-girder exceeds 4 mètres, we must consider the caso of two heavy vehicles passing in opposite directions. The wheels may vary in respective position, but as the ballast serves to distribute the pressure, we shall admit that the total weight of 12 tons for the four wheels, simultaneously situate upon the cross-girder, is distributed uniformly throughout its length. Table VII. gives for these conditions, some sections ready calculated. The weights include gussets similar to those in the figures already referred to, and suppose that the distance between the girders exceeds by $0 \cdot 40$ mètre their bearing.

Cross-girders of more than 8 mètres will generally be divided in their length by one or more of the principal intermediate girders.
32. The figures referred
to in 30 represent bridges of three different breadths :-
6 mètres. composed of 4 mètres of roadway and two footways of 1.00 mètre each

| 10 | $"$ | $"$ | 7 | , | $"$ | , | $"$ | 1.50 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 14 | $"$ | $"$ | 10 | $"$ | , | $"$ | $"$ | 2.00 |

And for each of these breadths we may distinguish the cases of, -1 , small span, which requires only a moderate thickness between the level of the road and the under-side of the girders; 2 , wide span with unlimited height; and 3, wide span with limited height. This gives occasion
for nine different transverse sections, or rather for twelve, for Figs. 1626 to 1630 are double, and represent either girders loaded on the lower side or tubular bridges. Cross-girders supporting the road are always placed $1 \cdot 80$ mètre apart; but crossgirders used only as tie-
1629.
Road Bridge.

Roadway, $7^{\mathrm{m}}$; fontway, $1^{\mathrm{m}} \cdot 50$. Upper cross-girder for tubular bridge. beams, struts, and braces,
are placed only at double intervals. In Figs. 1607 to 1611 there is, therefore, one out of every two crossgirders untrussed; but the want of this is compensated by an additional plate in the flanges. In Figs. 1608 , and 1626 to 1630 , the cross-girders, which are very long, are held in the middle of their length by longitudinal bars in double T-iron of 20 kilogrammes to thé lineal mètre. In bridges of a rather-wide span we add a horizontal cross-bracing of flat bars of

cross-girders, serves to prevent their lateral deflection. Tubular bridges require the cross-bracing both on the upper and the lower side.

The following Table gives the weight of iron the lineal mètre of flooring, not including the principal gitders, for each of the transverse sections referred to above:-


Those kinds which are applicable to wide spans admit, in the case of unlimited height, of truss-bars, the length of which increases with the height of the girders: it is for this reason that we have indicated the particular height for which the calculation has been made.

The weight includes the cross-bracing adopted in the cases of wide span ; it includes also the parapets of 30 kilogrammes the lineal mètre, except in those cases in which the girders themselves form parapets.
33. Loads applicable to the principal Girders.-Proportion between the Span, the Bearing, and the total Length.-Bearing-rollers.-A girder of a considerable bearing is always calculated on the hypothesis that the loads are uniformly distributed, namely:-

1. A permanent load of $p$ kilogrammes the lineal mètre, throughout the whole length.
2. An accidental load of $p^{\prime}$ kilogrammes the lineal mètre, throughout a certain length of the bearing.

Loads $p$ and $p^{\prime}$ applicable to Railvay Bridges.-Sometimes the dead weight $p$ is purposely increased by means of a thick layer of earth or ballast, for the purpose of deadening the vibrations caused by the rolling load $p^{\prime}$; this, besides, allows us to make the lines of rails independent of the flooring, by placing them upon sleepers not fixed to the metal part of the bridge. But this is an exceptional arrangement; usually the dead weight of the indispensable parts is reduced, for the reason that the iron is capable of resisting the vibrations which are produced by the passage of the trains, when the girders are calculated for a strain of 6 kilogrammes the square millimètre. An increasc of the permanent load may no doubt be favourable to the power of resistance in the material, but if it be adopted, it will only be logical to take for $R$ a greater value than 6000000 , for it is acknowledged that this coefficient offcrs perfect security in cases in which the vibrations are exerted in the usual manner. At any rate, we shall suppose only a thin layer of ballast to protect the planking from the burning coals dropped by the engine.

By adding the weight of the rails, of the timber, and the various picces of the flooring, we may take as the average the lineal mètre of each of the chief girders, not including its own weight, the following permanent loads:-
Bridges loaded
on the
upper part.
of 4 girders: $p_{1}=500$ to 600 kilogrammes. $800, ~$
of $3 "$,

Bridges loaded (outside girders: 800 to 700 kilogrammes, according as the thickon the upper part. of 3 girders of 2 girders: 1400 kilogrammes.

To obtain $p$, we must add to $p_{1}$, given above, the weight $p_{2}$ of the girder itself a lineal mètre, a weight which it is of small consequence to know exactly, and which may be corrected, if it be deemed necessary, by a second calculation. When we wish to obtain only the net value, we may get rid of the weight $p_{2}$, by remarking that in general the weight of the girder a lineal metre is, at least in the greater part, proportional to $\left(p+p^{\prime}\right) l$. Supposing, then, $p_{2}=\mathbf{D}\left(p_{1}+p_{2}+p^{\prime}\right) l$, we obtain $p_{2}=\frac{\mathbf{D}\left(p_{1}+p^{\prime}\right) l}{p_{1}-l \mathbf{D}}$; or by making $\mathbf{D}\left(p_{1}+p^{\prime}\right) l=p_{3}, p_{2}=\frac{\left(p_{1}+p^{\prime}\right) p_{3}}{p_{1}+p^{\prime}}-p_{3}$; we can, therefore, deduce the weight $p_{2}$, found from the weight $p_{3}$, which is obtained by neglecting in the dead weight the weight of the girder itself. The result will not be strictly true, because the proportion $q$ of the rolling load to the total load is increased by the reduction of the latter; but the error is small, $q$ having but a slight influence except upon a few lattice-bars.
34. Let us now consider the moving load $p^{\prime}$.

An engine with its tender may be placed upon a bridge of 10 mètres, and if the weight amounts to 60 tons, we shall have 6000 kilogrammes the lineal mètre for one line of rails, admitting, as we may for this length of span, that the load produces nearly the same effect as it would do if uniformly distributed.

Beyond a span of 60 mètres, we always suppose, in calculations, a load of 4000 kilogrammes.
Between 10 and 60 mètres, this assumed load should vary as follows:-
Spans $\ddot{i} \quad \ddot{\square}=10, \quad 20, \quad 30, \quad 40, \quad 50, \quad 60$ mètres, and above.
Load $\mathrm{P}^{\prime}$ (for a single line $)=6000,5000,4600,4300,4100,4000$ kilogrammes.
A girder may have to support two rails, one rail, or a fraction of a rail, according to the system of flooring. The following Table gives the necessary information regarding this matter, omitting altogether rails which are not loaded.

35. Loads $p$ and $p^{\prime}$ applicable to Road Bridges.-The following Table gives the loads applicable to the girders of various kinds of bridges. The dead weights $p$ are calculated on the hypothesis that there is a layer of earth, or ballast, $0 \cdot 20$ mètre deep, and the moving loads $p^{\prime}$ on the hypothesis that the roadway is loaded at the rate of 400 kilogrammes, and the footways at the rate of 200 kilogrammes, the square mètre.

| $p$ for one girder, not including) its own weight | Breadth <br> 6 Mètres Figs. 1607, 1625, 1627. | Breadth, 10 Mètres. |  |  | Breadth, 14 Mètres |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Outside Girder. | Middle Girder. | $\begin{aligned} & \text { Figs. } \\ & \text { 1608, } \\ & 1629 . \end{aligned}$ | Fig. 1609. |  | Fig. 1626. |  |
|  |  |  |  |  | Outside Girder. | Middle <br> Girder. | Outside Girder. | Middle <br> Girder. |
|  | kilos. $1300$ | $\begin{aligned} & \text { kilos, } \\ & 1100 \end{aligned}$ | $\begin{aligned} & \text { kilos. } \\ & 1900 \end{aligned}$ | $\begin{aligned} & \text { kilos. } \\ & 2300 \end{aligned}$ | kilos. $1600$ | $\begin{aligned} & \text { kilos. } \\ & 2800 \end{aligned}$ | $\begin{aligned} & \text { kilos. } \\ & 1450 \end{aligned}$ | $\begin{aligned} & \text { kilos. } \\ & 3800 \end{aligned}$ |
| $p^{\prime} \quad$ ditto $\quad . \quad . \quad .$. | 1000 | 1000 | 1400 | 1700 | 1400 | 2000 | 1050 | 2700 |

[^0] mentary prism $F M$, is $\frac{G H}{H I}=\frac{G M}{F I}=\frac{\sqrt{r^{2}-x^{2}}}{\epsilon}$. To find HI, for example, we deduce from this proportion $\frac{\mathrm{GH}+\mathrm{HI}}{\mathrm{HI}}=1+\frac{\sqrt{r^{2}-x^{2}}}{\epsilon}$; or $\mathrm{GH}+\mathrm{HI}$, or $\mathrm{GI}=\lambda-r+\sqrt{r^{2}-x^{2}}$, and, consequently, denoting the modulus of elasticity by E , the portion of the load which the elementary prism FM supports will be $d \mathrm{Q}=\mathrm{E} \cdot \frac{\mathrm{HI}}{\epsilon}=\mathrm{E} \frac{\lambda-r+\sqrt{\frac{r^{2}-x^{2}}{}}}{\epsilon+\sqrt{r^{2}-x^{2}}} d x$.

To determine $\lambda$, we remark that the metal must not be subjected to a strain greater than R the unity of section, so that $\lambda$ cannot exceed the value $\frac{R}{\bar{E}}(\epsilon+r)$, which value will be substituted in the integral $Q=2 \mathrm{E} \int_{0}^{\sqrt{\lambda(2 r-\lambda)}} \frac{\lambda-r+\sqrt{r^{2}-x^{2}}}{\epsilon+\sqrt{r^{2}-x^{2}}} d x$.

Now the value of the integral of the form $\int \frac{\mathrm{A}+\sqrt{r^{2}-x^{2}}}{\mathrm{~B}+\sqrt{r^{2}-x^{2}}} d x$ is

$$
\int \frac{\mathrm{A}+\sqrt{r^{2}-x^{2}}}{\mathrm{~B}+\sqrt{r^{2}-x^{2}}}=\mathrm{C}+x+2(\mathrm{~B}-\mathrm{A}) \tan ^{-1} \frac{\sqrt{r^{2}-x^{2}}}{x-r}+\mathrm{Z},
$$

C being the arbitrary constant, and the term Z being equal to

$$
\frac{\mathrm{B}(\mathrm{~A}-\mathrm{B})}{\sqrt{r^{2}-\mathrm{B}^{2}}} \log \cdot \frac{\frac{\mathrm{~B} \sqrt{r^{2}-x^{2}}}{x-}-r-\sqrt{r^{2}-\mathrm{B}^{2}}}{\frac{\mathrm{~B} \sqrt{r^{2}-x^{2}}}{x-r}-r+\sqrt{r^{2}-\overline{\mathrm{B}^{2}}}} \text {, in the case in which } r>\mathrm{B} \text {; }
$$

to $\frac{2(r-\mathrm{A})(x-r)}{r-x+\sqrt{r^{2}-x^{2}}}$, in the case in which $r=\mathrm{B}$; and to

$$
\frac{2 \mathrm{~B}(\mathrm{~A}-\mathrm{B})}{\sqrt{\mathrm{B}^{2}-r^{2}}} \tan _{\cdot}^{-1} \frac{\frac{\mathrm{~B} \sqrt{r^{2}-x^{2}}}{x-r}-r}{\sqrt{\mathrm{~B}^{2}-r^{2}}}, \text { when } r<\mathrm{B} .
$$

Usually, we have $\mathrm{E}=0.100$ mètre, and the diameter $2 r$ of the rollers $=0.150$ mètre. It will, therefore, be the last expression of $Z$ that we shall have occasion to apply, and by calculating the definite integral which gives the value of $Q$, we obtain

$$
\begin{aligned}
& Q=2 \mathrm{E}\left\{\sqrt{\lambda(2 r-\lambda)}-2(\epsilon+r-\lambda)\left[\tan ^{-1} \frac{r-\lambda}{r-\sqrt{\lambda(2 r-\lambda)}}\right.\right. \\
& \left.\left.-\frac{\pi}{4}-\frac{\epsilon}{\sqrt{\epsilon^{2}-r^{2}}}\left(\tan ^{-1} \frac{r+\frac{\epsilon(r-\lambda)}{r-\sqrt{\lambda(2 r-\lambda)}}}{\sqrt{\epsilon^{2}-r^{2}}}-\tan .^{-1} \frac{\epsilon+r}{\sqrt{\epsilon^{2}-r^{2}}}\right)\right]\right\} .
\end{aligned}
$$



For $\mathrm{R}=6,000,000$ and $\mathrm{E}=8,000,000,000$, a value applicable to thick cast-iron plates, the formula gives $Q=36370$ kilogrammes the lineal mètre of roller, say 24 kilogrammes the square centimètre of diametrical section: this value we shall adopt as a maximum.

But when there are several rollers it is difficult to load them all equally. In order that no roller may exceed its limit of elasticity, it is essential that the bed-plates be fixed with care; the keys or bolts which hold them should, as far as possible, be tightened in an equal degree while the trial load is upon the bridge. When the distance of the load has modified the deflection of the girder, the pressure will be unequally distributed between the rollers; but this is no longer of consequence, because the rollers are greatly eased of the total weight. But in practice, a certain latitude must be allowed; and we should consider that the pressure has always a tendency to concentrate itself towards the facing of the abutment. If we admit that the centre of pressure, under a full load, may move forwards along 0.60 of the bearing upon the abutment, reckoning from the end, the mean pressure will be $\frac{5}{8}$ of the maximum 24 kilogrammes, say 15 kilogrammes the square centimetre of diametrical section of the rollers. If this pressure be referred to the total surface of the plates, it will be necessary to deduct the play of the rollers, and we must then reckon upon about 13 kilogrammes.

The evil of imperfect bed-plates cannot be remedied by increasing indefinitely the number of the rollers; for, under a full load, the extreme rollers support hardly anything. And, even if a roller should be subjected to a strain a little too great, the consequence will, in general, be of small importance.

In bridges of several spans, or bays, upon very high piers, the rollers should turn freely in order that the expansion of the iron may not cause a great horizontal strain upon the masonry. The bedplates should be as equal as possible in all respects, both when the bridge has no load upon it and when it is fully loaded; and there is but one case in which the resultant of the pressures will, in any considerable degree, be diverted from the middle of the face of the support, namely, when the load is applied to one only of the contiguous bays. This case allows us to reckon upon the maximum pressure of 24 kilogrammes, or near it, for the full load, say 20 kilogrammes after having made the deduction for the play between the rollers.
37. Relation between the Span, the Bearing, and the total Length.-From what precedes, reckoning upon 13 kilogrammes the square centimètre, the bearing $c$, Fig. 1632, of a girder upon the abutment may be calculated by the formula $c=\frac{\left(p+p^{\prime}\right) \mathrm{L}}{260000 \beta}, \mathbf{L}$ being the total length of the girder (single bay), and $\beta$ the breadth of the plates or the bearing length of the rollers, which usually exceeds by $0 \cdot 14$ metre the breadth of the flange of the girder, in the case of moderate span. For very large bridges, it will be preferable to increase $\beta$, and perhaps also the diameter of the rollers, in order that they may offer more elasticity to lessen the effect of the inclination produced by the deflected girder.

In general, supposing that the dimension $b$ of the figure is 0.60 of $c$, the total length $L$ of the girder should be made $=1.045 \mathrm{l}$.

As to the ratio of the real bearing $l$ to the span $d$, it depends upon the dimension $a$, which should, in the first place, be at least equal to 0.40 c , and, in the second place, sufficient to keep the resultant of the pressures at a suitable distance from the facing of the abutment, and to prevent a crushing of the masonry. This condition depends itself upon the dimensions and talus of


L, Length of the girder. I, Real bearing $r$, Apparent bearing. d, Span. P, Appar
$0 \cdot 025 \mathrm{~d}$. the abutment; but we shall generally do right by making $a=0.025 d$.

Thus, to recapitulate, in calculations for bridges, we shall adopt $l=1.050 \mathrm{~d}$ and

$$
\mathbf{L}=1 \cdot 045 l=1.0973 d
$$

We shall usually consider the mean weight of the lineal mètre of girder in reference to the total length $L$. If we obtain the total weight $P$ by a direct measurement including the ends of the girder, $\frac{\mathrm{P}}{\mathrm{L}}$ will be the mean weight; we shall take, on the contrary, $\frac{\mathrm{P}^{\prime}}{l}$, if the weight $\mathrm{P}^{\prime}$ results from a theoretical formula which supposes the length of the girder to be $l$, subject to the corrections necessary to take into account, according to the case, the ends of the girder. To obtain the weight in reference to the span, we must multiply the weight found for the mètre of total length, by $1 \cdot 0973$ or $1 \cdot 10$.
38. In Skew bridges, the normal span being $d$, and the angle of the skew being $a$, the skew span will be $\frac{d}{\sin . \alpha}$, the real bearing $1 \cdot 05 \frac{d}{\sin . \alpha}$, and the total length of the girders $1 \cdot 0973 \frac{d}{\sin . \alpha}$. This
case presents no difficulty; the cross-girders are normal to the chief girders; some of them at the extremities are short, and rest upon an oblique piece which holds together the ends of the girders along the abutment. The first complete cross-girder forms with this oblique piece and the principal girder an unyielding triangle, which allows us to have recourse to a less expensive system of counter-bracing than would be required for a straight bridge of the same span. Beyond this difference, in the matter of calculating the weight and the resistance, a skew bridge and a straight bridge of the same span are alike.

The bridge, Figs. 1633, 1634, erected near Calcutta, was constructea according to the system of bridge-building introduced and practised by A. Sedley. The span, Fig. 1633, is 75 ft ., and the structure was calculated to support a moving load of 120 lbs . to the sq. ft . of roadway. The total weight, including rods, girders, and Mallet's buckled-plates, is 30 tons. The iron was proved to 20 tons to the sq. in. of cross-sectional area; the tension or suspension bars are of the best Bowling iron. The cantilevers have merely the effect of increasing the range of the piers, and if the suspension-rods support the lattice-girder without destroying its perfect contact across and on each of the two brackets, the system is reduced to a lattice-girder resting on supports.

Hydraulic Swing Bridye over the River Ouse.-The information that we give respecting this bridge is taken from 'Proceedings of I. M. E.,' August, 1869.

This bridge was designed by Harrison, the engineer of the North-Eastern Railway. Near the bridge there was no supply of hydraulic power, and the total power required was too great to be supplied by hand labour. It was further necessary, on account of the position of the swing bridge, either to convey the power to the centre pier by a pipe under the bed of the river, or to produce it upon the pier by placing a steam-engine within the pier itself; and the latter plan was the one adopted.

The construction of the Ouse Bridge is shown in Figs. 1635 to 1646. Figs. 1635, 1636, show an elevation and plan of the swinging portion. Fig. 1637 is a vertical transverse section at the centre pier, showing the engine-room and accumulator, which are situated within the centre pier ; and Fig. 1639 is a plan of the centre pier at the level of the engine-room, showing the arrangement of the driving-gear with the steam-engines and hydraulic engines. Fig. 1640 is an enlarged section of one half of the engine-room, and Fig. 1641 a sectional plan of the accumulator. Figs. 1647, 1648, are a plan and elevation of the hydraulic engines for turning the bridge ; and Figs. 1643 to 1646 show the gear at the two extremities of the bridge for working the adjusting-supports and the locking bolts.

The total length of the bridge, fixed and movable, is 830 ft . The fixed portions consist of five spans of 116 ft . each from centre to centre of piers. The bridge being for a double line of railway, each span is composed of three wrought-iron plate-girders, the centre girder having a larger section to adapt it for its greater load; these girders have single webs, and are 9 ft . deep in the centre. The total width of the bridge from outside to outside is 31 ft . Each of the piers for the fixed spans consists of three cast-iron cylinders of 7 ft . diameter and about 90 ft . length. The depth from the under-side of the bridge to the bed of the channel in the deepest part is about 61 ft . The headway beneath the bridge is 14 ft .6 in . from high-water datum, and 30 ft .6 in . from low water.

The swinging portion of the bridge, Figs. 1635, 1636, consists of three main wrought-iron girders, 250 ft . long, and 16 ft .6 in . deep at the centre, diminishing to 4 ft . deep at the ends. The centre girder is of larger sectional area than the side girders, and instead of being a single web is a box-girder 2 ft. 6 in. in width, Fig. 1640, with web-plates $\frac{7}{16}$ to $\frac{5}{1} \frac{\mathrm{in} \text {. in thickness, and the }}{}$ top and bottom booms contain together about 132 sq . in. section. The roadway is carried upon transverse wrought-iron girders resting upon the bottom flanges of the main girders, Figs. 1640 and 1644. In the centre of the bridge the main girders are stayed by three transverse wroughtiron frames securely fixing them together; and over the top of these frames a floor is laid, from which the bridgeman controls the movements of the bridge.

An annular box-girder A A, 32 ft . mean diameter, is situated below the centre of the bridge and forms the cap of the centre pier, Figs. 1637 and 1640 ; this girder is 3 ft .2 in . in depth and 3 ft . in width, and rests upon the top of six cast-iron columns, each 7 ft . diameter, which are arranged in a circle and form the centre pier of the bridge. Each of these columns has a total length of 90 ft ., being sunk about 29 ft . deep in the bed of the river. A centre column B B, 7 ft . diameter, is securely braced to the six other columns by a set of cast-iron stays, which support the floor of the engine-room. This centre column contains the accumulator C, Fig. 1637, and forms the centre pivot on which the bridge rotates.

The weight of the swing bridge is 670 tons. There is no central lifting-press, and tne entire weight rests upon a circle of conical live rollers E E, Figs. 1637 and 1640. These are twenty-six in number, as shown in the plan, Fig. 1638; they are each 3 ft . diameter with 14 in . width of tread, as shown in Fig. 1642, and are made of cast iron hooped with steel. They run between the two circular roller-paths D D, 32 ft . diameter and 15 in . broad, which are made of cast iron faced with steel ; the axles of the rollers are horizontal, and the two roller-paths are turned to the same bevel.

The turning motion is communicated to the bridge by means of a circular cast-iron rack $G$, Fig. 1640, $12 \frac{1}{2} \mathrm{in}$. wide on the face and $6 \frac{1}{2} \mathrm{in}$. pitch, which is shrouded to the pitch-line and is bolted to the outer circumference of the upper roller-path. The rack gears with a vertical bevelwheel H, which is carried by a steel centre-pin J supported in the lower roller-path; and this wheel is driven by a pinion connected by intermediate gearing with the hydraulic engine. There are two of these engines, duplicates of one another, which are situated at KK in the engine-room, Figs. 1639, 1640; and either of them is sufficient for turning the bridge, the force required for this purpose being equal to about 10 tons applied at the radius of the roller-path. Each hydraulic engine is a three-cylinder oscillating engine, as shown in Figs. 1647, 1648, with simple rams of $4 \frac{1}{4} \mathrm{in}$. diameter and 18 in , stroke. These engines work at forty revolutions a minute with a pres-

sure of water of 700 lbs . an inch, and are estimated at 40 horse-power each. The steam-engines for supplying the water-pressure are also in duplicate, situated at L L, Fig. 1639, and are doublecylinder engines driving three-throw pumps of $2 \frac{3}{4} \mathrm{in}$. diameter and 5 in . stroke, which deliver

in centre pier.

into the accumulator. The steam-cylinders are 8 in. diameter and 10 in . stroke, each engine being 12 horse-power.

The accumulator C, Fig. 1637, shown also in the sectional plan, Fig. 1641, has a ram $16 \frac{1}{2} \mathrm{in}$. diameter with a stroke of 17 ft .; it is loaded with a weight of 67 tons, composed of castiron segments suspended from a cross-head and working down inside the cylindrical casing formed by the centre cylinder of the pier. A pair of cross-beams M M are fixed to limit the rise of the weight.

For the purpose of obtaining a perfectly solid roadway when the bridge is in position for the passage of trains, and also for securing the perfect continuity of the line of rails, the following arrangement is adopted, shown in Figs. 1643 to 1646. Each extremity of the bridge
is lifted slightly by a horizontal hydraulic press N, Figs. 1643, 1644, acting upon the levers PP which form a toggle-joint; the press has two rams acting in opposite directions upon two toggle-joint levers, which are counected by a horizontal bar Q, and this bar is confined to a
1637.

Section of centre pier and engine-room.

vertical movement by a stud sliding in a vertical guide, so as to ensure an exactly parallel action of the two toggle-joint levers, in order thereby to lift the bridge end exactly parallel. While the end of the lridge is thus held lifted, the three resting-blocks $R R$, one under each

1640
Enlarged section of engine-room.

1641.

Sectional plan of accumulator.

1642.

Vertical section of roller-paths and roller-frame.

1643.

Locking-gcar, \&c., at ends of Bridge.
Plan of bridge end, showing locking-bolt and lifting-gear.


## BRIDGE.

girder are pushed home by means of three separate hydraulic cylinders S S, Figs. 1645, 1646; the bridge is then let down upon these resting-blocks by the withdrawal of the toggle-joint levers P P, and the bridge ends are then perfectly solid for trains to pass over. The hydraulic cylinders $N$ and $S$ for working this fixing-gear at the two ends of the bridge are controlled by valves placed upon the centre platform in reach of the bridgeman, the pipes from the valves to the cylinders passing along the side of the roadway of the bridge.

For the purpose of enabling the bridgeman to stop the turning movement of the bridge at the right place, an indicator is provided consisting of a dial with two pointers which are actuated by the motion of the bridge. One of these pointers makes two revolutions and the other forty-two revolutions for one complete rotation of the bridge, they are similar to the hour and minute hands of a watch. The bridge has no stop to its turning movement, and would swing clear past its right position if the turning power were continued; but the bridgeman being gudded by the indicator, knows when to stop and reverse the hydraulic engines for the purpose of stopping the bridge at its right place. When this is done, a strong locking-bolt T, Figs. 1643 to 1645, 3 in. thick, pressed outwards by a spiral spring, is shot out at each end of the bridge into a corresponding slot in the fixed girder, so as to lock the bridge; and when the bridge is required to be opencd, these two bolts are withdrawn by a wire cord U, Fig. 1645, leading to the platform on which the bridgeman is

1647.

Plan of hydraulic engines

stationed. In consequence of the line of the bridge lying in a north and south direction, the heat of the sun acting alternately on the opposite sides of the bridge produces a slight lateral warping; and in order to bring the ends back into the straight line after swinging the bridge, so as to enable the two locking-bolts to enter their slots, the feet of the toggle-joint lifting-levers P P are bevelled off at $55^{\circ}$ on their inner faces, as shown at I I in Fig. 1644, and bear against corresponding bevels $V$ V on the bed-plates. By this means the ends of the bridge when warped are forced back into the correct centre line, in which they are then held secure by the locking-bolts.

As the accumulator is stationary in the centre pier, while the fixing-gcar at the ends of the bridge travels with the bridge in swinging, the communication of water-power is effected by a central copper pipe W, Fig. 1640, passing up in the axis of the bridge through the middle of the centre girder, and having a swivel joint at the lower end. Also as the bridgeman's hand-gear rotates with the bridge while the hydraulic turning engines are stationary, the communication for working the valves is made by a central copper rod X, Fig. 1640, passing down through the

centre of the pressure-pipe $W$ in the axis of the bridge. The hydraulic engines are reversed in either direction by the action of a small hydraulic cylinder, which is governed by the movement of a three-port valve actuated by the rod X from the bridgeman's platform.

The time required for opening or closing the bridge, including the locking of the ends, is only fifty seconds, the average speed of motion of the bridge ends being 4 ft . a second. For the purpose of ensuring safety in the working of the railway line over the bridge, a system of self-acting signals is arranged, which is actuated by the fixing-gear at the two ends of the bridge; and a signal of all right is shown by a single semaphore and lamp at each end of the fixed part of the bridge; but this cannot be shown until each one of the locking-bolts and resting-blocks is secure in its proper place.
39. Calculation of simple Lattice-girders.-General Methods.-The triangle is the only articulate polygon which is unyielding; in other words, a triangle composed of inflexible bars cannot yield except by the extension or the contraction of its sides under the influence of the forces applied to them. We shall give the name of simple lattice to a network composed of a series of triangles arranged as in Fig. 1649: the whole collection of bars being regarded as a simple figure, the strain upon all the pieces will be determined by the elementary rules of statics. The load consists of distinct weights $P, P_{1}$, applied to the several summits. The reaction $\mathbf{Q}$ of the support $\mathbf{A}$ is easily obtained by considering the equilibrium of the whole figure. To find the strain borne by any single piece CE, imagine a plane $\mathrm{N}^{\prime} \mathrm{N}^{\prime}$ cutting only this piece and two others BC, EG, and seek the conditions of equilibrium of the portion of the system included between the plane $\mathbf{N ~ N}^{\prime}$ and the support A. We obtain three distinct equations-two of projections, and one of moments-which determine the strains upon the three pieces crossed by $\mathrm{N} \mathrm{N}^{\prime}$; for these strains appear as reactions of the right
 portion of the lattice upon the portion under consideration, and the other forces Q, P . . are known.

But it is easy to avoid any elimination. If we make the secant plane pass through the point C, the strains upon the pieces B C, E C, will not appear in the equation of moments, and we shall have at once the strain upon the section E G of the side, by dividing the moment of rupture about the opposite summit C, by the perpendicular let fall from C upon EG. The strain upon BC is found in like manner, in terms of the moment of rupture taken with respect to E . Returning to the section $\mathrm{N} \mathrm{N}^{\prime}$, we have only to state the equation of the vertical projections of the forces, to obtain the strain upon the bar EC in terms of the strains, already determined, of EG and BC.

If we take the upward direction of the forces as positive, the stress will be positive as long as the sum $P+P_{1}+\ldots$ of the loads between $A$ and $N^{\prime}$ is less than $Q$, and the moment of rupture will be positive if it tends to turn the figure in the same direction as Q . If we find, for example, the strain T of the bar EC positive, this will indicate that the portion $\mathrm{C} O$ of the bar tends to raise the portion EO, and, consequently, the piece CE is subject to a tensile strain; it will be, on the contrary, compressed, if T is negative. These conclusions would change if the bar were placed according to the other diagonal B G.
40. We may also determine the strains by a graphical process, founded upon the observation that at each summit of the triangles the strains upon the pieces which join them are in equilibrio, otherwise this summit would be displaced. Thus, beginning at the abutment, we shall decompose Q in the directions A B and AE, which will give the strains upon these two sides. Then in B we shall take the resultant of the known strain upon A B, and the weight $P$ which is also known, and decompose it between BE and BC; this will give, without the sign, the strains upon these latter pieces. This operation must be continued for the other pieces.
41. A mixed process consists in calculating beforehand the strains upon the exterior sides or sections of flange by means of the moments of rupture, as explained in 39. Then we can at once in any summit, without passing through the preceding ones, determine graphically the strains upon the two inner bars, the function of which is to destroy the known resultant of the strains upon the two sections of flange and the particular load which may be applied to the summit in question.
42. To pass from these general considerations to formulæ more precise, we must begin by fixing the form of the system. Let us consider the case of straight and horizontal flanges. On applying the foregoing process to this case, we see immediately that, to find the strain of an interior bar, it is useless to know the absolute strains upon the two sections of flange which join at one of its extremities; it is enough to find the difference of their strains; the function of the two lattice-bars being to counterbalance that one of the two sections which exerts a predominant horizontal thrust, and to support, if necessary, the load which may be applied directly to the summit under consideration.

Now, this difference of strain depends upon the increase of the moment of rupture for a given interval, or, which is the same thing, of the transverse strain. The equation of equilibrium between the vertical forces gives $\frac{F}{\sin . a}$ for the stress upon any interior bar, a being the angle of this bar with the horizon, and F the stress in the interval which it occupies.

Thus, let CD N be the locus of the transverse strains, Figs. 1650, 1651, the strain upon any bar EG of the girder ABH will be represented graphically by the length of a right line $\mathrm{E}^{\prime} \mathrm{G}^{\prime}$, parallel to E G, inscribed between the axis of the abscisse and the horizontal line representing the stress in the interval occupied by the bar in question.

If the inclination $\alpha$ of the bars is constant, and we reduce in a constant proportion the ordinates of the locus CD N, these latter will represent directly to a given scale, either the strains or the sections of the several bars. In the absence of a special plan of strains, the locus $a b c d$ of the moments of rupture is sufficient to determine the strains upon the lattice-bars. For example, the stress relative to the bar EG is nothing but the tangent of inclination of the right line $c d$ with respect to the axis of the $x$ 's, taking into account, of course, the proportion between the scales of the abscissæ and of the ordinates.
43. But the locus of the moments of rupture is specially constructed with regard to the flanges of the girder: it is a polygon the summits of which correspond with the loads $\mathrm{P}, \mathrm{P}^{\prime}$. The volume or the total weight of the lower flange will be proportional to the area comprised between the axis of the $x$ 's and the broken line a ef ghiklm; that of the upper flange will be given by $a^{\prime}$ no $p q r s$;

$$
1651 .
$$ and it will be seen from the figure

that the whole of the two flanges will be proportional to the area $a b c d$. . . of the locus of the moments of rupture.

Suppose the girder loaded with equal weights $\left(p+p^{\prime}\right) \delta$ placed at equal intervals $\delta$, and the bearing $l$ expressed by $\mathrm{N} \delta$. The polygon of the moments of rupture will be parabolic, and its area will be given by the formula obtained in 6 . The double of this area, multiplied by $\frac{t}{h}, t$ being the ratio $\frac{7800}{\mathrm{R}}=0.0013$ of the weight of the cubic mètre of iron to the admitted resistance, and $h$ the height of the girder, will give for the total weight of the two flanges the value $\frac{\left(p+p^{\prime}\right) l^{3} t\left(\mathrm{~N}^{2}-1\right)}{6 h} \mathrm{~N}^{2}$, say the $\frac{2\left(\mathrm{~N}^{2}-1\right)}{3 \mathrm{~N}^{2}}$ of the weight they would have in a girder loaded in a continuous manner, if the greatest section necessary in the middle were preserved throughout the whole length.

This expression does not change whether N be even or odd. But if N be even and we substitute $2 m$ for it, the expression becomes $\frac{\left(p+p^{\prime}\right) l^{3} t}{24 h} \frac{4 m^{2}-1}{m^{2}}$, whilst for N odd and equal to $2 m+1$, it assumes the form $\frac{2\left(p+p^{\prime}\right) l^{3} t}{3 h} \frac{m(m+1)}{(2 m+1)^{2}}$.

In practice, this weight of flange should be multiplied by a coefficient $\mathbf{U}$ designed to include the joint-plates and other accessories (see 63 and following sections).

In the more general case, when the extreme intervals have a length $a$ different from $\delta$, the bearing being expressed by $2 a+\mathrm{N} \delta$, the weight of the two flanges together will be

$$
\frac{\left(p+p^{\prime}\right) t}{6 h}\left(l^{3}-\mathrm{N} \delta^{3}-2 a^{3}\right) .
$$

The total theoretical weight of the lattice-bars may also be represented by the area of a geometrical locus, namely, that having for equation $y=\frac{\mathrm{F} t}{\sin \cdot \alpha \cos \cdot \alpha}$, the inclination $\alpha$ being liable to vary from one bar to another, as well as the stress F . But if there were vertical bars, they would elude this mode of calculation, and we should have to consider them apart.

We are now about to state the formulæ giving the maximum strains and the weight of the several parts of straight girders of regular shape. We call struts the bars which are compressed by a load covering all the bridge, and braces or ties those which are subject to tensile force. But it may happen that a partial load changes the direction of the strains. The struts, or at least some of them, usually require an excess of section to give them the necessary rigidity. We shall take this into account by affecting the theoretical weight with a coefficient the mean value of which is denoted by the letter V (see 68 and following sections).
44. Girder loaded in one point only.-The load consists of a single weight P , applied to a point C, Fig. 1652. From A to C the stress has the constant value $\frac{\mathrm{P} b}{a+b}$; the struts, inclined at an angle $\alpha$, bear in this interval a constant compression $\frac{\mathrm{P} b}{(a+b) \sin \cdot \alpha}$, and the braces, inclined at an angle $\beta$, a tension $=\frac{\mathrm{P} b}{(a+b) \sin \cdot \beta}$. In the second interval, from C to B , the stress has the negative value $-\frac{\mathrm{P} a}{a+b}$; the struts and braces affect, therefore, directions opposite to those which they had between A and $\mathbf{O}$, and their strains are found in like manner by dividing the stress by the sine of angle of inclination.

When a girder supports a single weight, economy requires that the height should be increased and that the number of triangles formed by the bars should be reduced. For example, the most economical means of supporting a weight in the middle of the bearing consists theoretically of two bars
 at $45^{\circ}$ resting against each other at their summits, and having their lower extremities supported upon abutments and held by a tie.

If the single weight $P$ is a moving one, the maximum strain upon the bars having the same inclination will vary uniformly according to the same law as the maximum whole stress, which reaches the value P towards the support, and is reduced to $\frac{1}{2} \mathrm{P}$ in the middle.
45. Girders uniformly loaded on the upper side.- If the load rests directly upon the upper flange, the several sections of the latter will undergo slight partial deflections; but these may be neglected when we regard the load as distributed among the upper summits proportionally to the intervals. This load is of two kinds: one portion denoted by $p$ per lineal mètre remains permanently throughout the bearing ; the other portion, $p^{\prime}$ per metre, is moving, that is, it may affect either the whole bearing or only any portion of it; the letter $q$ denotes the ratio $\frac{p^{\prime}}{p+p^{\prime}}$. Always supposing that the loaded points are equidistant, we shall consider various particular figures, symmetrical with respect to the middle of the bearing.

First arrangement, Fig. 1653. The intermediate upper summits support each a weight $p \delta$, and

accidentally another weight $p^{\prime} \delta$; but in the two extreme summits $C$ and $D$, these weights are reduced to $\frac{p}{2}\left(\delta+\frac{h}{\tan . \alpha}\right)$ and $\frac{p^{\prime}}{2}\left(\delta+\frac{h}{\tan . \alpha}\right)$.

According to 5 , the stress will reach its maximum when the load extends over a portion of the bridge only, and we easily find the following expressions. The several pieces are numbered from the middle to each abutment, for any two symmetrical pieces are in perfectly identical conditions.

## BRIDGE.

A lengthening A C, D B, of the flanges, with upright stays at the ends, will be found useful in practice for a load above; but the formula do not include this, unless, indeed, it be included in the coefficient U.

Anystrut, the order of which with respect to the others is expressed by $n$, supports the following maximum strain of compression :
$(2 n-1) \frac{p \delta}{2 \sin . \alpha}+\frac{p^{\prime}}{4 \sin . \alpha\left(m \delta+\frac{h}{\tan . \alpha}\right)}\left\{(m+n)(m+n-1) \delta^{2}+\left[\frac{h}{\tan . \alpha}+\left(2 m+2 n-1 \delta^{\prime}\right)\right] \frac{h}{\tan . \alpha}\right\}$,
except the end strut, the order of which is $m+1$, and which will support only

$$
\frac{\left(p+p^{\prime}\right)}{\sin . \alpha}\left(m \delta+\frac{h}{2 \tan \cdot \alpha}\right)
$$

As to the braces, the tension of the $n$th reckoning from the middle is given without exception by $2 n-1 \frac{p \delta}{2 \sin . \beta}+\frac{p^{\prime}}{4 \sin . \beta\left(m \delta+\frac{h}{\tan \cdot \alpha}\right)}\left\{(m+n)(m+n-1) \delta^{2}+\left[\frac{h}{\tan \cdot \alpha}+(2 m+2 n-1) \delta\right] \frac{h}{\tan \cdot \alpha}\right\}$.

The flanges require a full load to produce in them the maximum strain. The compression of the $n$th upper section is $\frac{\left(p+p^{\prime} h\right)}{2 \tan .^{2} \alpha}+\frac{\left(p+p^{\prime}\right) \delta}{\tan . \alpha}\left(m-n+\frac{1}{2}\right)+\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left[m^{2}-(n-1)^{2}\right]$; which gives for the first which is the most heavily loaded $\frac{p+p^{\prime}}{2}\left[\frac{h}{\tan .^{2} \alpha}+\frac{(2 m-1) \delta}{\tan . \alpha}+\frac{m^{2} \delta^{\ell}}{h}\right]$.

Tension of the $n$th lower section $=\frac{p+p^{\prime}}{\tan \cdot \alpha}\left(m \delta+\frac{h}{2 \tan . \alpha}\right)+\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left[m^{2}-(n-1)^{2}\right]$; say for the first, $\frac{p+p^{\prime}}{2}\left[\frac{h}{\tan .{ }^{2} \alpha}+\frac{2 m \delta}{\tan \cdot \alpha}+\frac{m^{2} \delta^{2}}{h}\right]$.

Multiplying the strain upon a strut by its length and by the quantity $t \mathrm{~V}$, we shall have its weight. Then, summing the expression obtained, in which $n$ takes the successive values $1,2,3, \ldots m$, doubling the result, and adding the two extreme struts (the order of which is $m+1$ ) calculated anart, we find: Total weight of the $2(m+1)$ struts

$$
\begin{aligned}
= & \frac{p h t \mathrm{~V}}{\sin ^{2} \alpha}\left[m(m+2) \delta+\frac{h}{\tan \cdot \alpha}\right]+\frac{m p^{\prime} h t \mathrm{~V}}{2 \sin .^{2} \alpha\left(m \delta+\frac{h}{\tan \cdot \alpha}\right)} \\
& \left\{\frac{\delta^{2}}{3}\left(7 m^{2}+12 m-1\right)+\frac{h}{\tan \cdot \alpha}\left[\frac{(m+2) h}{m \tan \cdot \alpha}+3(m+2) \delta\right]\right\} .
\end{aligned}
$$

We have next, without a coefficient of stiffness: Total weight of the $2 m$ braces

$$
=\frac{m^{2} p \delta h t}{\sin .^{2} \beta}+\frac{m p^{\prime} h t}{2 \sin ^{2} \beta\left(m \delta+\frac{h}{\tan . \alpha}\right)}\left[\frac{\delta^{2}}{3}\left(7 m^{2}-1\right)+\frac{h}{\tan \cdot \alpha}\left(\frac{h}{\tan \cdot \alpha}+3 m \delta\right)\right]
$$

Then with coefficient U , for joint-plates, and so on :
Total weight of the upper flange $=m\left(p+p^{\prime}\right) \delta t \mathrm{U}\left[\frac{h}{\tan .^{2} \alpha}+\frac{m \delta}{\tan . \alpha}+\frac{\delta^{2}}{6 h}(4 m-1)(m+1)\right]$ :
Total weight of the lower flange

$$
=\frac{\left(p+p^{\prime}\right) h^{2} t \mathrm{U}}{\tan .^{3} \alpha}+m\left(p+p^{\prime}\right) \delta t \mathrm{U}\left[\frac{3 h}{\tan .^{2} \alpha}+\frac{3 m \delta}{\tan \cdot \alpha}+\frac{\delta^{2}}{6 h}(4 m+1)(m-1)\right] ;
$$

Total weight of the two flanges together $=\frac{\left(p+p^{\prime}\right) t \mathrm{U}}{3}\left(\frac{l^{3}}{2 h}-\frac{m \delta^{3}}{h}-\frac{h^{2}}{\tan .^{3} \alpha}\right)$.
This last result may be obtained directly by means of the area of the locus of the moments of rupture (see end of 6).
46. Particular case $\alpha=\beta$, and consequently $\frac{h}{\tan . \alpha}=\frac{8}{2}$, and the bearing $l=(2 m+1) \delta$.

Strain upon the $n$th strut or the $n$th brace $=\frac{(2 n-1) p \delta}{2} \frac{p^{\prime} \delta\left[4(m+n)^{2}-1\right]}{8(2 m+1) \sin . \alpha}$;
Except the extreme strut, of order $m+1$, which supports $\frac{(p+p) \delta}{\sin . \alpha}\left(m+\frac{1}{4}\right)$;
Compression of the $n$th end of upper flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{2}-n^{2}+m+n-\frac{1}{4}\right)$;
Maximam strain of upper flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{2}+m-\frac{1}{4}\right)$;
Tension of the $n$th and of lower flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{2}-n^{2}+m+n-\frac{1}{4}\right)$;

Maximum strain upon lower flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{2}+m+\frac{1}{4}\right)$.
Total weight of the $2(m+1)$ struts $=\frac{p h \delta t \mathrm{~V}}{\sin .^{2} \alpha}\left(m^{2}+2 m+\frac{1}{2}\right)+\frac{p^{\prime} \delta h t \mathrm{~V}\left(28 m^{2}+18 m-1\right)}{12 \sin .^{2} a(2 m+1)}$;
Total weight of the $2 m$ braces $=\frac{m^{2} p h \delta t}{\sin .^{2} \alpha}+\frac{m p^{\prime} h \delta t\left(28 m^{2}+18 m-1\right)}{12 \sin .^{2} \alpha(2 m+1)}$.
If we substitute for the coefficient V , with which the struts alone are affected, a coefficient $\frac{1+\mathrm{V}}{2}$ applied to the whole of the lattice, we shall have:
Total weight of the lattice $=\frac{(2 m+1)^{2}\left(p+p^{\prime}\right) \delta h t(1+\mathrm{V})}{4 \sin .^{2} \alpha}\left[1+\frac{m\left(4 m^{2}+6 m_{6}-1\right)}{3(2 m+1)^{3}} q\right]$.
And for the flanges, we have:
Weight of the upper flange $=\frac{m\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{12 h}\left(8 m^{2}+12 m+1\right)$;
Weight of the lower flange $=\frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{12 \bar{h}}\left(8 m^{3}+12 m^{2}+7 m+\begin{array}{l}3 \\ 2\end{array}\right) ;$
Weight of the two flanges together $=\frac{\left(p+p^{\prime}\right)}{6 h} \delta^{3} t \mathrm{U}\left(8 m^{3}+12 m^{2}+4 m+\frac{3}{4}\right)$.
If $\alpha$ were of $45^{\circ}$, we should substitute $\frac{1}{\sqrt{2}}$ for sin. $a$, and $2 h$ for $\delta$; we might, besides, substitute for $m$ the value $\frac{6-2 h}{4 h}$.
47. Another particular case : $\beta=90^{\circ}$. We have $\sin . \beta=1, \frac{h}{\tan . \alpha}=\delta$; the bearing becomes $2(m+1) \delta$, say $2 m^{\prime} \delta$, making $m=m^{\prime}-1$. In the following values, $\frac{h}{\sqrt{h^{2}-\delta^{2}}}$ may be substituted
for $\sin$.
Strain upon the $n$th strut (including the last, of order $m^{\prime}$ )

$$
=\frac{(2 n-1) p \delta}{2 \sin . \alpha}+\frac{\left(m^{\prime}+n\right)\left(m^{\prime}+n-1\right)}{4 m^{\prime} \sin . \alpha} p^{\prime} \delta ;
$$

Strain upon the $n$th brace $=\frac{(2 n-1) p o}{2}+\frac{\left(m^{\prime}+n\right)\left(m^{\prime}+n-1\right) p \delta}{4 m}$;
Strain upon the $n$th end of the upper flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{\prime 2}-n^{2}\right)$;
Maximum strain upon the upper flange $=\frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}\left(m^{\prime 2}-1\right)$;
Maximum strain upon the $n$th end of lower flange $=\frac{\left(p+p^{\prime}\right) \delta^{3}}{2 h}\left(m^{\prime}+n-1\right)\left(m^{\prime}-n+1\right)$;
Maximum strain upon the lower flange $=m^{\prime 2} \frac{\left(p+p^{\prime}\right) \delta^{3}}{2 h}=\frac{\left(p+p^{\prime}\right) l^{2}}{8 h}$;
Total weight of the $2 m^{\prime}$ struts $=\frac{h \delta t \mathrm{~V}}{\sin .^{2} \alpha}\left[m^{\prime 2} p+\frac{7 m^{\prime 2}-1}{6} p^{\prime}\right]$;
Total weight of the $2\left(m^{\prime}-1\right)$ braces $=\left(m^{\prime}-1\right) \delta \hbar t\left[\left(m^{\prime}-1\right) p+\frac{7 m^{\prime}-5}{6} p^{\prime}\right]$;
Total weight of the upper flange $=\frac{m^{\prime}\left(m^{\prime}-1\right)\left(4 m^{\prime}+1\right)\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U} \text {; } ; 6 h}{}$
Total weight of the lower flange $=m^{\prime}\left(m^{\prime}+1\right)\left(4 m^{\prime}-1\right) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{6}$;
Total weight of the two flanges together $=m^{\prime}\left(4 m^{\prime 2}-1\right) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{3 h}$.
48. In the particular case $\alpha=90^{\circ}$, the two middle struts will become one. For $l$ and $\delta$ constant, this case will be a little less economical than the preceding.
49. Second arrangement, Fig. 1654. This arrangement differs from the preceding only in the fact that the loaded point falls in the middle of the girder. The dotted triangle in the middle resists only beneath the action of a partial load; it offers a defect of symmetry if $\alpha$ is not equal to $\beta$. The weights applied to the extreme summits are inferior to the others, as in the preceding arrangements (see 45).
Strain upon the $n$th strut
$=\frac{p n \delta}{\sin . \alpha}+\frac{p^{\prime}}{2 \sin . \alpha\left(2 m \delta-\delta+\frac{2 h}{\tan . \alpha}\right)}\left\{\delta^{2}(m+n)(m+n-1)+\frac{h}{\tan . \alpha}\left[\frac{}{\tan . \alpha}+\delta(2 m+2 n-1)\right]\right\} ;$
Except the last (the $m$ th), which supports $\frac{p+p^{\prime}}{\sin . \alpha}\left[(2 m-1) \delta+\frac{h}{\text { tan. } \alpha}\right]$;


Strain upon the $n$th brace

$$
=\frac{p n \delta}{\sin \cdot \beta}+\frac{p^{\prime}}{2 \sin . \beta\left(2 m \delta-\delta+\frac{2 h}{\tan \cdot \alpha}\right)}\left\{\delta^{2}(m+n)(m+n-1)+\frac{h}{\tan \cdot \alpha}\left[\frac{h}{\tan \cdot \alpha}+\delta(2 m+2 n-1)\right]\right\}
$$

Strain upon the $n$th section of the upper flange

$$
=(m-n+1)(m+n-2) \frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}+\frac{p+p^{\prime}}{2 \tan \cdot \alpha}\left[\delta(2 m-2 n+1)+\frac{}{\tan \cdot \alpha}\right]
$$

Which gives for the middle section, the most heavily loaded,

$$
\frac{m(m-1)\left(p+p^{\prime}\right) \delta^{2}}{2 h}+\frac{p+p^{\prime}}{2 \tan . \alpha}\left[(2 m-1) \delta+\frac{h}{\tan . \alpha}\right] ;
$$

Strain upon the $n$th section of the lower flange

$$
=(m+n-1)(m-n) \frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}+\frac{p+p^{\prime}}{2 \tan . \alpha}\left[(2 m-1) \delta+\frac{h}{\tan . \alpha}\right]
$$

Which gives in the middle the same maximum strain as in the upper flange.
Total weight of the $2 m+1$ struts, inclined at an angle $\alpha$,

$$
\begin{gathered}
=\frac{p h t \mathrm{~V}}{\sin ^{2} \alpha}\left[\frac{h}{\tan \cdot \alpha}+\delta\left(m^{2}+m-1\right)\right]+\frac{p^{\prime} h t \mathrm{~V}}{6 \sin ^{2} \alpha\left(2 m \delta-\delta+\frac{2 h}{\tan . \alpha}\right)} \\
\left\{\delta^{2}\left(14 m^{3}+3 m^{2}-17 m+6\right)+\frac{3 h}{\tan . \alpha}\left[\frac{(2 m+3) h}{\tan . \alpha}+\delta\left(6 m^{2}+6 m-5\right)\right]\right\}
\end{gathered}
$$

Total weight of the $2 m-1$ braces, inclined at an angle $\beta$,

$$
\begin{gathered}
=\frac{m(m-1) p \delta h t}{\sin ^{2} \beta}+\frac{p^{\prime} h t}{6 \sin ^{2} \beta\left(2 m \delta-\delta+\frac{2 h}{\tan . \alpha}\right)} \\
\left\{\delta^{2}\left(14 m^{3}-21 m^{2}+7 m\right)+\frac{3 h}{\tan \cdot \alpha}\left[\frac{(2 m-1) h}{\tan \cdot \alpha}+\delta\left(6 m^{2}-6 m+1\right)\right]\right\}
\end{gathered}
$$

Total weight of the upper flange

$$
=m(4 m+1)(m-1) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{6 h}+\left[\frac{(2 m-1) h}{2 \tan \cdot u}+\left(m^{2}-m+\frac{\mathbf{1}}{2}\right) \delta\right] \frac{\left(p+p^{\prime}\right) \delta t \mathrm{U}}{\tan . \alpha} ;
$$

Total weight of the lower flange

$$
\begin{aligned}
&=m(m-1)(4 m-5) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{6 h}+\left[\frac{3(2 m-1) h}{2 \tan \cdot \alpha}+\left(3 m^{2}-3 m+\frac{1}{}\right) \delta\right] . \\
& \frac{\left(p+p^{\prime}\right) \delta t \mathrm{U}}{\tan \cdot \alpha}+\frac{\left(p+p^{\prime}\right) h^{2} t \mathrm{U}}{\tan \cdot{ }^{\circ} \alpha} ;
\end{aligned}
$$

Total weight of the two flanges together

$$
=(2 m-1)\left(p+p^{\prime}\right) \delta t \mathbb{U}\left[\frac{2 m(m-1) \delta^{2}}{3 h}-\frac{l}{\tan \cdot \alpha}\right]+\frac{\left(p+p^{\prime}\right) h^{2} t \mathbf{U}}{\tan .^{3} \alpha} .
$$

50. Particular case $\alpha=\beta$, and consequently $\frac{h}{\tan \cdot \alpha}=\frac{\delta}{2}$. The bearing is then $2 \mathrm{~m} \delta$.

Strain upon the $n$th strut $=\frac{p n \delta}{\sin . \alpha}+\frac{p^{\prime} \delta}{4 m \sin . \alpha}\left[(m+n)^{2}-\frac{1}{4}\right]$
Except the last ' $m$ th), which supports $\left(p+p^{\prime}\right) \frac{(4 m-1) \delta}{4 \sin . \alpha}$;
Strain upon the $n$th brace $=\frac{p n \delta}{\sin . \alpha}+\frac{p^{\prime} \delta}{4 m \sin . \alpha}\left[(m+n)^{2}-\frac{1}{4}\right]$;
Strain upon the $n$th section of the upper flange $=\left(m^{2}-n^{2}+2 n-\frac{5}{4}\right) \frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}$;

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Strain upon the $n$th section of the lower flange $=\left(m^{2}-n^{2}+n-\frac{1}{4}\right) \frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}$ :
Maximum strain in the middle of the flanges $=\left(m^{2}-\frac{1}{4}\right) \frac{\left(p+p^{\prime}\right) \delta^{2}}{2 h}$;
Total weight of the $2 m+1$ struts (including one side of the middle triangle)

$$
=\frac{\left(2 m^{2}+2 m-1\right) p \delta h t \mathrm{~V}}{2 \sin .^{2} \alpha}+\frac{\left(56 m^{3}+48 m^{2}-26 m+3\right) p^{\prime} \delta h t \mathrm{~V}}{48 m \sin .^{2} \alpha} ;
$$

Total weight of the $2 m-1$ braces (including one side of the middle triangle)

$$
=\frac{m(m-1) p \delta h t}{\sin .^{2} \alpha}+\frac{\left(56 m^{2}-48 m^{2}-2 m+3\right) p^{\prime} \delta h t}{48 m \sin .^{2} \alpha} ;
$$

Total weight of the whole lattice, applying the coefficient $\frac{1+\mathrm{V}}{2}$,

$$
=\frac{\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \delta h t(1+\mathrm{V})}{4 \sin .^{2} \alpha}\left[1+\frac{8 m^{3}-2 m+3}{12 m\left(4 m^{2}-1\right)} q\right] ;
$$

Total weight of the upper flange $=\left(16 m^{3}-10 m+3\right) \frac{\left(p+p^{\prime}\right)}{24} \frac{\delta^{3} t \mathrm{U}}{2}$;
Total weight of the lower flange $=m\left(8 m^{2}+1\right) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{12 h}$;
Total weight of the two flanges together $=\left(32 m^{3}-8 m+3\right) \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{24 h}$.
The weight of the lower flange exceeds by $\frac{\left(4 m^{2}-1\right)}{8 h} \frac{\left(p+p^{\prime}\right) \delta^{3} t \mathrm{U}}{8 h}$ that of the upper flange.
If the inclination of the bars is $45^{\circ}$, the value of the strains upon them will be

$$
h \sqrt{2}\left[2 p n+\frac{p^{\prime}}{2 n}\left(m+n+\frac{1}{2}\right)\left(m+n-\frac{1}{2}\right)\right]
$$

with the exception of the last or $n$th strut, which will support $\left(2 m-\frac{1}{2}\right)\left(p+p^{\prime}\right) h \sqrt{2}$. The total weight of the lattice is $=\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) h^{2} t(1+\mathrm{V})\left[1+\frac{8 m^{3}-2 m+3}{12 m\left(4 m^{2}-1\right)} q\right]$. In the strains upon the flanges $\frac{\delta^{2}}{2 h}$ should be changed into $2 h$, and in their weight $\frac{\delta^{3}}{h}$ into $8 h^{2}$. And in all the expressions the quantity $\frac{l}{4 h}$ may be substituted for $m$.
51. The particular case $\beta=90^{\circ}$, will offer a defect of symmetry in the middle, like the general case. The bearing $l$ becomes $(2 m+1) \delta$.

The strain upon the $n$th vertical brace is $p n \delta+p^{\prime} \delta \frac{(m+n)(m+n+1)}{2(2 m+1)}$, and that of the $n$th strut is given by the same formula divided by $\sin . \alpha$.

The $n$th section of the upper or the lower flange supports

$$
(m+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}
$$

Total weight of the $2 m+1$ struts $=\frac{m(m+1) \delta h t \mathrm{~V}}{\sin ^{2} \alpha}\left(p+\frac{7}{6} p^{\prime}\right)$;
Total weight of the $2 m-1$ braces $=m \delta h t\left((m-1) p+\frac{7 m-5}{6} p^{\prime}\right)$;
Total weight of the upper flange $=m(m+1)(4 m-1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h}$;
Total weight of the lower flange $=m(m+1)(4 m+5)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h}$;
Total weight of the two flanges together $=\frac{2}{3} m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t U}{h}$.
52. Third arrangement, Fig. 1655. This arrangement differs from the first, in the fact of the struts being equal in number to the braces, which will simplify the formulæ. As the girder ends with a brace, it may be supported on the ends of the upper flange, as shown in the figure.

The weights applied to the upper summits are all equal to $p \delta$ for the permanent portion, and to $p^{\prime} \delta$ for the moving load. The maximum reaction of the abutment, not including the weight which it bears directly has the value $\left(m-\frac{1}{2}\right)\left(p+p^{\prime}\right) \delta$.


Strain upon any $n$th strut $=\frac{\delta}{2 \sin . \alpha}\left[(2 n-1) p+(m+n)(m+n-1) \frac{p^{\prime}}{2 m}\right] ;$
Strain upon the $n$th brace $=\frac{\delta}{2 \sin . \beta}\left[(2 n-1) p+(m+n)(m+n-1) \frac{p}{2 m}\right]$;
Strain upon the $n$th section of the upper flange $=\frac{\left(p+p^{\prime}\right) \delta}{2}\left[\left(m^{2}-n^{2}\right) \frac{\delta}{h}+\frac{(2 n-1)}{\tan . \beta}\right]$;
Which gives for the first towards the middle, $\frac{\left(p+p^{\prime}\right) \delta}{2}\left[\left(m^{2}-1\right) \frac{\delta}{h}+\frac{1}{\tan \cdot \beta}\right]$;
Strain upon the $n$th section of the lower flange $=(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Which gives for the maximum strain in the middle, $\frac{m^{2}\left(p+p^{\prime}\right) \delta^{2}}{2 h}$, or $\frac{\left(p+p^{\prime}\right) l^{2}}{8 h}$;
Total weight of the $2 m$ struts $=\frac{\delta h t \mathrm{~V}}{\sin ^{2} \alpha}\left[m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right]$;
Total weight of the $2 m$ braces $=\frac{\delta h t}{\sin ^{2} \beta}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$;
Total weight of the upper flange $=m\left(p+p^{\prime}\right) \delta^{2}\left[\frac{(4 m+1)(m-1) \delta}{6 h}+\frac{m}{\tan . \beta}\right] t \mathbf{U}$;
Total weight of the lower flange $=m\left(p+p^{\prime}\right) \delta^{2}\left[\frac{(4 m-1)(m+1) \delta}{6 h}-\frac{m}{\tan . \beta}\right] t \mathbf{U}$ :
Total weight of the tro flanges together $=m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathbf{U}}{3 h}$.
The difference between the weights of the upper and lower flanges is expressed by $m^{2}\left(p+p^{\prime}\right) \delta^{2} t \mathbf{U}\left(\frac{2}{\tan . \beta}-\frac{\delta}{h}\right)$.
53. Particular case in which $\alpha=\beta$. The quantity $\frac{h}{\tan . \beta}$ becomes $\frac{\delta}{2}$.

Strain upon the $n$th struts and braces $=\frac{\delta}{2 \sin . \alpha}\left[(2 n-1) p+(m+n)(m+n-1) \frac{p^{\prime}}{2 m}\right]$;
Strain upon the $n$th section of the upper flange $=\left(m^{2}-n^{2}+n-\frac{1}{2}\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Strain upon the $n$th section of the lower flange $=(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Total weight of the $2 m$ braces $=\left(p+p^{\prime}\right) \frac{m^{2} \delta h t}{\sin .^{2} \alpha}\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)$;
Total weight of the 2 m struts $=$ the preceding expression multiplied by $\mathbf{V}$;
Volume of the upper flange $=$ volume of the lower $=m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h}$.
If the inclination of the bars is $45^{\circ}$, the strains upon them will be calculated by the formula $h \sqrt{ } 2\left[(2 n-1) p+(m+n)(m+n-1) \frac{p^{\prime}}{2 m}\right]$, and their total weight, struts and braces together, by $4 m^{2}\left(p+p^{\prime}\right) h^{2} t(1+\mathrm{V})\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)$.

The total weight of the two flanges will be $8 m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{h^{2} t \mathrm{U}}{3}$, and in the expression of their strains $\frac{\delta^{2}}{2 h}$ should be changed into $2 h$. We may also substitute $\frac{l}{4 h}$ for $m$.
54. Particular case in which $\beta=90^{\circ}$. The strain upon the extreme ends of the upper flange is rendered nul, and the extreme braces are simple suspension-rods, necessary only on the hypothesis that the point of support upon the abutments is taken at the height of the upper

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flange. But if these suspension-rods be suppressed and the girder supported by the bottom, we shall have exactly the case in 47. Our present formulæ would give $\delta h t\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ as the total weight of the braces, including the suspension-rods; but the suppression of these latter will reduce the weight to $(m-1)^{2} \delta h t\left(p+\frac{7 m-5}{6(m+1)} p^{\prime}\right)$, as in the section above referred to.

In the case in which $\alpha=90^{\circ}$, the two struts in the middle become one. The hypotheses $a=90^{\circ}$ or $\beta=90^{\circ}$, introduced into the general formulæ of 52 , both give the same total weight for the flanges and for the lattice.
55. Fourth arrangement, Fig. 1656. This figure differs from the preceding, in the fact that the middle of the girder does not coincide with one of the loaded points, but falls in the middle

of an interval. The dotted triangle, in the middle, offers a defect of symmetry, if $\alpha$ differs from $\beta$; it does not resist when the load extends throughout the length; one of the bars which compose it is regarded as a strut, the other as a brace.
Strain upon the $n$th strut $=\frac{\delta}{\sin . \alpha}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right] ;$
Strain upon the $n$th brace $=\frac{\delta}{\sin . \beta}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right]$;
Strain upon the $n$th upper horizontal section

$$
=(m-n+1)(m+n)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}+(n-1)\left(p+p^{\prime}\right) \frac{\delta}{\tan . \beta}
$$

Strain upon the $n$th lower horizontal section $=(m-n+)(m+n)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Maximum strain in the middle of the flanges $=m(m+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Total weight of the $2 m+1$ struts $=\frac{m(m+1)}{\sin ^{2} \alpha} \frac{\delta h t \mathbf{V}}{}\left(\mu+\frac{7}{6}\right)$;
Total weight of the $2 m+1$ braces $=\frac{m(m+1) \delta h t}{\sin .^{2} \beta}\left(p+\frac{7}{6} p^{\prime}\right)$;
Total weight of the upper flange $=m(m+1)\left(p+p^{\prime}\right) \delta^{2} t \mathrm{U}\left[\frac{(4 m-1) \delta}{6 h}+\frac{1}{\tan . \beta}\right]$;
Total weight of the lower flange $=m(m+1)\left(p+p^{\prime}\right) \delta^{2} t \mathrm{U}\left[\frac{(4 m+5) \delta}{6 h}-\frac{1}{\tan . \beta}\right]$;
Total weight of the two flanges together $=\frac{2}{3} m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{h}$.
56. Case in which $\alpha=\beta$.

Strain upon the lattice-bars $=\frac{\delta}{\sin \cdot \alpha}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right] ;$
Strain upon the $n$th upper horizontal section $=\left(m^{2}-n^{2}+m+2 n-1\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Strain upon the lower horizontal section $=(m+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$;
Total weight of the lattice $=\frac{m(m+1) \delta h t(1+\mathrm{V})}{\sin .^{2}}\left(p+\frac{7}{6} p^{\prime}\right)$;
Total weight of the upper flange $=$ weight of the lower flange

$$
=m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{3 h}
$$

If the inclination is $45^{\circ}, \frac{1}{\sqrt{2}}$ may be substituted for $\sin$. $\alpha$, and $2 h$ for $\delta$. The quantity $m$ is then equal to $\frac{l}{4 h}-\frac{1}{2}$.
57. The case in which $\beta=90^{\circ}$ is similar to the corresponding case of the second arrangement in 51. There will be nothing changed except the total weight of the braces, which will amount

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here to $m(m+1) \delta h t\left(p+\frac{7}{6} p^{\prime}\right)$, on account of their number being $2 m$ instead of $2(m-1)$, the extreme braces serving only to suspend the girder from the upper points of support.
58. Girders uniformly loaded on the lower side.-If we turn upside down the various systems which we have been examining, the load will be transposed to the lower side, and we shall have merely to apply the same formulæ with the signs of the strains changed; the lower flange will have become the upper, the struts braces, and vice versâ. We must also change $\alpha$ into $\beta$, and $\beta$ into $\alpha$, the symbol $\alpha$ being always applied to struts and $\beta$ to braces.

We shall consider only the two arrangements shown in Figs. 1657 and 1658. The first is obtained by reversing the third arrangement of girders loaded on the upper side, Fig. 1655, and the second by reversing the fourth arrangement, Fig. 1656.


The formulæ applicable to these two arrangements and to their particular cases will be found in Table VIII.
59. Inclination of the Lattice-bars.-When the inclination $\alpha$ of the bars is quite arbitrary, it should be taken equal to $45^{\circ}$, for a bar crossing an interval $\delta$, will weigh $\frac{F t \delta_{1}}{\sin . a \cos a}$, which amounts to $\frac{F t}{\sin . \alpha \cos . \alpha}$ the lineal mètre of girder : now this expression is a minimum when $\alpha=45^{\circ}$.

But simple lattice-girders are usually divided into intervals of a given length $\delta$, which form the base of triangles the other sides of which are a strut inclined to an angle $\alpha$ and a brace inclined to $\beta$, so that these two angles $\alpha$ and $\beta$ are an implicit function one of the other, for they are connected by the relation $h(\cot . \alpha+\cot \beta)=\delta$, whence $\frac{d \beta}{d \alpha}=-\frac{\sin ^{2} \beta}{\sin ^{2}{ }^{2} \alpha}, \delta$ and $h$ being regarded as given constants. The weight of the two bars of one of the triangles has the value $\mathrm{F} h t\left(\frac{\mathrm{~V}}{\sin .^{2} \alpha}+\frac{1}{\sin .^{2} \beta}\right)$, and it becomes a minimum when we have $\frac{\mathrm{V} \cos . \alpha}{\sin .^{3} \alpha}+\frac{\cos \cdot \beta}{\sin .^{3} \beta} \cdot \frac{d \beta}{d \alpha}=0$, whence $\frac{\tan . \alpha}{\tan . \beta}=\mathrm{V}$. It would, therefore, be necessary to give the struts an inclination nearer the vertical than the braces.
60. It is easy to solve the question from a more general point of view, by attributing distinct coefficients to each of the two flanges and to each of the two systems of bars, as if, for example, these different parts were made of different materials. On this hypothesis let $\xi$ denote, for the upper flange, the value of the lineal mètre of a prism capable of bearing a net strain of 1 kilogramme, taking into account the joint-plates and other accessories. Let $\xi_{1}, \xi^{\prime}$, and $\xi^{\prime \prime}$ be the analogous quantities relative to the lower flange, the struts, and the braces. And let us take, by way of example, a girder loaded on the lower side of the system of Fig. 1657, in which $b$ is expressed by $2 m \delta$.

The total value $\mathbf{P}$ of the girder will be given by the formulæ of weights of Table VIII., by suppressing the factor $t$ and the coefficients U V , and substituting for them the new coefficients $\xi$, and so on. If we put

$$
\left(1+\frac{m^{2}-1}{6 m^{2}} q\right) \xi^{\prime}=a,\left(1+\frac{m^{2}-1}{6 m^{2}} q\right) \xi^{\prime \prime}=a^{\prime}, \frac{4 m^{2}-1}{6 m}\left(\xi+\xi_{1}\right)+\frac{1}{2}\left(\xi_{1}-\xi\right)=c,
$$

and $\frac{4 m^{2}-1}{6 m}\left(\xi+\xi_{1}\right)-\frac{1}{2}\left(\xi_{1}-\xi\right)=c^{\prime}$, and substitute the value $\frac{\delta}{\cot \alpha+\cot . \beta}$ for $h$, and $\frac{l}{2 m}$ for $\delta$, we shall have $\mathrm{P}=\frac{1}{4}\left(p+p^{\prime}\right) l^{2}\left\{\frac{1}{\cot \alpha+\cot . \beta}\left(\frac{a}{\sin ^{2} \alpha}+\frac{a^{\prime}}{\sin ^{2} \beta}\right)+c \cot . \alpha+c^{\prime} \cot . \beta\right\}+\Omega$;
$\Omega$ is the additional term intended to include accessory pieces, such as gussets, and so on, regarded as independent of $h$, as well as of the angles $\alpha$ and $\beta$. The quantities $\alpha, a^{\prime}, c, c^{\prime}$, are also constants, for $\delta$, and consequently $m$, is given; the height $h$ is arbitrary, but as it has been eliminated, there remain only the two independent variables $\alpha$ and $\beta$. For the minimum expense, the partial derivatives of P , relative to $\alpha$ and to $\beta$, must be nul separately, which will give the equations

$$
\begin{aligned}
& \quad(a+c) \operatorname{cot.}^{2} \alpha+\left(c-a^{\prime}\right) \cot ^{2} \beta+2(a+c) \cot \cdot \alpha \cot \beta-a-a^{\prime}=0 \\
& \text { and } \quad(c-a) \cot ^{2} \alpha+\left(a^{\prime}+c^{\prime}\right) \cot ^{2} \beta+2\left(\alpha^{\prime}+c^{\prime}\right) \cot . \alpha \cot \beta-a-a^{\prime}=0 .
\end{aligned}
$$

Subtracting one from the other, member by member, we obtain an equation of the second degree to determine the proportion of the cotangents or of the tangents. We deduce from it $\frac{\tan \cdot \alpha}{\tan . \beta}=-1$, first value inadmissible, and $\frac{\tan . \alpha}{\tan . \beta}=$

$$
\frac{2 a+c-c}{2 a^{\prime}-c+c^{\prime}}=\frac{2 \xi^{\prime}\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)+\xi_{1}-\xi}{2 \xi^{\prime \prime}\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)-\xi_{1}+\xi}
$$

Elimination leads to

$$
\tan . \alpha=\frac{\mathrm{Z}}{2 \xi^{\prime \prime}\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)+\xi-\xi_{1}} \text { and } \tan , \beta=\frac{\mathrm{Z}}{2 \xi^{\prime}\left(1+\frac{m^{2}}{6 m^{2}} q\right)+\xi_{1}-\xi},
$$

Z denoting the radical

$$
\begin{gathered}
\sqrt{4 \xi^{\prime} \xi^{\prime \prime}\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)^{2}+2\left(\xi_{1}-\xi\right)\left(\xi^{\prime \prime}-\xi^{\prime}\right)\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)-\left(\xi_{1}-\xi\right)^{2}} \\
+\frac{2\left(4 m^{2}-1\right)}{3 m}\left(\xi+\xi_{1}\right)\left(\xi^{\prime}-\delta^{\prime \prime}\right)\left(1+\frac{m^{2}-1}{6 m^{2}} q\right) .
\end{gathered}
$$

Consequently, the dependent variable $h$ will have, as a minimum, the value

$$
h_{1}=\frac{l Z}{4 m\left(\xi^{\prime}+\xi^{\prime \prime}\right)\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)}
$$

In a girder consisting entirely of iron we have not to distinguish different materials, but the distinction of coefficients may exist from other causes. If, for example, we wish to impose upon the upper flange $\frac{1}{\mathrm{~K}}$ of the coefficient R imposed upon the lower one, we must substitute $\mathrm{K} \mathrm{U} t$ for $\xi, \mathrm{U} t$ for $\xi_{1}, \mathrm{~V} t$ for $\xi^{\prime}$, and $t$ for $\xi^{\prime \prime}, \mathrm{P}$ then denoting the total weight of the girder. But it must be remarked that the preceding calculations suppose that we may regard as constant the coefficients $U$ and $V$, whilst it would be more exact to consider them as functions of the height. With this observation, when both flanges are affected with the same coefficient $(K=1)$, we shall have

$$
\frac{\tan . \alpha}{\tan . \beta}=\mathrm{V}, h_{1}=\frac{1}{2 m(\mathrm{~V}+1)} \sqrt{\mathrm{V}+2 m \frac{4 m^{2}-1}{6 m^{2}+\left(m^{2}-1\right) q}(\mathrm{~V}+1) \mathrm{U}}
$$

61. Analogous calculations for the case in which $l=(2 m+1) \delta$, Fig. 1539, would give as a minimum

$$
\frac{\tan . \alpha}{\tan . \beta}=\frac{2 \xi^{\prime}\left(1+\frac{1}{6} q\right)+\xi_{1}-\xi}{2 \xi^{\prime \prime}\left(1+\frac{1}{6} q\right)+\xi-\xi_{1}}, \text { and } h_{1}=\frac{l \mathrm{Z}}{2(2 m+1)\left(\xi^{\prime}+\xi^{\prime \prime}\right)\left(1+\frac{1}{6} q\right)^{\prime}}
$$

Z denoting the radical

$$
\sqrt{4\left(1+\frac{1}{6} q\right)\left[\xi^{\prime} \xi^{\prime \prime}\left(1+\frac{1}{6} q\right)+\xi \xi^{\prime}+\xi_{1} \xi^{\prime \prime}+\frac{2}{3}(4 m-1)\left(\xi+\xi_{1}\right)\left(\xi^{\prime}+\xi^{\prime \prime}\right)\right]+\left(\xi-\xi_{1}\right)^{2}}
$$

For bridges constructed of plate iron, we should make $\xi=\xi_{1}=\mathrm{U} t, \xi^{\prime}=\mathrm{V} t$ and $\xi^{\prime \prime}=t$; and, consequently, we slould take, U and $V$ being regarded as nearly constant,

$$
\frac{\tan . \alpha}{\tan . \beta}=\mathrm{V}, \text { and } h_{1}=\frac{l}{(2 m+1)(\mathrm{V}+1)} \sqrt{\mathrm{V}+\frac{(16 m-1) \mathrm{U}(\mathrm{~V}+1)}{3+\frac{1}{2} q}}
$$

But as a function varies little when near a minimum, we need not bind ourselves to follow strictly the rules obtained for the best proportion of the inclinations $\alpha$ and $\beta$. Thus they are generally made equal to each other, which is of a more satisfactory aspect.
62. Deflection.-The deflection of a simple lattice under the influence of a certain load will, in general, be easy to determine, when we have the sections of all the pieces and the exact figure of the system under a determinate load given. Then the load, the effect of which we wish to consider, will produce upon the several parts extensions or centractions easy to calculate, so that we can know the length of all the pieces after deflection, and the angles may afterwards be found by trigonometrical formulæ. The determining figure being known, we may deduce the curve of the flanges, the inclination at their extremities, and so on.

Cocfficient U applicable to the Flanges.- Coefficient of stiffness $V$.-Supplementary term $\Omega$.-General form of the Formula for Weight.-63. Coefficient U. This coefficient is intended to take into account the joint-plates of the flanges, and, beyond this, the impossibility, in practice, of reducing the latter exactly to the dimensions given by calculation.

1. Joint-plates.-The narrower an iron plate is, the longer it may be, so that small girders will cost proportionally less in joint-plates than large ones. But even for these latter, with plates of 0.80 mètre broad, for example, we may reckon upon lengths of 6 mètres. If at each joint we have a distinct joint-plate of 0.80 mètre long ( 0.40 mètre on each side of the joint), the quantity to bo added the mètre of plate would be $\frac{0.80}{6}=0.134$; but we may reduce this figure to 0.11 , in consideration of the fact that the angle-irons of the flange may reach lengths of more than 12 mètres, and, consequently, will cost less in joint-plates than the plates of the flanges. This additional quantity of 0.11 may be regarded as the highest limit of the cost of joint-plates in the case of flanges of inconsiderable section composed of a single horizontal plate.

But when the maximum moment of rupture requires a strong section of flange, they are composed of several plates placed one upon another, and in that case a single joint-plate may serve for several joints placed after each other at intervals of 0.40 mètre. For example, a joint-plate of $1 \cdot 60$ mètre may cover three joints, which will reduce the additional quantity to 0.089 . If we consider as an hypothetical limit an extremely low girder, such that the number of flange-plates shall be very great, the joint-plates would be as long as possible, and would have a tendency to weigh, for each joint covered, only the half of the weight of a distinct joint-plate of 0.80 mètre. We may thus include the joint-plates in our calculations by applying to the flanges a coefficient varying from $1 \cdot 06$ to $1 \cdot 11$, according as these latter are of great thickness (as if the height were nul), or, on the contrary, reduced to the minimum section that may practically be adopted without depriving the girder of its necessary rigidity, having regard to its other dimensions. These figures may be reduced a little in the case of plates 0.30 or 0.40 mètre broad, for example, which would allow us to reckon upon lengths greater than 6 metres.

In heavy girders the number of plates diminishes towards the extremities, and it will be only in the middle that the coefficient may reach the limit 1.06 mètre. But, to compensate this, it often happens that at the extremities the first joints of the first plates may be covered by increasing slightly the length of the strengthening plates.
2. Excesses of Section.-The section of the flanges cannot be, throughout the length, reduced strictly to the value required by calculation, chiefly on account of the facts that the variation takes place by redans or gradations, and that at those points in which the moment of rupture is inconsiderable, we cannot practically reduce the section below a certain minimum. Instead of a small number of very thick plates, we shall find it a saving of material to employ a larger number of less thickness. But to abstract these thicknesses, let us consider, as above, the limiting cases. If, in the first place, the height were nul, the number of plates of ordinary thickness would be very great, and the redans would be only an imperceptible fraction of the total weight. If, on the contrary, the height reaches a value $H$, such that the weakest section that we wish to give to tho flanges shall be sufficient in the middle, the flange will be prismatic instead of forming a parabolic volume, as the formulæ for the case of a continuous load suppose. To establish the prism in its integrity, we must multiply the value of the formula by a coefficient of 1.50 , affecting generally the joint-plates which increase in the same proportion.
64. Thus the total coefficient $U$ to be applied to the flanges for joint-plates and excesses of section may be taken equal to 1.06 for $h=0$, and to $1 \cdot 11 \times 1 \cdot 50=1.66$ for the limiting height H , which renders necessary a constant section in the flanges. We may, therefore, approximatively adopt, for any value of $h, \mathrm{U}=1 \cdot 06+0 \cdot 60 \frac{h}{\mathrm{H}}$.

Usually the volume of the flanges is inversely proportional to $h$, so that the second term of U will give a quantity independent of $h$. Consequently, in seeking the most economical height, this constant term will have no influence, and we shall have simply to substitute $1 \cdot 06$ for U .

65 . When the load is applied to certain intervals only, we may still admit the preceding expression of $U$. Yet, from the observation in 43 , it will be more exact, if the bearing is divided into N intervals, to multiply the theoretical weight by $1.50 \frac{\mathrm{~N}^{2}}{\mathrm{~N}^{2}-1}$, instead of 1.50 , when there is occasion to return to the prismatic form in the case of the limiting height $H$. The coefficient will then be expressed by the function $\mathrm{U}=1 \cdot 06+\left(\frac{1 \cdot 66 \mathrm{~N}^{2}}{\mathrm{~N}^{2}-1}-1 \cdot 06\right) \frac{h}{\mathrm{H}}$

In simple lattice bridges, $U$ has a tendency to increase slightly on account of the vertical rods serving to fix the bars, for these rods give a certain rigidity to the flanges which affects the resistance by easing the oblique bars, but at the same time by producing in the flanges deflecting strains and an unequal distribution of pressure. The best way to take this into account is to consider the two flanges as forming a solid whole resisting by its summit of resistance.

In girders loaded in a discontinuous manner, theory itself requires that the flanges should vary by sudden leaps from one section to another, but we cannot wholly get rid of the loss occasioned by the redans, on account of the difficulty of making the extra thicknesses prescribed by calculation coincide exactly with the usual thicknesses of plate iron.
66. The limiting height $H$ to be considered in calculating U , may be taken equal to $\frac{\left(p+p^{\prime}\right) l^{2}}{8 \mathrm{R} s_{0}}, s_{0}$ being the minimum section that we wish to give, or that we may give, to the flanges. Consequently the value of $U$. considering only the case of a continuous load, will be $U=1 \cdot 06+4 \cdot 80 \frac{\mathrm{R} s_{0} h}{\left(p+r^{\prime}\right) l^{2}}$.

In a girder composed of triangles, we may regard as nearly constant the ratios $\frac{h}{l}$ and $\frac{s_{0}}{h}$; or at least if $s_{0}$ has a tendency to be proportionally a little stronger in small girders than in large ones, so that U will decrease slightly when $h$, and consequently $l$, increases; on the other hand, small girders cost less in joint-plates, and we may, therefore, regard U as expressed by $1.06+\frac{c}{p+p^{\prime \prime}}$, $c$ being a constant independent of $l$.
67. The preceding remarks concern straight girders. In bow-string girders, the joint-plates will have the same proportional value, but if the lower flange be straight, it will have a constant section, and there will be no loss for redans; besides, the section of the flanges is utilized quite to the extremities, whilst in straight girders there is always an excess of section at the ends. On the other hand, the bow and the chord, or stringer, in a bow-girder are fixed together at the ends by iron plates, which may be considered as forming an integral part of the flanges. When the height is inconsiderable, the strain between the bow and the chord will be great, and the gusset must be very long, but its mean breadth will be less than in a girder having a versed sine or rise of great magnitude. The coefficient $U$ will vary but little in bow-string girders, and may be taken from $1 \cdot 25$ to 1.35 .
68. Coefficient of stiffness V.-The pieces subject to compression have a tendency to deviate from the straight line when they have a weak section and great length; therefore, we are obliged in such cases to give them an excess of section, that is, instead of calculating them for a pressure of 6000000 kilos. the square mètre, we shall calculate them for a less pressure $R$; or having calculated them for 6000000 kilos., we must multiply the section so obtained by a coefficient $\mathrm{V}=\frac{6000000}{\mathrm{R}}$.

For flat bars, simply abutting against others at their extremities, we may, from the results of experience, admit the following figures:-

| $\left.\begin{array}{c}\text { Ratio of the length to } \\ \text { the thickness } \frac{\lambda}{e}\end{array}\right\}=$ | 15 | 20 | $25$ | 30 | 35 | 40 | $45$ | $50$ | 55 | 60 | 65 | 70 | 75 | 80 | 90 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left.\begin{array}{l}\text { Resistance the square } \\ \text { millimètre .. .. }\end{array}\right\}=$ | $6 \cdot 00$ | $5 \cdot 30$ | $4 \cdot 75$ | 4-25 | $3 \cdot 90$ | $3 \cdot 55$ | $3 \cdot 25$ | $3 \cdot 00$ | $2 \cdot 70$ | $2 \cdot 50$ | 2.30 | $2 \cdot 10$ | 2.00 | 1'80 | 1-50 | $1 \cdot 20$ |
| Coefficient of increase $\mathrm{V}=$ | $1 \cdot 000$ | 1-132 | $1 \cdot 263$ | 1.412 | $1 \cdot 538$ | 1.690 | $1 \cdot 846$ | $2 \cdot 000$ | $2 \cdot 222$ | $2 \cdot 400$ | 2.609 | $2 \cdot 857$ | $3 \cdot 000$ | $3 \cdot 333$ | $4 \cdot 000$ | $5 \cdot 000$ |

The bars of lattice-girders being usually attached at their extremities by several rivets, may be regarded as welded on, and we shall obtain additional security by observing the preceding data.

But flat bars have precisely the worst possible form to resist a strain of compression. This obliges us to increase the number and to place them near together, in order that each isolated part may have the necessary rigidity without giving to V too high a value. But the rivetings at the points where the bars cross each other are not and-sufficient, for the compressed bars are badly supported by being fixed to other bars as flexible as themselves; when the number of lattice-bars becomes great and, in consequence, the sections become very inconsiderable, the struts may yield by numerous deflections, twisting the braces, some in one direction, some in another, an effect similar to the wrinkles which show themselves upon a thin membrane stretched in a frame, when the angles of the frame are distorted. It follows, that we ought to employ flat bars only for girders of small height and heavily loaded, unless, indeed, the rigidity of the vertical middle part of the girder be sufficiently provided for by upright stays. If these stays serve no other purpose than to stiffen the part in question, it will generally be better to suppress them and to strengthen directly the resisting lattice-bars, either by increasing the section, or better, by adopting another form, such as that of angle or Tiron. The braces, if nothing prevents should be constructed in the same way, because their rigidity, useless to themselves, lends a support to the struts. Sometimes these upright pieces are needed to fix tie-beams on one side only of the girder; it will be sufficient, in this case, if they project upon this same side, which will allow us to construct half of the oblique bars of stiff iron, the mouldings of which shall be turned in the opposite direction, to avoid cutting them away at the points where they cross each other.
69. The influence of the form of the sections being established, it remains to be seen how we can consider the rigidity in the various cases which may present themselves.

Now the curve which a piece under the force of compression tends to assume is a real deflection, which requires, not a certain area of section, but a sufficient moment of resistance. In plate iron we may regard the thickness as being nothing but the ratio of the moment of resistance $M$ expressed by the coefficient $R=6000000$, to the section $S$ expressed in square millimètres. Thus, to generalize the Table in the preceding section, it is sufficient in the ratio $\frac{\lambda}{e}$, applied to any section, to substitute the ratio $\frac{M}{\mathrm{~S}}$ for $e$, which we may call reduced thickness, or imaginary thickness, in the direction in which deflection is to be feared, and in which the moment of resistance $M$ is taken.
70. When compressed bars are unsupported throughout their length, as is the case, for example, in girders formed of simple triangles, they should have a section possessing a rigidity equal in all directions. Such would be the round form, but it offers practical difficulties.

The section in the form of a cross with four equal branches, may be advantageously employed; it offers a moment of inertia constant in all directions. As to the moment of resistance, which depends on the distance of the fibre subject to the greatest strain from the centre of gravity, it is

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$\sqrt{2}$ times greater in the direction of the lines bisecting the angles than in the direction of the arms; but as deflection will always take place in the weakest part, we can reckon only upon a reduced thickness, exceeding but little the dimension of one of the four arms, when these latter have a constant thickness. Figs. 1659 to 1667 represent some sections in the form of a cross,

composed of angle-iron and plate; this mode of construction causes irregularities of thickness, and the moment of resistance is affected by it according as the material is accumulated towards the centre or towards the extremities. Under each of the sections in the figures will be found the reduced thickness, taken in the direction of least stiffness when the branches are not identical.
71. The single $T$ is more convenient for fixing than the section in the form of a cross, and it offers also a slight advantage from the point of view of resistance in the direction of the web. If we consider only the moment of inertia, it is easy to make it rigorously equal in all directions. To effect this, we have merely to proportion the arms and the web so that the moment of inertia in their respective directions CD and A B, Fig. 1668, shall remain the same. In fact, on account of the symmetry, the directions at $45^{\circ} \mathrm{MN}$ and $\mathrm{M}^{\prime} \mathrm{N}^{\prime}$ give equal moments of inertia : the central ellipse of inertia will, therefore, offer two systems of equal rectangular diameters, which is found only in the circle. But, as is the case with the cross, the moment of resistance will be smaller in the principal directions than in the others; and if the reduced thickness is made
 equal in these two directions, we shall reckon only upon the value which it will so possess as if it remained constant in all directions.

The double $T$ cannot be made to resist equally in its two chief directions, because this condition would require flanges of excessive breadth.
72. For bars of small dimension we may employ single angle-iron. This form offers least stiffness in the direction of the line bisecting the angle. If the angle-iron is thin and its branches have a breadth $c$, we may regard the minimum reduced thickness as equal to $\frac{c}{\sqrt{2}}$ or $0.7 c$. The central ellipse of inertia of such an angle-iron has its major axis double the minor ; the minimum moment of inertia, a quarter of the maximum, is $\frac{2}{5}$ of the moment of inertia in the direction of one of the branches.

As an application of the foregoing, suppose a bar compressed by a strain of 5 tons, upon a free length of 3 mètres. An angle-iron of 0.080 mètre broad will give a minimum reduced thickness of about 0.056 mètre, and the ratio of the length to this thickness will be $\frac{3}{0 \cdot 056}=53$. This value authorizes the adoption of $R=2^{k} \cdot 8$ per square millimètre, which leads to a section of 1790 square millimètres: this section will be obtained by giving to the angle-iron a mean thickness equal to 0.012 mètre. If this thickness did not suit, we should try an angle-iron of a different breadth.

We must proceed thus tentatively for every question of the same kind. We may calculate in advance the compressions which a certain number of model sections will bear, under various lengths, and these data will serve as a guide in determining, at sight, the modified sections which will suit given cases.

When the load is great and the length inconsiderable, we may adopt thicker sections and suppress the coefficient of stiffness. It is advisable, however, in such cases, to preserve an excess of rigidity.
73. In multiple lattice-girders, a compressed bar is fixed, in the points where they cross, to other bars subject to tension, but which generally act more effectually against a deflection in the plane of the vertical because they cannot resist a deviation normal to this plane; in fact, if the braees are flexible bars, they will resist perfectly a strain of tension directed in the direction of their length, but very badly a transverse strain, which would bend them in a very marked manner. Thus, with regard to deflection in the plane of the vertical, we may consider the compressed bar as subdivided into shorter portions which may be stiffened separately; and as to deviation out of the plane of the vertical, it will be better not to reckon upon the tension of the braces, but only upon their rigidity which will supply what is wanting to the strut. The connection at the points where they cross each other may, therefore, improve the relation of the length to the reduced thickness, in the plane of the vertical because it reduces the length, and in the direction normal to
this plane because the rigidity of the braces assists that of the struts. However, this must not induce us, from the point of view of rigidity, to multiply the lattice as much as possible, for so we weaken each bar taken individually, and we should at length produce a girder the vertical portion of which would be liable to crumple up like a thin sheet of iron.
74. Let us consider the case of a simple cross formed by two bars of the same length and the same section: the strain of deviation which tends to throw the centre of the cross out of the plane of the vertical meets with an equal resistance from each of the two bars, omitting the special effect due to the tension of the brace ; consequently the strut will be required to furnish only a moment of resistance or a reduced thickness, less by half than in the case in which it is isolated. Although its resisting section is not increased, it will be regarded as having the same reduced thickness as a single piece formed by the superposition of the two branches of the cross.

When the brace has less rigidity than the strut, the latter must be more rigid in the direction normal to the vertical. We may give it a reduced thickness equal to the sum of its own and that of the brace. Tie
75. The vertical bars which are sometimes applied to a lattice to consolidate it, act in an analogous manner. Having no longitudinal strain to bear, the area of their section is of little consequence; they act only by their moment of resistance.

Suppose, for example, Fig. 1669, a flexible bar A B, held in its middle C by a rigid vertical bar D E . If the latter were
 absent, the bar A B could be subjected to a compression of 6 kilogrammes a square millimètre only under the condition that the ratio of its length $\lambda$ to its reduced thickness $\frac{M}{\mathrm{~S}}$ (see 69) did not exceed 15 ; whicn is equivalent to saying that it shoula have a moment of resistance at least equal to $\frac{\lambda S}{15}$ (the section $S$ being expressed in square millimètres), precisely as if the point C were subjected to a transverse strain equal to $\frac{4 \mathrm{~S}}{15}$; but as the bar is, of itself, incapable of resisting this strain, we may impose it upon the vertical bar which on this account should possess a moment of resistance equal to $\frac{\lambda^{\prime} \mathrm{S}}{15}$, the section S being, not its own, but that of the bar to be strengthened. Thus, by its less length, the vertical bar may furnish a determinate rigidity with a less moment of resistance: it is, however, generally better to increase the section itself of the bar and to suppress the vertical, for by that means we diminish the pressure per unity of surface, which requires less rigidity.

If the bar A B already possesses of itself a certain rigidity, it may take a portion $x$ of the strain of deflection in C. It will then be deflected by a quantity $\frac{x \lambda^{3}}{48 \mathrm{EI}}(\mathrm{E}=$ modulus of elasticity, $I=$ moment of inertia) which, on account of their being fixed together, must be equal to the versed sine of the arc assumed by the vertical bar; this condition gives the equation $x\left(\frac{\lambda^{3}}{I}+\frac{\lambda^{\prime 3}}{I^{\prime}}\right)=\frac{48 \lambda^{\prime 3}}{15 I^{\prime}}$, whence we deduce $x$. Consequently the vertical bar will be required to furnish only a supplementary moment of resistance equal to $\left(\frac{4 \mathrm{~S}}{15}-x\right) \frac{\lambda^{\prime}}{4}$.

We might apply analogous considerations to the less simple case in which C is not the middle either of the oblique or of the vertical bar.

For a cross formed of two bars of the same section and the same free length $\lambda$, we shall have $x=\frac{2 \mathrm{~S}}{15}$, and each bar will furnish, as we have already seen, a moment of resistance equal to the half of that which the compression-bar ought to have, if it were deprived of all assistance.
76. In a multiple lattice with flexible braces; the struts must be capable of greater transverse resistance in the direction normal to the vertical part of the girder than in the direction of the vertical part. Figs. 1670 to 1684 give as examples various sections, with their reduced thicknesses in the direction of greatest resistance; it will be seen that the forms in $U$ or double $T$ are very good, for they give reduced thicknesses which exceed much the apparent dimension.

If, for instance, we have to construct a piece 8 mètres long, capable of resisting a maximum strain of compression of 10 tons only, the fifth section gives as the ratio of the length to the reduced thickness $\frac{8}{0 \cdot 159}=50$. According to the Table in 68 , the piece ought to bear a strain of only 3 kilogrammes a square millimètre, and it is capable of bearing $3500 \times 3=10500$ kilogrammes, which is something more than sufficient. Adopting this section, the coefficient of stiffiness would be $2 \cdot 10$; but it would be more advantageous to adopt a $\bigcup$ iron of less weight.
77. The principle of increased section in the middle applied to the connecting-rods of machines, may also be adopted for the struts of very important girders. This may be effected by applying plates to the middle of the piece, precisely as in the case of girders loaded transversely.

It is always useful, in multiple lattices, to give to the tension-bars sections with projecting mouldings, at least as much as possible without making the area of the section exceed the value corresponding to a strain of 6 kilogrammes the square millimètre. When the length becomes so great that this condition of area, applied to the struts and breees, does not allow us to realize a sufficient

rigidity, we must hive recourse to a coefficient of increase V , which may be exclusively applied to the struts, or divided between these and the braes when nothing prevents us from giving to the


Sometimes accessory arrangements contribute to stiffen the vertical portion of girders; such is the case, for instance, Figs. 1580, 1585, where the cross-girders are placed in the middle of the height.

These various considerations no doubt are not exhaustive, but they will be sufficient to enable us to choose, in every case, the best forms; and when it may be done at a small cost, it will always be prudent to have an excess of rigidity.
78. When we wish to find as near as possible, without going through a complete calculation, the weight which a girder fulfilling certain given conditions would have, we may proceed as follows to find the value of the mean coefficient V to be applied to the parts of the lattice under compression.

We calculate the strains $Q_{1}$ and $Q_{2}$ upon the strut which bears the least strain (in the middle), and upon the one which bears the greatest strain (at the end); then, having regard to their length, we consider what sections it would be necessary to give them, and we deduce the coefficients of stiffness $v_{1}$ and $v_{2}$, special for these two bars. The total weight of the struts will then be nearly proportional to the mean $\frac{1}{2}\left(\mathrm{Q}_{1} v_{1}+\mathrm{Q}_{2} v_{2}\right)$ whilst if the question of rigidity did not interfere, it would be proportional to the mean $\frac{1}{2}\left(Q_{1}+Q_{2}\right)$ of the limiting strains; we may thence conclude approximatively $\mathrm{V}=\frac{\mathrm{Q}_{1} v_{1}+\mathrm{Q}_{2} v_{2}}{\mathrm{Q}_{1}+\mathrm{Q}_{2}}$.
79. If some of the end struts could support a strain of 6 kilogrammes, the general coefficient would be smaller still. We should then seek the least section under which a strut can bear 6 kilogrammes, having regard to its free length and other circumstances, and determine the load $Q_{3}$ which it would carry with this section; then we should make approximatively

$$
\mathrm{V}=\frac{\mathrm{Q}_{2}{ }^{2}-\mathrm{Q}_{1} \mathrm{Q}_{3}+\mathrm{Q}_{1} v_{1}\left(\mathrm{Q}_{3}-\mathrm{Q}_{1}\right)}{\mathrm{Q}_{2}{ }^{2}-\mathrm{Q}_{1}{ }^{2}}
$$

80. The mean coefficient V may also be expressed by means of the variable coefficient $v$ applicable to the successive bars, by making $\mathrm{V}=\frac{\Sigma \mathrm{F} v}{\Sigma \mathrm{~F}}$. We might obtain from section 6 the values of $\mathbf{F}$ and $\Sigma \mathrm{F}$; but to calculate $\Sigma \mathrm{F} v$, we should have to express the law of variation of $v$ according to that of the strains. Now this law would be too complicated or too arbitrary to lead by this method to a satisfactory result, and it is preferable to trust to the approximative formula in section 78.
81. By increasing the height of a girder we weaken its middle, or vertical portion, and this obliges us to raise the value of the coefficient V in order to preserve a sufficient rigidity.

To find the mode of variation of $V$ with the height $h$ of the girder, we may remark in the first place that, according to the values of the Table in 68, as long as the ratio $\frac{\lambda}{e}$ does not exceed 50 or 55 , V will be accurately enough represented by the function $0.57+\frac{1}{35} \frac{\lambda}{c}$, with the condition, however, that it cannot become less than unity. In this expression, the reduced thickness $e$ depends on the form and area of the section, and this area is a function of the load $Q$ which the piece supports and of the coefficient V : it is expressed by $\frac{\mathrm{Q} V}{\mathrm{R}}$.

If the section vary in such a way that on dividing it by the reduced thickness, the quotient $b$ is constant, we shall have $e=\frac{\mathrm{QV}}{\mathrm{R} b}$, and consequently $\mathrm{V}=0.57+\frac{1}{35} \frac{\mathrm{R} \lambda b}{\mathrm{QV}}$, whence

$$
\mathrm{V}=0.29+\sqrt{0.082+\frac{\mathrm{R} \lambda b}{35 \mathrm{Q}}}
$$

For example, for girders composed of triangles or crosses, expressing the length $\lambda$ of the bars and their strain $Q$ by means of the height $h$ of the girder and the whole strain $F$, we shall have
$\mathbf{V}=0.29+\sqrt{0.082+\frac{\mathrm{R} b}{35 \mathrm{~F}}} . h$. For a multiple lattice of the $i$ degree, we have only to change F into $\frac{F}{i}$.

If the section varied while remaining similar to itself, the quotient of the square of the section by the reduced thickness would be constant, and $V$ would be determined by an equation of the third degree.

Thus, keeping to the simplest case first considered, V will be expressed in terms of the height by formulæ of the form $a+\sqrt{b+c h}$. But as there is nothing absolute in the value of this coefficient, nor in the relation of the section to the reduced thickness, on account of the great diversity of form, we may content ourselves, in the investigation of the most economical heights, with replacing V by a rational function of the first degree $a+b h$, determined by means of tro particular values of this coefficient for heights differing not too widely, but including between them the height sought. This is equivalent to replacing an are by its chord.
82. Circumstances other than the condition of rigidity may oblige us to increase the weight of the vertical portion of a girder beyond the limits prescribed by theory. For example, in a multiple lattice in which the bars are very numerous, their section is not made to vary in a perfectly continuous manner, but only.by successive groups; so that the greater portion of them will possess a slight excess of weight. Then again in the parts in which the stress is inconsiderable, it often happens that we cannot in practice make the bars so slender as they are required to be by calculation. Besides, when the section is inconsiderable, a single rivet-hole weakens it in a proportionally great degree. This is another reason for increasing the dimensions. These several augmentations might give occasion for a new coefficient applied to the whole of the vertical portion of the girder, but it will simplify the matter to include them in the coefficient V by raising its value sufficiently high.
83. Supplementary term, $\Omega$.-To complete the computation of the weight of a girder, we must include certain accessory pieces the dimensions of which are nearly independent of the height of the girder, and which may be represented by a constant additional term $\Omega$. Such are, for example, in lattice-girders, the gussets at the points where the bars cross each other, joint-plates and other contrivances for making the rivets offer a double section to the strain upon them. On the other hand, these special pieces are partly compensated by the fact of the lattice-bars ending at the edge of the longitudinal angle-irons of the flanges, so that their length is slightly inferior to that supposed in the formulæ. Sometimes certain accessory pieces may be considered as increasing in a certain proportion with the height; in these cases we may include them in the coefficients U or V according as they belong to the flanges or to the vertical portion.
84. General form of the Formule giving the Weight of Girders, and the most advantageous Heights.The total weight of a girder is obtained by adding the terms referring respectively to the flanges, to the vertical portion and to the accessories independent of the height. Dividing by the length of bearing, we shall have the mean weight of the lineal metre.

In this mean weight, the flanges will appear as a quantity expressed by $\frac{2 \mathrm{G} t \mathrm{U}}{h}, \mathrm{G}$ denoting the mean ordinate of the locus of the maximum moments of rupture (that is, $\frac{1}{12}\left(p+p^{\prime}\right) l^{2}$ in the case of a single bay and a continuous load). As U is of the form $a+b h$ (64), the flanges are divided into a part inversely proportional to $h$, and a part which is constant.

In girders composed of simple triangles or crosses, $\delta$ being constant, the inclination of the bars is a function of the height, and the pieces of the vertical portion generally make, a mètre of length, a weight of the form $\frac{\mathrm{G}^{\prime} t(1+\mathrm{V})}{l \delta}\left(h+\frac{\delta^{2}}{4 h}\right)$, $\mathrm{G}^{\prime}$ denoting the mean ordinate of the locus of the maximum stress, the value of which ordinate is $\frac{1}{4}\left(p+p^{\prime}\right) l\left(1+\frac{1}{6} q\right)$ in the case of a continuous load. Admitting that $1+\mathrm{V}$ is expressed by $a+b h(81)$, the weight in question will give rise to terms in $h^{2}$, in $h$, in $\frac{1}{h}$, and to a constant term.

In multiple lattice-girders in which the inclination of the bars is $45^{\circ}$, or else keeps a certain other constant value, as in the case of solid girders, the theoretical vertical position would have a constant weight. But, in such cases, the rigidity is usually obtained by the addition of special stays, vertical or inclined, the section and length of which increase with $h$, so that their weight will be expressed under the form $\left(\kappa+\kappa^{\prime} h\right) h$.
85. Thus, the weight of the lineal metre of girder will be generally given by formulæ of the following form: $\frac{\mathrm{P}}{l}=\frac{\mathrm{A}}{h}+\mathrm{B}+\mathrm{C} h+\mathrm{D} h^{2}$. The corresponding height at the minimum expense
will be given by the equation of the third degree: $\mathbf{A}=(\mathrm{C}+2 \mathrm{D} h) h^{2}$, whence we deduce the real value:


But as it is always enough to know this height within 1 décimètre, it will be a saving of time to solve the equation tentatively. As a function varies but little when near a minimum, we may without much increasing the expense adopt heights less than those given by calculation, and we find an additional reason for doing this in the fact that the ties or cross-pieces generally increase with the height. We might, it is true, seek directly the conditions of the minimum of weight for the whole flooring, instead of a single girder.
86. Sometimes, for the sake of simplicity, we may regard V as a fixed coefficient which may conveniently be forced a little; in this case, V being constant, we shall arrive for simple lattices at formulæ of the form $\frac{\mathrm{P}}{l}=\frac{\mathrm{A}}{h}+\mathrm{B}+\mathrm{C} h$, and taking $h=\sqrt{\frac{\mathrm{A}}{\overline{\mathrm{C}}}}$, we shall have as the minimum value, $\frac{\mathrm{P}}{l}=2 \sqrt{\mathrm{AC}}+\mathrm{B}$. It is evident that an alteration of height, even if considerable, has but a small influence; for if, by way of example, instead of $\sqrt{\overline{\mathrm{A}}}$, we take $h=\kappa \cdot / \frac{\overline{\mathrm{A}}}{\overline{\mathrm{C}}}$, the term $2 \sqrt{\overline{\mathrm{AC}}}$ will be merely changed into $2 \sqrt{\frac{\overline{\mathrm{~A}}}{\overline{\mathrm{C}}} \frac{1+\kappa^{2}}{2 \kappa} \text {. }}$

| No | $\frac{1+\kappa^{2}}{2 \kappa}=1 \cdot 0056$ |  |  |  | And $\kappa=1 \cdot 1$ gives $\frac{1+\kappa^{2}}{2 \kappa}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| " | $0 \cdot 8$ | " | " | $1 \cdot 0250$ | " | $1 \cdot 2$ |  | , | 1.0167 |
| ", | $0 \cdot 7$ | " | " | $1 \cdot 0643$ | " | $1 \cdot 3$ |  | " | 1.0346 |
| " | $0 \cdot 6$ | " | " | $1 \cdot 1333$ | " | $1 \cdot 4$ |  | ", | 1.0571 |
| " | $0 \cdot 5$ | " | " | $1 \cdot 2500$ | " | $1 \cdot 5$ |  |  | 1.0833 |
| " | $0 \cdot 4$ |  |  | $1 \cdot 4500$ | " | $2 \cdot 0$ |  |  | $1 \cdot 2500$ |

An increase of height above $\sqrt{\frac{\bar{A}}{\bar{C}}}$ produces a less loss than an equal diminution, but in almost every case it will be a diminution that we shall have to make.

It will be well to make a verification afterwards to ascertain whether the height $\sqrt{\overline{\mathrm{A}}}$, or at least the reduced height adopted, does not require a value of V superior to that which we have introduced into the calculation.

This method gives too great height, and, therefore, the reduction which it will be necessary to make will be more considerable than when we employ the more exact method of the preceding section.
87. Girders of Simple Triangles.-Formulce of the Weight a lineal mètre, in the case of Bars of Equal Inclination.-To find the total weight P of girders of simple triangles, formed by bars of equal inclination, we have only to add together the weights of the flanges, the struts, and the braces, given by the formulæ $p$, for the particular cases in which $\alpha=\beta$. As the total length of the girder exceeds its bearing, we shall give only the weight per lineal mètre, found by dividing $P$ by the bearing $l$, and it is understood that when the total weight, including the ends, is required, the weight of the lineal mètre is to be multiplied, not by the bearing, but by the total length, say by $1.045 l$ or $1.097 d, d$ being the free span, according to what was said in 37.

1. Load on the lower side.-If $l=2 m \delta=\mathrm{N} \delta$, we shall have, $\Omega$ being an additional term for the accessories independent of $k$,

$$
\frac{\mathrm{P}}{l}=\frac{1}{2} m\left(p+p^{\prime}\right) t\left(1+\frac{m^{2}-1}{6 m^{2}} q\right)\left\{\frac{\delta^{2}}{h}\left[\frac{\left(4 m^{2}-1\right) \mathrm{U}}{3 m+\frac{m^{2}-1}{2 m} q}+\frac{1+\mathrm{V}}{4}\right]+h(1+\mathrm{V})\right\}+\Omega
$$

or

$$
\frac{\mathrm{P}}{l}=\frac{1}{4}\left(p+p^{\prime}\right) \operatorname{tl}\left(1+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}^{2}} q\right)\left\{\frac{l}{\mathrm{~N} h}\left[\frac{2}{3} \cdot \frac{\mathrm{~N}^{2}-1}{\mathrm{~N}} \cdot \frac{\mathrm{U}}{1+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}^{2}} q}+\frac{1+\mathrm{V}}{4}\right]+\frac{\mathrm{N} h}{l}(1+\mathrm{V})\right\}+\Omega
$$

a formula applicable when N is even.
When $l=(2 m+1) \delta=\mathrm{N} \delta$, we have

$$
\frac{\mathbf{P}}{l}=\frac{m(m+1)}{2 m+1}\left(p+p^{\prime}\right) t\left(1+\frac{1}{6} q\right)\left\{\frac{\delta^{2}}{h}\left[\frac{2}{3}(2 m+1) \cdot \frac{\mathrm{U}}{1+\frac{1}{6} q}+\frac{1+\mathrm{V}}{4}\right]+h(1+\mathrm{V})\right\}+\Omega
$$

or

$$
\frac{\mathrm{P}}{l}=\frac{1}{4} \cdot \frac{\mathrm{~N}^{2}-1}{\mathrm{~N}}\left(p+p^{\prime}\right) t l\left(1+\frac{1}{6} q\right)\left\{\frac{l}{\mathrm{~N} h}\left(\frac{2 \mathrm{U}}{3+\frac{1}{2} q}+\frac{1+\mathrm{V}}{4 \mathrm{~N}}\right)+\frac{h}{l}(1+\mathrm{V})\right\}+\Omega
$$

a formula applicable when N is odd.

The flanges do not change whether N be even or odd; but the lattice decreases slightly when N is odd, in the proportion of $\mathrm{N}^{2}-1$ to $\mathrm{N}^{2}$.
2. Load on the upper side.-For $l=2 m \delta=\mathrm{N} \delta$, we shall have

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{1}{48 m}\left(p+p^{\prime}\right) t\left[6\left(4 m^{2}-1\right)+\left(4 m^{2}-1+\frac{3}{2 m}\right) q\right] \\
\left\{\frac{\delta^{2}}{h}\left[\frac{\left(32 m^{2}-8 m+3\right) \mathrm{U}}{6\left(4 m^{2}-1\right)+\left(4 m^{2}-1+\frac{3}{2 m}\right) q}+\frac{1+\mathrm{V}}{4}\right]+h(1+\mathrm{V})\right\}+\Omega
\end{gathered}
$$

which amounts nearly to

$$
\frac{\mathrm{P}}{l}=\frac{\mathrm{N}^{2}-1}{4 \mathrm{~N}}\left(p+p^{\prime}\right) \operatorname{tl}\left(1+\frac{1}{6} q\right)\left\{\frac{l}{\mathrm{~N} h}\left[\frac{2 \mathrm{U}}{3+\frac{1}{2} q}+\frac{1+\mathrm{V}}{4 \mathrm{~N}}\right]+\frac{h}{l}(1+\mathrm{V})\right\}+\Omega
$$

a formula identical with the last of the case in which the load is on the lower side, except that here N is even.

And if $l=(2 m+1) \delta=\mathrm{N} \delta$, we have

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{1}{12}\left(p+p^{\prime}\right) t\left[3(2 m+1)^{2}+m \cdot \frac{4 m^{2}+6 m-1}{2 m+1} q\right] \\
\left\{\frac{\delta^{2}}{2 h}\left[\frac{32 m^{3}+48 m^{2}+16 m+3}{3(2 m+1)^{3}+m\left(4 m^{2}+6 m-1\right) q} \mathrm{U}+\frac{1+\mathrm{V}}{2}\right]+\frac{h(1+\mathrm{V})}{2 m+1}\right\}+\Omega
\end{gathered}
$$

or

$$
\frac{\mathrm{P}}{l}=\frac{1}{4}\left(p+p^{\prime}\right) t l\left(1+\frac{1}{6} q\right)\left\{\frac{l}{\mathrm{~N} h}\left[\frac{2}{3} \cdot \frac{\mathrm{~N}^{2}-1}{\mathrm{~N}} \cdot \frac{\mathrm{U}}{1+\frac{1}{6} q}+\frac{1+\mathrm{V}}{4}\right]+\frac{\mathrm{N} h}{l}(1+\mathrm{V})\right\}+\Omega
$$

a formula in which N is odd, and which is obviously the same as the first of case 1.
88. Examples.-Girder of 28 mètres bearing, supporting, on the upper side, a load of 3500 kilogrammes to a mètre, Figs. 1685 to $1689 ; p=1100^{\mathrm{k}}$ (the girder itself weighing 400), $p^{\prime}=2400^{\mathrm{k}}, q=0 \cdot 686, m=4$, and, consequently, $\delta=3^{\mathrm{m}} \cdot 50, \hbar=3^{\mathrm{m} \cdot 30}$.

As the girder is lightly loaded, we must seek to diminish as much as possible the least section of the flanges, without, however, reducing too much their breadth, which we will fix at $0^{\mathrm{m}} \cdot 400$. To diminish the breadth of the vertical ribs, projecting gussets are placed at intervals, to which the struts and braces may conveniently be attached. Those struts which are most heavily loaded have a section in the form of a cross, favourable to rigidity in all directions; one of the angle-irons passes behind the gusset to avoid the necessity of cutting it aray at the extremities; and, consequently, some of the rivets offer a double section. Instead of filling the whole of the space above the gusset to make this angle-iron solid with the rest of the bar, it is sufficient to fill it in part only The middle bars require fewer rivets, but it has been necessary to increase their theoretical section to obtain the required rigidity.

The lower vertical rib is only $0^{\mathrm{m}} \cdot 300$ broad • but the upper could not be reduced below $0^{\mathrm{m}} 350$, because the flange should be rigid to resist compression, because in the present case it might be required to serve as a minor longitudinal girder, and because we might have occasion to affix to it a light corbel, in which case this breadth of $0 \mathrm{~m} \cdot 35$ is barely sufficient. The brackets may be placed in the middle of the intervals to be out of the way of the bars. It is solely for the purpose of forming a minor longitudinal girder that the upper flange has been produced, and the vertical supports added at the ends.

We find by calculation the total weight of the girder to be 11200 kilogrammes, say 380 kilogrammes the lineal mètre.

The flanges weigh together 8070 kilogrammes, including the gussets for fixing the bars; but to determine the value of U , it is better to consider only the weight of 7400 kilogrammes, which we obtain by deducting the projection of the gussets, and reckoning with the rertical rib only the joint-plates which would be needed if it were not interrupted at short intervals by the gussets. Dividing by the whole length $29^{\mathrm{m}} \cdot 40$, we find that the flanges make together 252 kilogrammes the lineal mètre. The formula would give $177 \cdot 3 \mathrm{U}$; we shall, therefore, have $\mathrm{U}=\frac{252}{177 \cdot 3}=1 \cdot 42$, a value which includes the extensions at the ends.

However great the height of the girder might be, we could hardly compose the flanges of less than two angle-irons of $100 / 100 / 12$ millimètres, a vertical rib of $300 / 10$, and a horizontal plate of $350 / 8$ : this would form a sufficient section in the middle if the height were $5 \mathrm{~m} \cdot 50$, and it would remain constant throughout the bearing. By the formula of 64 , we should find again $\mathrm{U}=1 \cdot 06+0 \cdot 60 \frac{3 \cdot 30}{5 \cdot 50}=1 \cdot 42$.

The extra weight of 670 kilogrammes occasioned by the presence of the gussets, is reduced to 500 kilogrammes when diminished by the saving effected by the fact of the real length of the bars being a little less than their theoretical length, since they terminate at the edge of the angleirons of the flanges. Hence we conclude that $\Omega$ is here equal to $\frac{500}{29 \cdot 40}=17$ kilogrammes the lineal mètre of girder, say the 0.047 of the total weight of the principal pieces. This is a great deal, but we may reduce this value by giving the gussets a thickness of 8 millimètres only.

The oblique bars weigh 2600 kilogrammes, but the saving effected in their length has just been attributed to the term $\Omega$, we must, therefore, suppose them to possess the weight 2770

## BRIDGE.

ixilogrammes, which they would have with their theoretical length. Comparing with the formula, $\mathrm{V}=1 \cdot 34$.

If we wished to compute $V$ approximatively, without going through a complete calculation, we should have considered only one extreme and one middle bar. The latter, for a maximum strain of $\mathrm{Q}_{1}=9 \frac{1}{2}$ tons, gives $\mathrm{V}_{1}=226$ with the section adopted; and the other, for a strain $Q_{2}=52 \cdot 6$ tons, gives $\mathrm{V}_{\mathbf{8}}=1 \cdot 07$, including the filling. The formula of 78 then gives $V=\frac{Q_{2} V_{2}+Q_{1} V_{1}}{Q_{2}+Q_{1}}=1 \cdot 29$, a value which we must increase a little, because in the formulæ one of the bars of the middle triangle has been regarded as a brace, and, on this account, exempted from the coefficient V, whilst in reality both bars of this triangle should be able to serve, each in its turn, either as strut or as brace.

To discuss the height of the girder, we must first seek an approximative expression of V in terms of the height. To this end, let us consider, by way of example, a height of 5 mètres; we shall then have $Q_{2}=48 \cdot 6$ tons and $Q_{1}=8.7$ tons, and the free length of the bars between the gussets will be about 4 mètres. For the one which is the most heavily loaded, a section in the form of a cross with four arms of $150 / 10$, held together by four angle-irons of $70 / 70 / 10$, will give $\mathrm{V}_{\mathbf{3}}=$ about $1 \cdot 40$; and for the weakest bar, a cross composed only of a bar of 140/6 and nf two angle-irons of $70 / 70 / 10$, will give $\mathrm{V}_{1}=2 \cdot 40$. We thence conclude $\mathrm{V}=1 \cdot 55$, say $1 \cdot 60$ on account of the observation made above. Knowing the values of V for two different heights, it is evident that we may adopt nearly $1+\mathrm{V}=1 \cdot 70+0 \cdot 19 h$.

Substituting this expression in the formula of the preceding section (case 2 for N even), we find that the minimum expense is given by a height of about $4^{\mathrm{m}} \cdot 40$. In fact, with this height, we should have $1+\mathrm{V}=2 \cdot 54, \mathrm{U} 1 \cdot 54$, and, consequently, the weight of the lineal mètre descends to 350 kilogrammes, $\Omega$ included; whereas with $h=3^{\mathrm{m}} \cdot 30,1+\mathrm{V}=2 \cdot 33, \mathrm{U}=1 \cdot 42$, the formula gives 370 kilogrammes. These weights should be increased a little to take sufficiently into account the end upright pieces, which really form no portion of the girder, but serve only to support the rail upon the abutment. It is obvious that if the height allowed permits, it may be of some use to increase slightly the height of the plan, to carry it to $3^{\mathrm{m}} \cdot 60$, for instance, but it would be useless to go beyond that, for the cross-pieces would increase without giving any sensible advantage to the girder. We might, in a more general manner, have sought the most advantageous height, by considering the whole of the flooring, and expressing the cross-pieces themselves as a function increasing with $h$.
89. Girder of 56 mètres bearing, with a load on the lower side of 7000 kilogrammes the mètre, Figs. 1690 to $1700 ; p=3000^{k}$ (the girder itself weighing $1300^{k}$ ), $p^{\prime}=4000^{k}, q=0 \cdot 572$, $m=4$ or $\mathrm{N}=8, \delta=7^{\mathrm{m}}, h=7^{\mathrm{m}}$.

The load supposes both lines loaded simultaneously; otherwise, for one line only, $p^{\prime}$ would not exceed 3000 kilogrammes, and we might reduce by about $\frac{1}{7}$ the weight of the girder, allowing it a strain of 7 instead of 6 kilogrammes in the exceptional case of two heavily-laden trains crossing the bridge at the same time.

Cross-bracing behind the compression-bars.


The interval $\delta$ can be raised to 7 mètres only on the supposition that there are two cross-girders for each section or division of flange, which subjects the bottom flare to a deflecting strain; but this strain may be neglected, having regard to the form of the section and to the other arrangements adopted.

The following Table gives the strains upon the pieces:-

|  | Upper Flange. |  |  |  | Lower Flange. |  |  |  | Struts or Braces. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Numbers of the divisions, | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Maxinum strain, in tons.. | 392 | 369 | 294 | 172 | 386 | 337 | 239 | 92 | 51 | 94 | 141 | 192 |

The flanges are hollow and have a double vertical rib, which allows the bars to be very firmly fixed. The two triangles next the abutment are besides provided at their extremities with jointplates forming a kind of fork, which enables the rivets, or at least many of them, to offer a double section. By this means we may place upon each rib thirty rivets, twenty-four of which offer a double section. This gives for the foot of the end brace, for example, a total of 108 shearing sections, capable of resisting a strain of 216 tons if the rivets have $0 \mathrm{~m} \cdot 025$. The thickness of the double vertical ribs is $0 \mathrm{~m} \cdot 012$. In the last triangle it is well to strengthen them to increase their resistance to a shearing force; this is accomplished by adding broad gussets of $0 \mathrm{~m} \cdot 012$. At the foot of the end brace there is no projecting gusset; but the thickness of the rib is doubled by the rojecting of a joint-plate.

For the intermediate triangles, it is useless to have recourse to double shearing sections.
The struts have a hollow form, very favourable to resistance to compression, as the moment of resistance is rendered as great as possible in the two directions. The transverse plate of $400 / 8$ is not reckoned in the resisting section; it has been added to connect the other pieces of the bar so as to render them solid, and this plate or flat bar constitutes the principal element of the coefficient V. Its effect is completed by three triangles or rods placed on the other side, at equal intervals.

The braces are composed of two pieces left unconnected, except in the middle bar, which is as much a strut as a brace, and which, for this reason, has been stiffened by a continuous strip of plate iron connecting the two pieces. It would have been somewhat more economical to have substituted a light lattice for this continuous plate; but it has the advantage of giving more body to the pieces and of diminishing the pressure on each unit of surface.

The girder weighs 75 tons, say 1293 kilogrammes a lineal mètre of length ( 58 mètres).
The flanges with their joint-plates weigh 50 tons, say 862 kilogrammes the mètre, which gives $\mathrm{U}=1 \cdot 29$.

The plates or gussets added to the end bars to obtain a double section of rivet, are in part covered by the saving resulting from the fact of the lengths of the bars being a little less than their theoretical lengths.

The excess, 2050 kilogrammes, increased by the angle-iron forming a hand-rail, becomes 2400 kilogrammes; say $\Omega=42$ kilogrammes a lineal mètre, or the 0.032 of the whole weight.

The bars themselves make up a weight of 22600 kilogrammes, not including the accessories reckoned in $\Omega$. Hence we deduce $\mathrm{V} .=1 \cdot 25$.

Here the form favourable to rigidity given to the struts, would allow of the height being increased without much augmenting $V$.

It would be reckoning liberally to take, for example, $1+\mathrm{V}=1.55+0.10 h$ when near a height of 7 mètres. The best height is then theoretically given by the equation $h^{2}(h+7 \cdot 75)=1346$, and is about 9 mètres. Indeed, with this value we should have $V=1 \cdot 43, \mathrm{U} 1 \cdot 36$, and, consequently, the weight $=1 \cdot 250$ kilogramme per mètre, $\Omega$ included; whereas with $h=7, \mathrm{~V}=1 \cdot 25, \mathrm{U}=1 \cdot 29$, and $\Omega=42$, the formula gives 1295 kilogrammes, which agrees with the direct calculation.

These two weights, however, differ but little, which iustifies the adoption of the height 7 mètres. as being the more convenient.

90 . The object of formulæ of weight is to enable us to compute the cost of a girder subject to given conditions, without making a plan and a calculation of it. But, if we wish to attain a certain exactness, it will be well to consider approximatively arrangements which may be adopted, and hich affect U and V .
By way of example, let us suppose it is required to find the weight of a girder of the same form as that of 56 mètres which we have been considering, but the load of which is reduced by onehalf ( 3500 kilogrammes instead of 7000 ). It will be advisable to retain the hollow form of the struts and the double-ribbed flanges; but these ribs may be, for example, $0 \mathrm{~m} \cdot 400$ broad by $0 \mathrm{~m} \cdot 012$ thick, for only half the former number of rivets will be required. If the least section of flange comprises besides a horizontal plate of $550 / 8$ and four angle-irons of $100 / 100 / 12$, it will become sufficient in the middle when the height of the girder rises to 12 mètres; so that we may consider $\mathrm{U}=1 \cdot 06+0 \cdot 60 \frac{7}{12}=1 \cdot 41$.

The strongest strut supporting 96 tons $=\mathrm{Q}_{2}$, will have sufficient rigidity if we give it the hollow form composed of two side plates of $350 / 11$, a back plate of $350 / 8$, and four angle-irons of $100 / 100 / 12$; this would give to the bar a coefficient $v_{2}=1 \cdot 22$. The weakest strut, for the strain $\mathrm{Q}_{1}=25 \frac{1}{2}$ tons, may be formed of two side plates of $220 / 6$ and back plate of $350 / 6$, and four angleirons of $60 / 60 / 8$; this will make $v_{1}=1 \cdot 96$. Consequently we shall compute the general coefficient of stiffness at $\mathrm{V}=\frac{\mathrm{Q}_{2} v_{2}+\mathrm{Q}_{1} v_{1}}{\mathrm{Q}_{2}+\mathrm{Q}_{1}}=1.38$.

With the values thus found for U and V , the formula will give 680 kilogrammes as the weight
of the lineal mètre of girder, a value which must be increased a little to include $\Omega$, remarking, however, that this additional term will be considerably less than in the case of the double load.

The values of U and V are rather high, but they may be reduced a little by adopting a less height than 7 mètres, which will be warranted by a load reduced to 3500 kilogrammes.
91. Tables of Weights.The best height is an implicit function of the load, in virtue of V which diminishes when the load increases.

We must, therefore, distinguish the case of girders lightly loaded, supporting only one rail, for example, and that of girders heavily loaded, or supporting the whole line. In the former case, we will suppose $h=$ $0 \cdot 11 l$, and in the latter $h=\frac{1}{8} l$.

If in the formula of 66 , we suppose $\frac{R s_{0}}{h}=20000$, we shall have for a girder carrying 3500 kilogrammes, and having a height $0 \cdot 11$ of the bearing, $U=1 \cdot 06+$
$\frac{4 \cdot 8 \times 20000 \times \overline{0 \cdot 11}^{2}}{350}$
say $1 \cdot 40$; and for a girder carrying 7000 kilogrammes, and having a height equal to $\frac{1}{8} l ; \mathrm{U}=1 \cdot 28$, say $1 \cdot 30$. $V$ will be taken equal to 1.35 for light girders, and 1.25 for girders heavily loaded.

With these data we form the Tables IX. a and IX. B, giving the values of $\frac{\mathbf{P}}{\left(p+p^{\prime}\right) l^{2}}, \Omega$ not included: that is, the weight of the lineal mètre of girder, for the given values of N and $q$, will be obtained by multiplying the corresponding number of the Table, by the quantity $\left(p+p^{\prime}\right) l$, and adding a certain sum for $\Omega$, as will be shown hereafter.

If the girder were loaded on the upper side, the weight would be slightly diminished when N is even, and increased when N is odd.

There isevidently an advantage in taking N small; because in that case the flanges decrease in weight with the bars which approach the direction of $45^{\circ}$.

92. In large bridges, we may usually give sufficient dimensions to the vertical ribs of the flanges to enable us to reduce to a comparatively trifling matter, or to avoid altogether, the addition of special gussets. Thus the additional term $\Omega$, occasioned by the accessories, for 1 mètre of girder, will increase less rapidly than the bearing, whilst the weight of the mètre of girder increases as the bearing. It follows from this that the ratio of $\Omega$ to the total weight diminishes when the bearing increases. For light girders, we must add to the numbers of the first Table a sum varying, for example, from 5 to 2 per cent., according as the bearing varies from 20 to 80 mètres, and for the heavy girders of the second Table, from 6 to 3 per cent.

In the case of spans of 20,60 , and 80 mètres, we may allow the following loads for girders carrying either one or two rails:-

| $p\left\{\begin{array}{l}\text { Exterior dead weight ... } \\ \text { Weight of the girder itself .. }\end{array}\right.$ | $\begin{aligned} & l=20 \mathrm{~m} . \\ & \text { Girders carrying } \end{aligned}$ |  | $\begin{aligned} & l=60 \mathrm{~m} . \\ & \text { Girders carrying } \end{aligned}$ |  | $\begin{gathered} l=80 \mathrm{~m} . \\ \text { Girders carrying } \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | One Rail. | Two Rails. | One Rail. | Two Rails. | One Rail. | Two Rails. |
|  | kilos. 700 | kilos. 1400 | kilos. 700 | kilos. 1400 | kilos. 700 | kilos. 1400 |
|  | 300 | 500 | 900 | 1400 | 1200 | 2100 |
| Moving load $p^{\prime}$.. .. .. .. | 2500 | 5000 | 2000 | 4000 | 2000 | 4000 |
| Total load a mètre, $p+p^{\prime}$.. | 3500 | 6900 | 3600 | 6800 | 3900 | 7500 |

Let us suppose besides $\mathrm{N}=7$ for bridges of 20 mètres, 8 for those of 30 to 60 mètres, 9 for 70 mètres, and 10 for 80 metres; and let us add the sums for $\Omega$ indicated above. We shall have the following approximative weights:-

| Spans | $\begin{gathered} \text { Mètres. } \\ 20 \end{gathered}$ | $\begin{aligned} & \text { Mètres. } \\ & 30 \end{aligned}$ | Mètres. 40 | $\begin{gathered} \text { Mètres. } \\ 50 \end{gathered}$ | Mètres. 60 | Mètres. 70 | Mêtres. 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight a mètre of a girder carrying one rail $\left(\frac{h}{l}=0 \cdot 11\right)$ | $\begin{aligned} & \text { Kilos. } \\ & 265 . \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 405 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 545 \end{aligned}$ | Kilos. 680 | $\begin{gathered} \text { Kilos. } \\ 820 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 1000 \end{aligned}$ | Kilos. $1220$ |
| Weight a mètre of a girder carrying two rails $\left.\left(\frac{h}{l}=\frac{1}{8}\right) \quad . . \quad \text {.. } \quad . . \quad \text {.. .. } \quad . . \quad . . \quad . .\right\}$ | 460 | 685 | 910 | 1140 | 1365 | 1690 | 2090 |

These weights offer a saving of 0.20 to 0.40 upon those of solid girders satisfying the same conditions, see 119 to 135 . This saving is due to the fact that, by taking precautions to ensure the rigidity of the struts, we may have girders of great height without having a too heavy vertical portion.
93. Girders having Vertical Bars.-If we suppose $\beta=90^{\circ}$ in the girder, Fig. 1657, with a load on the lower side; and express the bearing by $\mathrm{N} \delta$ ( N being even), we shall have a girder with vertical braces, which will weigh for each mètre

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{24 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[4 \mathrm{NU}\left(\mathrm{~N}^{2}-1\right)+6 \mathrm{~N}^{2} \mathrm{~V}+\mathrm{V}\left(\mathrm{~N}^{2}-4\right) q\right]\right. \\
\left.+(1+\mathrm{V})\left[6 \mathrm{~N}^{2}+\left(\mathrm{N}^{2}-4\right) q\right] h\right\}+\Omega .
\end{gathered}
$$

If, on the other hand, $a$ be taken equal to $90^{\circ}$, we shall have a girder with vertical struts, weighing

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{24 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[4 \mathrm{~N} \mathrm{U}\left(\mathrm{~N}^{2}-1\right)+6 \mathrm{~N}^{2}+\left(\mathrm{N}^{2}-4\right) q\right]\right. \\
\left.+(1+\mathrm{V})\left[6 \mathrm{~N}^{2}+\left(\mathrm{N}^{2}-4\right) q\right] h\right\}+\Omega .
\end{gathered}
$$

In one respect, this latter weight will be less than the preceding because the coefficient V affects fewer terms, but in another respect, $\Omega$ will be greater, for it must include the lengthening of the lower flange, which, according to theory, would have a total length of only ( $\mathrm{N}-2$ ) $\delta$, whilst in practice it is absolutely necessary that it reach the abutments, even if the strain upon the extreme sections should be nul.

The other analogous cases in which N would be odd, or those in which the load would be applied to the upper part, will be easily calculated by the aid of the formulæ given, 39 to 62 . But, as these girders with vertical bars are less advantageous than those having bars at an equal inclination, we shall not consider them further.
94. Girders of Crosses, or Double Lattice.-Girders of simple crosses, with or without vertical stays, should be considered as a double lattice, and may be calculated by imagining that they result from the juxtaposition of two simple lattices. Now in every multiple system in which several pieces simultaneously combine to resist a certain distortion which may be prevented by a single piece, there takes place a distribution of strains that it is not always easy to discover exactly when the conditions are complex; it may be computed, however, with some degree of certainty when we take care to render the conditions of strain equivalent or comparable. The general rule is that the various pieces should undergo equal proportional extensions, and that,

## BRIDGE.

consequently, they distribute among each other the total strain proportionally to their sections; the strain a unit of section will be equal upon the several bars, with the exception of slight differences which may proceed from unequal tension given at the time of fixing. But in the case in which the strain acts by compression, the area of the sections does not alone influence the distribution, rigidity interferes, and if the forms of the sections are different, it is difficult to know to what degree a certain bar will yield, and another, and stronger one, to concentrate the load upon itself. The way to avoid these uncertainties is to make identical the multiple pieces which replace a single piece. However, with regard to multiple lattice-girders, this equal distribution must not be considered from bar to bar, but from system to system, because the pieces brought together being more or less distant from each other, the stress may vary from one to the other. If the whole is composed of $n$ simple lattices, each of them will be calculated as belonging to a girder supporting only the $\frac{1}{n}$ of the given load.
95. Girders of Crosses and Vertical Rods.-If we consider first a girder of crosses and vertical rods, for a load applied to the lower part, Fig. 1701, it is obvious that it may be resolved into two simple girders, one with vertical braces, the other with vertical struts, and the formulæ of which for the case of N even are given in 93. But the weight will be less than the arithmetical mean of these formulæ, for these vertical rods belonging at once to the two simple component lattices, are subject to a compressive strain on one side and to a tensile strain on the other; and the maximum strain upon them will be a constant tension $\frac{1}{2}\left(p+p^{\prime}\right) \delta$. The two end vertical

bars must be excepted, the strain upon them, which is one of compression, being $\frac{1}{4}(\dot{\mathrm{~N}}-1) \delta\left(p+p^{\prime}\right)$; so that these two bars alone make up the same theoretical weight as all the others, or rather more, for, resisting compression, they receive the coefficient V. We have thus as the total weight of the $\mathrm{N}+1$ vertical bars the value $\frac{1}{2}(\mathrm{~N}-1)(1+\mathrm{V})\left(p+p^{\prime}\right) \delta t h$, and the main weight of the lineal mètre of girder will be

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{24 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[4 \mathrm{NU}\left(\mathrm{~N}^{2}-1\right)+(1+\mathrm{V})\left(3 \mathrm{~N}^{2}+\frac{\mathrm{N}^{2}-4}{2} q\right)\right]\right. \\
\left.+(1+\mathrm{V})\left[3\left(\mathrm{~N}^{2}+4 \mathrm{~N}-4\right)+\frac{\mathrm{N}^{2}-4}{2} q\right] h\right\}+\Omega
\end{gathered}
$$

If the load be placed above the girder, the intermediate vertical bars will still support a maximum strain of $\frac{1}{2}\left(p+p^{\prime}\right) \delta$, but it will be one of compression which will necessitate the application of the coefficient of stiffness; and as to the two extreme bars, they, too, will be strengthened a little, for they will have to support directly the weight $\frac{1}{2}\left(p+p^{\prime}\right) \delta$ placed upright with the abutment, a weight which we had not to consider when the load was below, because it rested immediately upon the masonry without passing through the vertical bar. Consequently we shall have, for the case of a load on the upper side, and N even,

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{24 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[4 \mathrm{NU}\left(\mathrm{~N}^{2}-1\right)+(1+\mathrm{V})\left(3 \mathrm{~N}^{2}+\frac{\mathrm{N}^{2}-4}{2} q\right)\right]\right. \\
\left.+(1+\mathrm{V})\left(3 \mathrm{~N}^{2}+\frac{\mathrm{N}^{2}-4}{2} q\right) h+24 \mathrm{NV} h\right\}+\Omega
\end{gathered}
$$

When $N$ is odd, we have for the case of a load on the lower side,

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{(\mathrm{N}-1)\left(p+p^{\prime}\right) t}{24 \mathrm{~N}}\left\{\frac{(\mathrm{~N}+1) l^{2}}{2 \mathrm{~N}^{2} h}[8 \mathrm{~N} \mathrm{U}+(1+\mathrm{V})(6+q)]\right. \\
\left.+(1+\mathrm{V}) h\left[12+\frac{\mathrm{N}+1}{2}(6+q)\right]\right\}+\Omega
\end{gathered}
$$

and the changing of the load from the lower side to the upper will occasion an increase of expense, because the intermediate vertical bars will have to be stiffened, and the end bars will have the additional strain of a partial weight resting thr ugh them upon the abutment.

But we shall see presently that a saving may be effected by suppressing the intermediate vertical bars or rods, when possible; and if their presence is necessary, it will be for the purpose of affixing the cross-pieces, or for other purposes foreign to the resistance of the girder, so that it will be more logical to consider them as an accessory addition, their section being supposed as given according to the duty they have to fulfil (see 100).
96. The wooden girders of Howe's system have the form of girders of crosses with vertical rods; but an essential difference from the point of view of resistance is, that the wooden crosses, merely abutting the bearing-blocks, cannot resist by tension; under a given load, there is only one of the bars of each cross that resists, whilst the vertical rods, each composed of several smaller ones,
always take a tensile strain; the system resists as a simple yielding lattice, that is, its form changes according as the strain comes upon one or the other piece of each cross.
97. Girders of Crosses without Vertical Rods.-If we consider a lattice of crosses without intermediate vertical rods, loaded on the upper side, Fig. 1702, we see that it may be resolved into two simple systems given by Figs. 1703 and 170t. Now the system of Fig. 1703 corresponds to the case $\alpha=\beta$ in 53 by changing $\delta$ into $2 \delta, m$ into $\frac{N}{4}$, and reducing the load by half. Besides this, we must add the two end vertical bars, the strain upon which is nearly $\frac{\mathrm{N}}{4}\left(p+p^{\prime}\right) \delta$; they will weigh together $\frac{1}{4}(1+\mathrm{V})\left(p+p^{\prime}\right) l h t$, if we apply to them, instead o. V , the coefficient $\frac{1+\mathrm{V}}{2}$, which
 the bearing only.

The system of Fig. 1704 will come under the case of $\alpha=\beta$ of section 50 , but only under the condition that the extreme weight be made equal to the other weights, instead of being $\frac{3}{4}$ only. On this hypothesis, the strain upon the $n$th strut or the $n$th brace will be $\frac{\delta^{\prime}}{\sin . a}\left[n p+\frac{(m+n)^{2}}{4 m} p^{\prime}\right]$, a value which extends to brace No. 0 , as well as to the $n$th strut, and which supposes that the weights applied to the several summits are equal to $p \delta^{\prime}$ or to $\left(p+p^{\prime}\right) \delta$, according to the case. The total weight of the oblique bars will consequently be $\frac{2 m^{2}\left(p+p^{\prime}\right) \delta^{\prime} h t}{\sin ^{2} a}\left[1+\frac{2 m^{2}+1}{12 m^{2}} q\right]$ a formula in which $\delta^{\prime}$ will be changed into $2 \delta, m$ into $\frac{\mathrm{N}}{4}$, and in which the load will be divided by 2. For the flanges of this figure, we must also, in the formulæ of 50 , take into account the modification of the load in the extreme summit.

Adding together the weights of the two component lattices, and dividing by the bearing, we shall obtain as the weight of the lineal mètre of girder, Fig. 1702,

$$
\begin{aligned}
\frac{\mathrm{P}}{\mathrm{r}}=\frac{1}{24}(p & \left.+p^{\prime}\right) t\left\{\frac{l^{2}}{\mathrm{~N} h}\left[4 \mathrm{U} \cdot \frac{\mathrm{~N}^{2}-1}{\mathrm{~N}}+3(1+\mathrm{V})\left(1+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}^{2}} q\right)\right]\right. \\
& \left.+3 h(\mathrm{~N}+2)(1+\mathrm{V})\left(1+\frac{\mathrm{N}-2}{6 \mathrm{~N}} q\right)\right\}+\Omega
\end{aligned}
$$

This formula obviously remains the same when the load is on the lower side instead of the upper. It supposes N even, but may also be applied approximatively for N odd; however, in this latter case, it is better to employ the formula in 99.
98. The point of view from which we have considered the question in the preceding section offers the difficulty of a certain complication in calculating the strains upon the various pieces, and as a multiple system leaves the distribution of the loads arbitrary, we take advantage of it in this case to adopt a more convenient process. In the preceding method a brace bore the same strain as one of the struts to which it was fixed at the extremities, but now we shall suppose that both bars of a cross are subject to a strain $\frac{F}{2 \sin . \alpha}, F$ being the stress in the interval occupied by the cross.
It follows from this that in the two flanges, the sections or divisions situate perpendicularly to each other will support also the same strain (on account of the condition of equilibrium relative to the horizontal projections of the forces), which was not the case on the preceding hypothesis; this strain is calculated by the moment of rupture with respect to the centre of the cross; the total weight, however, of the two flanges together remains the same as before. As to the lattice, not including the end vertical bars, it is represented by $\frac{(1+\mathrm{V}) h t}{2 \sin ^{2}{ }^{2} \alpha} \Sigma \mathrm{~F}$, an expression in which $\Sigma \mathrm{F}$ denotes the sum of the absolute maxima strains in the successive intervals, for the whole bearing, the value of which sum is given in 6 for the two cases of N even or odd. Adding the end vertical bars, the
strain upon which is $\frac{1}{4}(\mathrm{~N}+1)\left(p+p^{\prime}\right) \delta$, including the weight they directly support, and which will, therefore, weigh together $\frac{\mathrm{N}+1}{4 \mathrm{~N}}\left(p+p^{\prime}\right) t l h$, we obtain as the total weight of the lattice per lineal mètre, in the case in which N is even, the following expression:

$$
\frac{1}{8}(1+\mathrm{V})\left(p+p^{\prime}\right) t\left\{\left(\mathrm{~N}+2 \frac{2}{\mathrm{~N}}+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}} q\right) h+\left(1+\frac{\mathrm{N}^{2}-4}{6 \mathrm{~N}^{2}} q\right) \frac{l^{2}}{\mathrm{~N} h}\right\}
$$

which differs only by a slight increase of the end vertical bars, from the value which enters into the formula of the preceding section. This difference proceeds from the fact of our having, in 97 , attributed to these bars a mean strain between the cases of a load on the upper and a load on the under side.
99. For N odd, we obtain the following weight of the lineal mètre of girder, the latter being calculated in the manner we have just alluded to :
$\frac{\mathbf{P}}{l}=\frac{\mathrm{N}^{2}-1}{24 \mathrm{~N}}\left(p+p^{\prime}\right) t\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[4 \mathrm{UN}+(1+\mathrm{V})\left(3+\frac{1}{2} q\right)\right]+3 h(1+\mathrm{V})\left(\frac{\mathrm{N}+1}{\mathrm{~N}-1}+\frac{q}{6}\right)\right\}+\Omega$.
This formula may be admitted for a load on the upper or on the under side, for the situation of the load occasions but a slight difference in the weight of the end vertical bars.

On comparing the formulæ of the lattice composed of crosses, witb those of girders of simple triangles with bars at an equal inclination, we see that the difference consists essentially in this fact, namely, that with crosses the weight of the lattice is modified as if the length of the intervals $\delta$ had been doubled, and, consequently, their number reduced by one-half. We ought, in general, to adopt that one of the two systems which brings the inclination of the bars nearest to $45^{\circ}$.
100. Vertical Rods considered as a supplementary term.-On comparing girders of crosses without intermediate vertical rods with those which possess them, we find that these latter have an excess of weight represented by these additional rods, which thus form, from the point of view of the resistance of the girder, a superfluous addition. Their use is, in fact, limited to distributing the load among all the summits both at the top and at the bottom, instead of affecting directly only one of the flanges; but there is no advantage in this, because the horizontal space between the points of application of the strains remains the same.

But these rods are useful in certain cases, chiefly in girders loaded on the lower side, to stay the vertical portion and maintain its perpendicularity. We are justified, therefore, in considering this kind of girder ; only, as the reason for introducing intermediate vertical rods is foreign to the theoretical resistance, we shall reckon them as a supplementary addition, represented for the mètre of girder by the term $\frac{r \mathrm{~N} h}{l}$, in which $r$ denotes the weight of the lineal mètre of rod. These rods, which usually need be placed upon one side only of the vertical portion of the girder, may have the form of a single $T$ composed of a projecting strip of plate iron and two angle-irons. If, for instance, for heights of 2 to 5 mètres, these sections are composed in the following manner :-

| Height $h$ of the girder | $2^{\text {m }}$ | $3^{\text {m }}$ | $4^{\text {m }}$ | $5^{\text {m }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Mean section of a vertical rod .. $\left\{\begin{array}{l}\text { Projecting plate of .. } \\ \text { Two angle-irons of .. }\end{array}\right.$ | $\begin{aligned} & 130 / 10 \mathrm{~mm} \\ & 80 / 80 / 10 \end{aligned}$ | $\begin{gathered} 200 / 10 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 260 / 10 \\ 110 / 70 / 10 \end{gathered}$ | $\begin{gathered} 260 / 10 \\ 130 / 75 / 12 \end{gathered}$ |

we shall conclude that $r$ may be represented by $8(2+h)$, and, consequently, the additional term for vertical rods will be $\frac{8 \mathrm{~N} h(2+h)}{l}$. The rest of the girder will be regarded as expressed by the formulæ for lattices composed of crosses without intermediate vertical rods.
101. Examples.-First Example.-Girder of Crosses without intermediate Vertical Rods, Figs. 1705 to 1707.-Data : $l=24^{\mathrm{m}}, \delta=3^{\mathrm{m}}, \mathrm{N}=8, h=2^{\mathrm{m}} \cdot 50, p=1000^{\mathrm{k}}$ (the girder itself weighing 300), $p^{\prime}=2400^{\mathrm{k}}$ (one rail), $p+p^{\prime}=3400, q=0.706$.

Following the method of calculation set forth in 98, the strains upon the successive divisions of the flanges will be, reckoning from the middle, $94 \cdot 9,82 \cdot 6,58 \cdot 2,21 \cdot 4$ tons; those upon each of the two bars forming the successive crosses will be $8 \cdot 2,14 \cdot 1,20 \cdot 6,27 \cdot 9$ tons; and that upon the end vertical bars 23 tons.

The braces are, like the struts, made of double angle-irons, in order that the rigidity of the compressed bar may be identified with that of a cross composed of four angle-irons by joining the sections of the brace and the strut. For a bar in the middle, for example, the reduced thickness of a cross of four angle-irons of $70 / 70 / 8$ millimètres will be $0 \mathrm{~m} \cdot 08$; the ratio of the free length to this thickness is $\frac{2 \cdot 80}{0 \cdot 08}$ or 35 , which requires a coefficient of stiffness $=1 \cdot 56$. With the section adopted, this coefficient is $1 \cdot 55$, a value which is all the more sufficient because the rivetings at the extremities of the bar amount to a joining of the metal.

The fixing of the extreme bars requires the application of joint-plates to give the rivets a double resisting section: we obtain by this means twenty sections, which is amply sufficient.

Direct calculation gives for the whole girder:-


Say 300 kilogrammes per mètre of girder.

Thefflanges weighing 224 kilogrammes the mètre of girder, whilst the formula in 97 gives them 167 U, we have U 1 34 .

The bars amount to 1900 kilogrammes, 140 of which are for the special pieces for fixing the end 1705.

Scale $0 \cdot 01$.

bars and the plates which are attixed to the bars at the points where they 1706. cross each other ; but these 140 kilogrammes are nearly compensated by the $a b$, min. see. cd , max, sec. saving resulting from the fact that the bars have not quite their theoretical length; so that we may suppose $\Omega$ nul, and take then $\mathrm{V}=1 \cdot 30$, at which we arrive from a comparison of the weight $\frac{1900}{25}=76$ kilogrammes. resulting from the calculation, with that of $33(1+\mathrm{V})$, furnished by the formula.

If we admitted $1+\mathrm{V}=1 \cdot 95+0 \cdot 15 h$, the best height would result from the equation $120=(2 \cdot 65+0 \cdot 408 h) h^{2}$, and would be about 5 mètres, a great value which proceeds from the extreme lightness of the lattice; but it is prudent not to exceed the height of $4^{\mathrm{m}} \cdot 50$, near which we cannot practically reduce the section of the flanges.

A greater diminution of the height is warranted by the small variation of
 the weight and by the advantage of diminishing the importance of the exterior tie-pieces. Yet, if the height allowed be unlimited, we shall do well to take a height of 3 mètres. instead of 2.50 as in the plan.

If the load were doubled and made 6800 kilogrammes the mètre, everything included, it would be easy to reduce U to $1 \cdot 25$, and V to $1 \cdot 15$, even if the height were carried up to 3 . $\cdot 50$.
102. Second Example.-Girder of Crosses and without Vertical Rods, for a Tubular Bridge of 72 mètres span, Figs. 1708 to 1717.-Data: $l=72^{\mathrm{m}}, \delta=8^{\mathrm{m}}, \mathrm{N}=9, h=9^{\mathrm{m}}, p=3000^{\text {k }}$ (the girder itself weighing 1700), $p^{\prime}=4000^{\mathrm{k}}, p+p^{\prime}=7000^{\mathrm{k}}, q=0.5714$.

When one line of rails only is loaded, $p^{\prime}$ amounts to only 3000 kilogrammes, which will allow us to reduce by $\frac{1}{7}$ the weight of the girder if we do not object to raise R to 7 kilogrammes the square millimètre for the very exceptional case of two heavily-laden trains crossing the bridge at the same time.

The strains upon the portions of the flanges, reckoning from the middle to the ends, are:-498, $473,398,274$, and 100 tons; the strains upon each of the two bars of the crosses $=24,52,82,115$, and 150 tons; the end vertical bar support 112 tons.

In the elevation of this girder and some others we have been obliged to exaggerate slightly the thicknesses of the horizontal plates of the flanges in order to render the arrangement of them perceptible.

The struts have a double T section, offering a great moment of resistance in the direction normal to the vertical portion of the girder. In the plane of this vertical portion, the free length being only half, the rigidity is sufficient with the sections adopted. We might, if necessary, take the sections of a hollow form, as in Figs. ${ }^{1} 690$ to 1700.

The braces are composed of two al and parallel pieces enclosing the strut, and are fixed externally to the vertical ribs of the flanges; but tine two bars of the middle cross are identical and in the form of a double $T$.

In fixing the end bars, the double rib allows us to place eighty rivets of $0^{\mathrm{m} \cdot 020}$; so that it is useless to double the shearing sections. The thickness of $0 \mathrm{~m} \cdot 012$ given to the ribs renders tearing a way impossible.

The gussets at the point where the bars cross each other serve not only to connect the brace and the strut, but as joint-plates for these pieces. But for several bars at the end, composed of two thicknesses of plate, this gusset can cover the joint of only one plate; the joint of the second plate is then made to fall in another place where it has special joint-plates. The angle-irons of the bars may be continuous throughout the length.

The following is the result of the calculation :-


## BRIDGE.

It follows from this that the mean weight of the lineal mètre of girder will be 1630 kilogrammes.

We might make $\stackrel{\circ}{-1}$ up an additional term $\Omega$ by taking, in the flanges, the border angle-irons which do not euter into the theoretical resistance, and the plates at the ends of the cross-girders; and in the bars, the plates at the points of crossing and the joint-plates; to this might be added the hand-rails. But we should have to deduct the saving in the length of the bars, which does not reach the theoretical length; and this saving more than compensates the weight of the light pieces of railing. The plates upon the bars and their joint-plates may enter into V , for these pieces are a function of $h$ in this sense that a less height would allow us to suppress the joint-plates and reduce the dimension of the plates. The accessories of the flanges may be introduced into the coefficient U. From this point of view, we shall regard $\Omega$ as nul, $U$ $=1 \cdot 36$, and $\mathrm{V}=1 \cdot 25$.

If we admit that when near the most economical height, 1 +V increases according to the law $1+$ $\mathrm{V}=1 \cdot 35+0 \cdot 10 h$, and that U is reckoned as only $1 \cdot 06$, according to the rule given in 64, the best height will result from the equation $0.807 h^{3}+$ $5 \cdot 45 h^{2}=2726$, and will be 13 mètres. Indeed we shall then have $V=1 \cdot 65, U$ $=150$, and the weight of the girder a metre will be brought down to 1510 kilogrammes, whilst for $h=9, \mathrm{~V}$ $=1 \cdot 25$ and $U=1 \cdot 36$, the formula gives 1626 kilogrammes.
103. ThirdExample. -Girder of Crosses and with Vertical Rods, of 40 mètres span, Figs. 1718 to 1725.-Data: $l=40^{\mathrm{m}}, \delta=4^{\mathrm{m}}$,

人
Scale 0.005 ，and the sections 0.025 ．

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 －－－－－－ $\qquad$
$\square$
Transverse section．




$\mathrm{N}=10, h=4^{\mathrm{m}} \cdot 50 ; p=2500^{\mathrm{k}}, \quad p^{\prime}=3200 \quad$ (one line of rails loaded), $p+p^{\prime}=5700^{\mathrm{k}}$ $q=0 \cdot 56$.

In the exceptional case in which both lines are simultaneously loaded at the rate of 4300 kilogrammes a mètre each, the bridge will be subjected to a strain of about $7^{\mathrm{k}} \cdot 2$, and this may be allowed.

Successive strains upon the portions of the flanges $=248,228,188,127$, and 46 tons;
Successive strains upon the bars $=16,28,41,54$, and 68 tons;
Strain upon the end vertical bars $=51$ tons.
In order to fix properly the struts in the double I form, whose greatest stiffness is in the direction normal to the vertical portion of the girder, double vertical ribs to each flange have been adopted, as in the preceding example. The braces are of flat, equal and parallel bars enclosing the struts, except in the two middle crosses, in which the flat bars are replaced by two angle-irons of $110 / 70 / 11$ millimètres, because, in certain positions of the load, these braces may have to bear slight compression.

The bars bearing the greatest strain are fixed without having recourse to double shearing sections, for 20 or 24 rivets may be placed upon each of the two ribs, and this is amply sufficient.

The intermediate vertical rods serve for affixing the cross-girders. The following is the result of calculation :-


Thas the girder weighs 815 kilogrammes per mètre, its whole length being $41^{\mathrm{m}} \cdot 80$.
The term $\Omega$ will be nul, for the bar-plates amounting to only 100 kilogrammes, do not even compensate the saving upon the length of the bars.

The flanges amount, according to calculation, to 563 a mètre, and according to the formula 435 U ; hence $\mathrm{U}=1 \cdot 30$.

The oblique bars $=207$ according to calculation, and $90(1+\mathrm{V})$ according to the formula. Therefore $\mathrm{V}=1 \cdot 30$.

Supposing $1+\mathrm{V}=1 \cdot 75+\frac{1}{8} h$, and expressing the vertical rods by an additional term
 struct a tubular bridge, and to suppress the intermediate vertical rods. Thus for bridges of two girders of more than 40 mètres span, we may adopt the tubular system in preference to girders loaded on the lower side, if we desire to obtain greater stability in the girders.
104. Tables of the Weights of Girders.-The best height to adopt is an implicit function of the load in virtue of the coefficient $V$, so that the height of heavy girders may be increased a little.

If for girders lightly loaded we suppose $h=0 \cdot 11 l, \mathrm{U}=1 \cdot 40, \mathrm{~V}=1 \cdot 35$, and for girders heavily loaded $h=\frac{1}{8} l, \mathrm{U}=1 \cdot 35$, and $\mathrm{V}=1 \cdot 30$, these coefficients will allow us to suppose $\Omega$ nul, and by applying the general formulæ ( 97 and 99 ), we may construct the Tables on p. 827, giving the values of $\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}$; that is, the weight of the lineal mètre of girder will be found by multiplying the numbers of the Tables by $\left(p+p^{\prime}\right) l$.

The least expense corresponds to $\mathrm{N}=8$ in the first Table, and to $\mathrm{N}=7$ in the second. These values give, having regard to the heights adopted, directions to the bars a little less than $45^{\circ}$ upon the horizontal, since the Tables decrease with N.

If there be intermediate vertical rods, they must be added as we have seen in 100 , which is equivalent to adding to the numbers in the Tables the quantity $\frac{8 \mathrm{~N} h(2+h)}{\left(p+p^{\prime}\right) l^{2}}$.
105. Supposing loads similar to those which were taken for girders of triangles (92), the weight of the girders themselves only being slightly reduced in a few cases, and taking the same values of N, namely, 7 for a span of 20 mètres, 8 for spans of 30 to 60 mètres inclusive, 9 for 70 mètres, and 10 for 80 mètres, we shall have the following approximative weights for girders without intermediate vertical rods.

| Spans .. .. | Mètres. $20$ | $\begin{array}{\|c} \text { Mètres. } \\ 30 \end{array}$ | $\begin{aligned} & \text { Mètres. } \\ & 40 \end{aligned}$ | $\begin{aligned} & \text { Mètres. } \\ & 50 \end{aligned}$ | $\begin{aligned} & \text { Mètres. } \\ & 60 \end{aligned}$ | $\begin{array}{\|c} \text { Mètres. } \\ 70 \end{array}$ | $\begin{gathered} \text { Mètres. } \\ 80 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight of the lineal mètre of girder carrying one rail ( $h=0 \cdot 11 l$ ) | $\begin{gathered} \text { Kilos. } \\ 260 \end{gathered}$ | $\begin{gathered} \text { Kilos. } \\ 385 \end{gathered}$ | $\begin{gathered} \text { Kilos. } \\ 520 \end{gathered}$ | $\begin{gathered} \text { Kilos. } \\ 650 \end{gathered}$ | $\begin{gathered} \text { Kilos. } \\ 780 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 940 \end{aligned}$ | Kilos. 1140 |
| Weight of the lineal mètre of girder carrying one line of rails $\left(h=\frac{1}{8} l\right) \quad$.. $\quad . \quad$.. ... $\}$ | 445 | 670 | 880 | 1100 | 1310 | 1590 | 1900 |

It will be seen that in large spans these girders of crosses without vertical rods offer a small saving over those of simple triangles (92), because the supposed values of N allow of the bars being placed nearer the inclination of $45^{\circ}$, and because in the double lattice the strains upon the
P
Table X. A.-Value of $\frac{P}{\left(p+p^{\prime}\right) l^{2}}$ for girders lightly loaded.

|  | $\mathrm{N}=5$. | $\mathrm{N}=6$. | $\mathrm{N}=7$. | $\mathrm{N}=8$. | $\mathrm{N}=9$. | $\mathrm{N}=10$. | $\mathrm{N}=11$. | $\mathrm{N}=12$. | $\mathrm{N}=13$. | $N=14$. | $\mathrm{N}=15$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0.40 | $0 \cdot 003674$ | $0 \cdot 003645$ | $0 \cdot 003623$ | $0 \cdot 003617$ | $0 \cdot 003622$ | $0 \cdot 003630$ | $0 \cdot 003649$ | $0 \cdot 003667$ | $0 \cdot 003694$ | 0.003718 | $0 \cdot 003750$ |
| $0 \cdot 50$ | $0 \cdot 003689$ | 0.003657 | $0 \cdot 003636$ | $0 \cdot 003629$ | $0 \cdot 003634$ | $0 \cdot 003643$ | $0 \cdot 003662$ | $0 \cdot 003680$ | $0 \cdot 003707$ | $0 \cdot 003732$ | $0 \cdot 003764$ |
| $0 \cdot 60$ | $0 \cdot 003703$ | 0.003669 | $0 \cdot 003649$ | $0 \cdot 003641$ | $0 \cdot 003647$ | $0 \cdot 003655$ | $0 \cdot 003675$ | 0.003693 | $0 \cdot 003721$ | $0 \cdot 003745$ | $0 \cdot 003778$ |
| $0 \cdot 70$ | $0 \cdot 003718$ | $0 \cdot 003682$ | 0.003661 | $0 \cdot 003653$ | $0 \cdot 003659$ | $0 \cdot 003667$ | 0.003688 | $0 \cdot 003706$ | $0 \cdot 003734$ | $0 \cdot 003759$ | $0 \cdot 003793$ |
| 0.80 | $0 \cdot 003732$ | $0 \cdot 003694$ | $0 \cdot 003674$ | $0 \cdot 003665$ | $0 \cdot 003672$ | $0 \cdot 003679$ | $0 \cdot 003700$ | 0.003719 | $0 \cdot 003748$ | $0 \cdot 003773$ | $0 \cdot 003807$ |


| Table X. b.—Value of $\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}$ for girders heavily loaded. |
| :--- |

struts and braces are less, and, therefore, the several parts may be fixed without the addition of supplementary pieces.
106. For girders with vertical rods calculated as in 100, the Table will become on the hypothesis of the same heights, although there may be occasion to reduce them a little in this case:-

| Spans .. | $\begin{array}{\|c} \text { Mètres. } \\ 20 \end{array}$ | Mètres. $30$ | $\begin{gathered} \text { Mètres. } \\ 40 \end{gathered}$ | Mètres. 50 | Mètres. $60$ | Metres. 70 | Mètres. 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight of the lineal carrying one rail | Kilos. | $\begin{gathered} \text { Kilos. } \\ 420 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 565 \end{aligned}$ | Kilos. 700 | $\begin{gathered} \text { Kilos. } \\ 840 \end{gathered}$ | Kilos. 1020 | Kilos. |
| metre of girder $\quad, \quad$ one line of rails | 475 | 720 | 940 | 1170 | 1390 | 1690 | 2020 |

being weights differing but little from those of girders of simple triangles (92).
107. Double Crosses.-In cases of limited height, or in those of small girders, if we happen to have $\zeta<\frac{\delta}{\sqrt{2}}$ we may place two crosses in an interval $\delta$ comprised between two successive points of applioation of the load. In this case, the weight of the diagonals will be modified, $\sin ^{2}{ }^{2} \alpha$ being changed into $\frac{1}{2} \sin .^{2} a^{\prime}, a^{\prime}$ representing the greater inclination obtained by the doubling of the crosses. The flanges depend only on the distance apart of the loaded points; the vertical rods, if there are any, will be placed square with the cross-girders, in every second cross, Fig. 1726.

Confining oursclves to the case of N odd, the weight of the mètre of girder may be
 computed by the formula

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(\mathrm{N}^{2}-1\right)\left(p+p^{\prime}\right) t}{48 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[8 \mathrm{NU}+3(1+\mathrm{V})\left(1+\frac{1}{6} q\right)\right]\right. \\
\left.+12 h(1+\mathrm{V})\left[\frac{\mathrm{N}+3}{\mathrm{~N}+1}+\frac{1}{6} q\right]\right\}+\Omega
\end{gathered}
$$

when the load is placed upon the lower part ; and by

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\frac{\left(\mathrm{N}^{2}-1\right)\left(p+p^{\prime}\right) t}{48 \mathrm{~N}}\left\{\frac{l^{2}}{\mathrm{~N}^{2} h}\left[8 \mathrm{NU}+3(1+\mathrm{V})\left(1+\frac{1}{6} q\right)\right]\right. \\
\left.+12 h\left[\frac{4 \mathrm{~N} \mathrm{~V}}{\mathrm{~N}^{2}-1}+(1+\mathrm{V})\left(1+\frac{1}{6} q\right)\right]\right\}+\Omega,
\end{gathered}
$$

if the load is on the upper part. The coefficient $V$ is applied to the end vertical bars as well as to the other bars under compression.
108. Multiple Lattice-girders.-General Observations.-A multiple lattice may usually be considered as resulting from the superposition of several simple lattices, at least if we omit to consider the connection at the points where they cross each other. The component simple lattices may be of different systems, as it happened in the case of girders of crosses; sometimes, in such cases, certain bars participate simultaneously in the composition of two elementary lattices; and if they are acted upon on one side by a force of compression and on the other by a force of tension, their resulting maximum strain may be limited to a small value, as in the case of vertical rods considered in section 95 .

We are now about to consider exclusively another kind of multiple lattice, resulting from the union of several simple lattices, identical in form, but placed one before another. Thus, in Fig. 1727, the system A BCD is a simple lattice, but we shall form a double one by superposing the parallel lattice $A^{\prime} B^{\prime} C^{\prime}$, and this duplication will enable us to distribute the load applied to the girder over a double number of points, without subjecting the portions of the flanges to a deflecting strain. The load may be applied to all the summits or only to some of them. But even if all the loaded summits should belong exclusively to one of the elementary lattices, all the bars will take part in the resistance,
 for in virtue of the rigidity of the portions of the flanges a parallelogram such as BCED cannot be distorted by the lengthening or shortening of the bars BC, DE, without producing a change in the intermediate bar $\mathbb{C}^{\prime} \mathrm{D}^{\prime}$. Thus it is usual to calculate first the strains in a simple system as A B C D, and then to divide them equally among the whole of the bars. The flanges are regarded as loaded in a continuous manner.

The connection at the points of crossing enables a bar to receive variable strains at the same instant in its several subdivisions, and the vertical portion of the girder thus possesses a certain moment of resistance. But these secondary considerations are never taken into account.

The theoretical weight of a multiple lattice must, therefore, be obviously the same as that of a simple lattice the bars of which have the same inclination. But in practice each system has its advantages and its disadvantages, and may modify the expense in one direction or in another. By multiplying the bars we incur excesses of section, because the theoretical strains become very
small and would require dimensions which the importance of the girder would not allow ; the plates, too, on the points where the bars cross each other increase in proportion. On the other hand, a multiple lattice may always be traced near $45^{\circ}$, whatever the interval between the crossgirders may be, whilst with simple bars we may be obliged to have recourse to less advantageous inclinations. From the point of view of rigidity, if the multiplicity of the points of crossing offers the advantage of subdividing the bars into portions of inconsiderable length, on the other hand the sections decrease, the vertical portion of the girder acquires a tendency to twist like a thin sheet of iron, and we are obliged to remedy this disadrantage by the addition of stiffening vertical rods. At the ends of the girder, a certain number of the elementary lattices terminate by fragments of bars which are fixed at different points in the height of the end vertical bar. This bar would, therefore, have a tendency to bend if the precaution were not taken of replacing it by a broad piece of solid plate iron stiffened by projecting ribs.

It seems, therefore, preferable not to multiply beyond a moderate degree the systems of lattice, but to construct them of bars possessing the greatest possible rigidity. It is only in cases in which the rivetings become difficult that we should have recourse to the multiple system, for then the bars, being weakened, may be fixed by a very small number of rivets. This is the principal consideration which should decide the degree of multiplicity.

As to the comparative security of simple and multiple lattices, provided the joints and rivetings are equally solid, they may be regarded from this point of view as equal. If the bars were of cast iron or of wood, it might be objected that a defect in a single piece would be sufficient to ruin a simple system, whilst in a multiple system the resistance never falls upon a single piece. But in bridges of malleable iron, this objection would be specious, for a bar should have such a section that it may be composed, in almost all cases, of several pieces riveted together ; consequently, a defect in one of the pieces will be protected by the other sound parts, and it may happen that all the elementary pieces may be defective without the whole breaking, if the defective portions are isolated by the rivetings; the only result in such a case being a greater strain upon the sound portions than was anticipated. In those parts in which the strains are inconsiderable, a multiple system will offer in general a great excess of resistance, but it will be at the price of an excess of material, and we might, if we chose, increase the sections in a simple system to obtain an equal additional security.
109. If the inclination of the bars remains constant when the height varies, the lattice will have, theoretically, a nearly constant weight; in order that the flanges may be supported at the same intervals, the multiplicity of the lattice must increase with the height. Indeed, even when the general rigidity of the vertical portion of the girder resides chiefly in the addition of projecting ribs, we must reckon upon a surplus expense for the local rigidity of the several portions of the compressed bars, and for other conditions which must be fulfilled, principally not to reduce the sections below a certain minimum, not to make a continuous variation from bar to bar, but only by successive groups, to take into account in certain cases the weakening occasioned by the rivetholes, and so on. We may affect the whole of the lattice with a coefficient V, but in analogy with the notations employed for girders of triangles or crosses, in which V was specially applied to the struts, we will denote the general coefficient by $\frac{1+\mathrm{V}}{2}$, (82).
110. The stiffening vertical rods may serve also for affixing the cross-girders, and the vertical cross-bracing. Their section increases with the height, but may be taken nearly constant with the load in the ordinary cases of railway bridges, for they will thus add, as they should do, a relative increase of strength which will be the greater as the lattice becomes weaker and more liable to distortion. For the same reason they may be made to vary but little in the several points in the length of a given girder.

Instead of placing only one vertical rod in a line with each cross-girder, it may be advantageous, if the distances $\delta$ are great, to have two for each interval, placing them alternatively on each side of the vertical portion of the girder when nothing prevents. Or we may place principal rods in a line with the cross-girders, and other weaker rods in the middle of the intervals.

Let us suppose adopted as a mean, for the case of one rod to the interval $\delta$, the sections given in Fig. 1728.


The weights given by this figure may be put into the formula $15 h$, so that the weight of the rods a lineal mètre of girder will be represented by $\frac{15 h^{2}}{\delta}$.
111. Formulce of Weight.-The flanges will make up a weight $\frac{\left(p+p^{\prime}\right) l^{2} t \mathrm{U}}{5 h}$ a lineal mètre of girder, regarding the load as continuous. The bars at $45^{\circ}$ give $\frac{1+\mathrm{V}}{24}\left(p+p^{\prime}\right)(6+q)$ it

The end vertical bars transmit to the abutments the strains upon the bars which are fixed at different points in their length; their section should theoretically increase from the top to the bottom, and as their mean compression is equal to half the reaction of the abutment, that is to $\frac{1}{4}\left(p+p^{\prime}\right) l$, their total weight will be $\frac{1}{2}\left(p+p^{\prime}\right) t l h$, say $\frac{1}{2}\left(p+p^{\prime}\right) t h$ per mètre of girder. But, in practice, this is insufficient, and, as we shall see by the examples hereafter, it is necessary to add to each extreme bar two vertical stiffening rods similar to those described in the preceding section; we thus obtain at once the necessary rigidity, and an increase such that the section will be everywhere superior to the theoretical value, and will, therefore, not need to vary from the bottom upwards.

This addition of four vertical rods will give, to a mètre of girder, a term $\frac{60 h^{2}}{l}$, admitting the sections of Figs. 1729 to 1731 ; and if there are already intermediate rods to the number of $\frac{l}{\delta}-1$, the term including all these additions will be $15 h^{2} \frac{l-3 \delta}{l \delta}$

To sum up, we shall have the weight of the lineal mètre of a lattice-girder with vertical rods, by the formula

$$
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) \frac{l^{2} t \mathrm{U}}{6 h}+\frac{1+\mathrm{V}}{24}\left(p+p^{\prime}\right)(6+q) l t+\frac{1}{2}\left(p+p^{\prime}\right) t h+15 h^{2} \frac{l+3 \delta}{l \delta}+\Omega
$$

112. Suppose $1+\mathrm{V}=\alpha+b h$, this formula becomes

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) \frac{l^{2} t \mathrm{U}}{6 h}+\frac{a}{24}\left(p+p^{\prime}\right)(6+q) l t+\frac{1}{2}\left(p+p^{\prime}\right) t \\
{\left[1+\frac{b l}{12}(6+q)\right] h+15 h^{2} \frac{l+3 \delta}{l \delta}+\Omega .}
\end{gathered}
$$

If the ratio $\frac{h}{\delta}$ were regarded as constant, and we made $\frac{h(l+3 \delta)}{\left(p+p^{\prime}\right) l \delta t}=\theta$, the most advantageous height would be $h_{1}=l \sqrt{\frac{\mathrm{U}}{3+\frac{1}{4} b l(6+q)+90 \theta}}$, and it would reduce the weight of the mètre of girder to the minimum value,

$$
\frac{\mathrm{P}_{1}}{l}=\left(p+p^{\prime}\right) l t\left\{\frac{1}{3} \sqrt{\mathrm{U}\left[3+\frac{1}{4} b l(6+q)+90 \theta\right]}+\frac{a}{24}(6+q)\right\}+\Omega
$$

If, instead of $h_{1}$ we adopted a different height $h=\kappa h_{1}$, the term of the radical in the preceding expression would be multiplied by $\frac{\kappa^{2}+1}{2 \kappa}$.

But usually $\delta$ is a fixed quantity, unconnected with $h$, and the most economical height is then determined by an equation of the third degree.
113. When the lattice-bars have sufficiently rigid forms to allow the vertical rods to be suppressed, the weight of the girder will be found by the formula

$$
\frac{\mathrm{P}}{l}=\frac{1}{2}\left(p+p^{\prime}\right) t\left[\frac{\mathrm{U} l^{2}}{3 h}+\frac{1}{12}(1+\mathrm{V})(6+q) l+h\right]+\frac{60 h^{2}}{l}+\Omega
$$

114. Examples.-First Example.-Flat-bar Lattice-girder with Vertical Rods, Figs. 1729 to 1731.Data: $l=27^{\mathrm{m}} \cdot 50, \delta=3^{\mathrm{m} \cdot 00}, p=1100^{\mathrm{k}}, p^{\prime}=2400^{\mathrm{k}}, p+p^{\prime}=3500^{\mathrm{k}}, q=0 \cdot 686$.

The flat form which the bars have in this case is the least favourable to rigidity, but it enables us to place the vertical rods in a projecting position upon each side of the vertical portion of the girder, without weakening in any part the section of the bars. The lattice is multiplied eight times; the bars of which it is composed vary from 130/14 millimètres to $60 / 12$, both for compression and tension; for here, the bars being weak, it is well to give, even to those subject to tension, an excess of section on account of the loss occasioned by the rivet-holes. In a line with the crossgirders, at intervals of 3 mètres, the lattice is enclosed by vertical rods formed of two angle-irons of $80 / 120 / 10$ millimètres, one on each side; and in the middle of the intervals there are single rods formed of one angle-iron. The weight of all these rods to each lineal mètre of girder is 42 kilogrammes; this result agrees with the formula which, for $\delta=3$, gives $\frac{15 h^{2}}{\delta}=45$, the weight 42 kilogrammes, not including certain pieces used to fill up the hollows in fixing the rods. The figure contains plans or diagrams of the resistance of the flanges and the lattice the former shows the arrangement of the horizontal plates and their joint-plates. In the diagram of the lattice the ordinates of the curve represent the strain upon a bar to the scale of $2 \frac{1}{2}$ millimètres a ton or the necessary theoretical section, abstracting the rigidity to the scale of $1 \frac{1}{2}$ millimètre a square centimètre; the broken line is the locus of the sections really adopted, or the strains to which the bars would be liable if they could be subjected to a stress of 6 kilogrammes a square millimètre.

The weight of a girder is made up of the following quantities:-



The formula of 111 will be made to agree with the results by supposing $\mathbb{U}=1 \cdot 39, \mathrm{~V}=2 \cdot 16$, and $\Omega=12$ kilogrammes say the $0 \cdot 026$ of the total weight. The term $\Omega$ includes the plates upon the bars at the points of crossing and the filling used in fixing the vertical rods. The value of $\mathbf{V}$ is high, because the tension-bars have been increased, as well as those subject to compression; the discontinuous variation of section by groups of bars is also a source of loss. Yet, notwithstanding this high value of V , the rigidity of the lattice is to us less satisfactory than in the girders of the following examples, in which the bars have a better form. If the load were double, $V$ could be reduced easily to $1 \cdot 90$. The vertical rods agree with the hypotheses of the formula by taking $\delta=3$ mètres, that is, by considering the single rods as borrowed from the double or principal rods, which are to an equal extent weakened.

The end solid vertical pieces are also conveniently represented by the formula, for the term $\frac{1}{2}\left(p+p^{\prime}\right) l t h$ gives 188 kilogrammes, four rods give $60 h^{2}$ or 540 , and the 860 kilogrammes of the calculation will be completed by the fact that the total weight of the lattice proper is calculated for the whole length of girder instead of for the bearing only.

If we suppose $\mathrm{V}+1=2 \cdot 35+0 \cdot 27 h$, the most adrantageous height will be obtained from the equation $608=(11 \cdot 69+13 \cdot 27 h) h^{2}$, whence $h=$ about $3 \cdot 30$. Reckoning in this case $\mathrm{U}=1 \cdot 42$, the minimum weight will be 452 kilogrammes, including 12 kilogrammes for $\Omega$.

It will be seen that the reduction of the height to 3 mètres has not increased the weight in any sensible degree.
115. Second Example.-Girder with Rigid Struts and with Vertical Rods, Figs. 1732 to 1741.Data: $l=64^{\mathrm{m}}, \delta=4^{\mathrm{m}}, h=8^{\mathrm{m}}, p=3000^{\mathrm{k}}, p^{\prime}=3000^{\mathrm{k}}$ (for the case of a single line loaded with $\left.4000^{\mathrm{k}}\right), p+p^{\prime}=6000^{\mathrm{k}}, q=0.50$.

Here we adopt a mixed system, better than the preceding ; the vertical rods are placed upon one side only of the lattice, which enables us to place on the other side oblique bars with projecting ribs, and arranged to resist compression; in the middle only, a few braces are made of angle-iron, because in certain cases they may be subject to compression. These angle-irons are then cut a way in a line with the vertical rods; but their resisting section remains intact in the portions of their length where they have of themselves to resist compression, producing a deflecting strain.


This system is of itself sufficiently rigid to allow the adoption of a more open lattice than in the case of flat bars; and, in fact, notwithstanding the considerable height of the girder, we have multiplied it only ten times. The vertical rods weigh only 145 kilogrammes a mètre of girder, instead of 240 kilogrammes, which the formula of 111 would allow. This gain is due to the rigidity which the struts already possess.

The following is the result of direct calculation:-
$\left\{\begin{array}{l}\text { Angle-irons and vertical }\end{array}\right.$
Flanges $\{$


| Vertical rods, with their filling | .. | 9300 |
| :--- | :--- | :--- | :--- |
| End solid vertical pieces | .. |  |

These quantities correspond with $\mathrm{U}=1 \cdot 25, \mathrm{~V}=1 \cdot 80, \Omega=15$ (for plates on the bars), or 0.01 of the total weight

For the end vertical pieces, the term $\frac{1}{2}\left(p+p^{\prime}\right) l h t$ gives 2000 kilogrammes, four vertical rods according to the formula will amount to 3840 ; these two amounts added to the weight furnished by the hypothesis that the lattice extends along the bearing upon the abutments, will give more than the 6000 kilogrammes obtained by direct calculation.

If the height be increased, and the same degree of multiplicity, and the same distance between the vertical rods retained, the latter being suitably strengthened, $V$ will vary but slowly on account of the rigid form of the compres-sion-bars. Suppose, for example, $1+\mathrm{V}=2 \cdot 40+0 \cdot 05 h$, $\mathrm{U}=1.06+0.024 h, \Omega=15$, and reduce to $10 h^{2} \frac{l+3 \delta}{l \delta}$ the torm for the rods, the weight of the mètre of girder with the present data, but for an indeterminate height, will be

$$
\frac{\mathrm{P}}{l}=5645 \frac{1}{h}+10 \cdot 66 h+2 \cdot 97 h^{2}+468
$$

The minimum is for $h=$ about $9^{\mathrm{m}} \cdot 30$, and its value is 1430 kilogrammes, whilst for the height of 8 mètres the same formula gives 1450 kilogrammes. The height may, therefore, be
 it at 7 mètres, for example, the weight would not exceed 1495 kilogrammes a mètre.
116. Third Example.-Rigid Lattice-girder without Vertical Rods, Figs. 1742 to 1747.-Data: $l=40^{\mathrm{m}}, h=4^{\mathrm{m}} \cdot 50, p=2200^{\mathrm{k}}$ (the girder itself weighing 900), $p^{\prime}=3800^{\mathrm{k}}$ (the contiguous line only being loaded at the rate of $4300^{\mathrm{k}}$ a mètre), $p+p^{\prime}=6000, q=0 \cdot 633$.

The lattice is only sixfold. All the bars are of a rigid form, and the connection at the points of crossing utilize the rigidity of the braces in favour of the struts. The bars on one side of the lattice keep the same direction throughout the length of the girder, so that no projection is cut away. Calculation gives

$$
\begin{aligned}
& \text { Flanges }\left\{\begin{array}{l}
\text { Vertical ribs and angle-irons with their joint-plates } \\
\text { Horizontal plates with their joint-plates } \\
7500^{\mathrm{k}} \\
\text {.. }
\end{array} \text {.. } 16900\right\} \\
& \text { Lattice-bars (including } 300^{\mathrm{k}} \text { for gussets at the points of crossing).. } 10300 \\
& \text { End vertical pieces .. .. .. .. .. .. .. .. .. .. } 2500 \text { girder. }
\end{aligned}
$$

These quantities correspond with $\mathrm{U}=1 \cdot 26, \mathrm{~V}=1 \cdot 90$, and $\Omega=7^{\mathrm{k}}$, say the $0 \cdot 008$ of the total weight; V might be reduced if, instead of having only four series of sections for the bars, we varied them in a continuous manner.

If, when $h$ varies, we suppose $\mathrm{U}=1.06+0.046 h$, and $1+\mathrm{V}=1.55+0.30 h$, the formula of 113 will become with the other data of the present example, $\frac{\mathrm{P}}{l}=\frac{2205}{h}+29 \cdot 8 \cdot h+1 \cdot 5 h^{2}+236$. The minimum for a height of $6^{\mathrm{m}} \cdot 70$ will be 831 kilogrammes a metre, instead of 890 as for the girder in the plan.
117. Tables of Weights.-We will suppose generally the following loads, which differ from those of 92 only by a small increase in the weight of the girders:-

| Girders carrying one rail. |  |  |  | Girders carrying two rails. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $l=20{ }^{\text {m }}$ | $60{ }^{\text {m }}$ | $80^{\text {m }}$ | . | $20^{\text {m }}$ | $60^{\text {m }}$ | $80^{\text {m }}$ |
| $+p^{\prime}=3600^{\mathrm{k}}$ | $3600^{\text {k }}$ | $4000^{\text {k }}$ |  | $7000^{\text {k }}$ | $6900{ }^{\text {k }}$ | $7600{ }^{\text {k }}$ |
| $q=0 \cdot 695$ | 0.556 | $0 \cdot 500$ | .. | $0 \cdot 714$ | $0 \cdot 580$ | $0 \cdot 576$ |

Let us suppose again for girders carrying one rail, $h=0 \cdot 11 \mathrm{l}, \mathrm{U}=1 \cdot 40, \mathrm{~V}=2 \cdot 20, \Omega=10$ and for girders loaded with two rails, $h=\frac{1}{8} l, \mathrm{U}=1 \cdot 30, \mathrm{~V}=1 \cdot 90$, and $\Omega=15$. The formula of 113 will give us the following Table.

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| Span .. | $\begin{aligned} & \text { Mètres. } \\ & 20 \end{aligned}$ | Mètres. 30 | Mètres. $40$ | $\begin{aligned} & \text { IIetres. } \\ & 50 \end{aligned}$ | $\begin{gathered} \text { Hiètres. } \\ 60 \end{gathered}$ | $\begin{array}{\|c} \text { Mètres. } \\ 70 \end{array}$ | $\begin{array}{\|c\|} \hline \text { Mètres. } \\ 80 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Girders carrying only one rail.. | $\begin{gathered} \text { Kilos. } \\ 315 \end{gathered}$ | Kilos. $465$ | Kilos. <br> 610 | $\begin{gathered} \text { Kilos. } \\ 760 \end{gathered}$ | $\begin{array}{r} \text { Kilos. } \\ 910 \end{array}$ | Kilos. $1125$ | Kilos. 1335 |
| Girders carrying two rails | 510 | 750 | 990 | 1230 | 1465 | 1800 | 2135 |

Generally girders carrying two rails may be relieved of a portion of their load, if we allow a strain greater than 6 kilogrammes the square millimètre for the accidental case of two heavily-laden trains crossing the bridge at the same time.

The weights in this Table exceed those of girders with simple crosses (105), but are considerably below those for girders with a solid rib (129, 130, 133).
118. For lattice-girders of flat bars, with stiffening vertical rods, V can receive hardly any reduction, and the weight will increase by the addition of the rods. The latter form a rather arbitrary element; but on the hypotheses made in section 110, and by giving to $\delta$ a value
 varying from 2 to 5 mètres, according as the span varies from 20 to 80 mètres, we may admit the following figures, which suppose the adoption of the same heights as in the preceding section

Weight of the mètre of a flat lattice-girder, stiffened by vertical rods.


We may, in the case of vertical rods, adopt heights a little less; but the weights will change only in a very small degree.

When the vertical rods are on one side only of the lattice, as in Fig. 1732, we may make half the bars at $45^{\circ}$ rigid, and reduce in consequence the weight of the rods. The value of the girder will then be between those of the Tables in 117 and 118.
119. Solid Girders.-Vertical Portion.-The solid rib or web is exclusively employed for all kinds of girders of small dimensions, such as cross-girders and minor longitudinal girders. We have already given in 13 to 19 the rules applicable to these kinds of pieces.

We purpose now to examine girders of greater importance, for there is an appreciable interest, in this case, in submitting the plates of the vertical portion and the pieces which give it rigidity to a more minute discussion.

Denoting the height of the girder by $h$, and the variable thickness of the vertical portion by $e$, the theoretical volume of the latter will be expressed by $h \int e d x$, or by $\frac{1}{\mathrm{R}_{1}} \int \mathrm{~F} d x$, according to the value of $e$ given in section 1. Replacing the area $\int \mathbf{F} d x$ of the locus of the stress by the value in section 4, we shall have for the whole bearing: theoretical volume of the vertical portion $=\frac{1}{4 \mathrm{R}_{1}}\left(p+p^{\prime}\right)\left(1+\frac{1}{6} q\right) l^{2}$.
120. Comparing this expression with the theoretical volume of a multiple lattice at $45^{\circ}$ without vertical rods, namely, $\frac{1}{4 \mathrm{R}_{1}}\left(p+p^{\prime}\right)\left(1+\frac{1}{6} q\right)(1+\mathrm{V}) l^{2}$, we see that if $\mathrm{R}_{1}$ were equal to R , the solid rib would have only a volume equal to that of the braces in a multiple lattice, or equal to half the volume of this lattice, supposing V reduced to unity.

We may conceive this fact by imagining the solid rib split into a great number of strips at $45^{\circ}$, placed so as to form the tension-bars of a multiple lattice: these bars in mutual contact will press each other laterally, and these pressures will render useless the addition of struts; for a rod subject to a tensile strain at the rate of $R$ kilogrammes the unit of section, may also be compressed at the same coefficient normally to its length without the resulting pressure exceeding $\mathbf{R}$ in any direction.

But the coefficient $R_{1}$ of resistance to shearing force is usually taken equal to 4000000 , whilst $\mathbf{R}$ rises to 6000000 . The vertical portion, or rib, will therefore already reach the $\frac{3}{4}$ of the theoretical volume of a lattice at $45^{\circ}$. Besides this, the joint-plates and the pieces added to give rigidity, joined to the circumstance that the vertical portion itself cannot be strictly reduced everywhere to the theoretical thickness, render it in reality more expensive than the lattice, at least when the girder acquires a certain importance. Stiffening pieces, or the application of a sufficient coefficient $\mathrm{V}_{1}$, are, it is true, necessary for a multiple lattice also; but if it be a very open one, and bars of a proper form be adopted, a certain fraction of the required rigidity will be already furnished by the theoretical volume, which will reduce in an equal degree the supplementary additions.

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The continuity of the rib, or vertical part, prevents it from resisting exactly as a lattice ; the strains, too, vary in a continuous manner from one point to another, instead of remaiuing constant throughout the length of the imaginary strips of which we have spoken. If the plate were very thin, compression would produce corrugations, while tension would be concentrated in certain points; the plate would wrinkle in directions near $45^{\circ}$, and the wrinkles thus formed would show the direction of the greatest tensile strains, or the positions of the braces in the lattice offering the greatest resistance, which might be cut out of the vertical plate, having regard to the mode of loading.

This continuity of the vertical portion allows it to be introduced into the calculation of the moment of resistance; but generally it will raise it only by a very small value, especially as the thickness is reduced to a minimum precisely in the middle, where the moment of resistance should be great. It is, therefore, not necessary to compute exactly the part taken by the vertical portion in the moment of resistance; it is well even to undervalue it slightly, on account of the influence of the joints, which diminish the resistance a little.
121. Flanges.-The variable section $s$ of the flanges is given by the following equation, which expresses the equality of the moment of rupture $\mathbf{M}$ to the moment of resistance: $\mathbf{M}=\mathbf{R} h\left(s+\frac{\mathbf{F}}{6 \mathrm{R}_{1}}\right)$, see section 14. Substituting for $M$ and $F$ the values given in section 4 , we deduce

$$
s=\frac{1}{2}\left(p+p^{\prime}\right)\left(\frac{x(l-x)}{\mathrm{R} h}-\frac{l-2 x}{6 \mathrm{R}_{1}}\right)-\frac{p^{\prime} x^{2}}{12 \mathrm{R}_{1}{ }^{\prime}} .
$$

The maximum for $x=\frac{1}{2}$ will be $s_{1}=\frac{\left(p+p^{\prime}\right) l^{2}}{8 \mathrm{R} h}-\frac{p^{\prime} l}{48 \mathrm{R}_{1}}$.
From this maximum value, the theoretical section $s$ decreases as the ordinate of a parabola as we approach the abutments; it becomes nul at the point where the vertical portion alone possesses a sufficient moment of resistance to be in equilibrio with the moment of rupture, and beyond this it becomes negative even. But it cannot in reality be thus: not only must $s$ not become nul, it must not even descend below a certain minimum $s_{2}$ which may be fixed in each case. If $x_{2}$ is the abscissa (reckoned from the abutment) which gives to $s$ this value $s_{2}$, the total volume of the flanges will be $2\left[l s_{2}+\frac{2}{3}\left(s_{1}-s_{2}\right)(l-2 x)\right]$.
122. But this formula would be inconvenient, and it is better to consider simply the volume of the two flanges as equal to $\frac{4}{3} l s_{1}$, that is, as if the section varied parabolically from $s_{1}$ in the middle to 0 at the extremities, and the coefficient $U$ may serve to correct nearly the divergence between the form thus conceived, and the one occurring in practice. And, as the least thickness of the vertical rib in the middle is never less than $0^{m \cdot 006, ~ w e ~ m a y ~ c o m p u t e ~ i t s ~ m o m e n t ~ o f ~ r e s i s t-~}$ ance for this thickness, neglecting the surplus if the plate is stouter; $s_{1}$ will then have the value $\frac{\left(p+p^{\prime}\right) l^{2}}{8 \mathrm{R} h}-0.001 h$, and consequently, total weight of the two flanges

$$
=\frac{t l \mathrm{U}}{3}\left(\frac{\left(p+p^{\prime}\right) l^{2}}{2 h}-0.004 \mathrm{R} h\right)
$$

123. Accessories.-To complete the weight of the girder, there yet remain to be computed the accessories of the vertical portion, namely, 1 , the excess of material occasioned by the fact that the thickness varies by redans or gradations and not in a continuous manner, and also by the
 joint-plates of the rib; and 3, the pieces intended to give it rigidity.

In girders heavily loaded, we may suppose that the redans and the surplus thickness, indispensable in the middle, are equivalent to an extra continuous thickness of about $0^{\mathrm{m} \cdot 0027 \text { applied }}$ to the whole vertical rib.
124. As to the joints, they may be placed widely apart when the height of the girder does not exceed the breadth of the iron plates, and this is one of the causes which make the solid rib economical for small girders. But when the height becomes considerable, a case which we are now discussing, we must not reckon upon distances greater than $0^{\mathrm{m} \cdot} 90$ between the successive joints. In order that the rivets may offer a double shearing section, for the purpose of reducing the breadth of the joint-plates, the latter are composed of double plates placed on each side of the vertical rib; these plates will nearly always be of $0^{\mathrm{m} \cdot 006}$, for a less thickness is seldom employed in the construction of bridges, and on the other hand that of $0^{\mathrm{m}} \cdot 006$ is sufficient when the rib consists of stout plate of $0^{\mathrm{m}} \cdot 012$. The breadth of the joint-plates may, at least over a certain portion of the girder, be limited to $0 \mathrm{~m} \cdot 160$, placing only one row of rivets upon each side of the joint. But if the stress reaches a high value near the supports, it may be necessaiy to place a double row of rivets, and to increase the breadth of the joint-plates to $0^{\mathrm{m} \cdot 300}$, so that there may always de on each side of the joint a number of rivets at least equal to $\frac{\mathrm{F}}{2} \mathrm{R}_{1} \sigma, \mathrm{~F}$ being the stress, $\mathrm{R}_{1}$ the resistance to a shearing force, and $\sigma$ the section of a rivet. If, as a mean, we reckon $0^{\mathrm{m}} \cdot 200$ for the breadth of the joint-plates, they will be equivalent to an extra continuous thickness of $0^{\mathrm{m}} \cdot 0027$, which, added to that of the preceding section, gives $0^{\mathrm{m}} \cdot 0054$.

This amount may be retained for girders lightly loaded; for if on the one hand the breadth of all the joint-plates is reduced to $0 \mathrm{~m} \cdot 160$, which produces an extra thickness of only $0^{\mathrm{m}} \cdot 0022$; on the other hand, in small girders especially, calculation will assign thicknesses too far below the limit.
125. The pieces added to give rigidity are usually vertical and are applied to the joint-plates;

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they thus form vertical rods, which are made use of in fixing the cross-girders, or brackets. Their section is not determined by rigorous laws, it is necessary only that their projecting ribs should increase with the height, to ensure rigidity. It is often advantageous to increase those situate in a line with the cross-girders at the expense of the intermediate ones. Let us, for example, suppose adopted, for heights of 2 to 5 mètres, the arrangements shown by Figs. 1748 to 1751, which gives the horizontal section of a certain length of vertical rib.


With these forms, the angle-irons and transverse plates, not including the joint-plates, will weigh 12 h kilogrammes a square mètre of vertical rib, and this is equivalent to a reduced thickness equal to 0.00154 h . We will put it at 0.002 h , to be able to increase the rigidity of the vertical rib again towards the ends. This value is considered as applicable to strong as well as to weak girders; it will constitute for these latter an addition of strength relatively greater, as, indeed, should be the case, for it is the weak rertical ribs which are most liable to twist. We may, within certain limits, modify the distance of the rertical stiffening pieces apart by modifying inversely their section. Sometimes we shall do well to fix the transrerse plates by means of a single angle-iron, and to rivet to the exterior edge a second angle-iron. The intermediate stiffening pieces will have a $U$ form instead of a $T$, and the border angle-iron will give a gratuitous increase of rigidity.
126. Formula and Table of Weights for Solid Girders.-Adding the weight of the theoretical vertical rib (119) and that of the flanges (122), reduced to the lineal mètre of girder, as well as
 the following formula as the expression of the weight of the girder a lineal mètre:

$$
\frac{\mathbf{P}}{l}=\frac{\mathbf{1}}{6}\left(p+p^{\prime}\right) l t\left\{\frac{\mathrm{U} l}{h}+\frac{3}{8}(6+q)\right\}+h(42 \cdot 12-8000 t \mathrm{U}+15 \cdot 6 h) .
$$

As $q$ has but a small influence, we may substitute for it the value 0.60 from which it will generally not differ much, and we shall then have

$$
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) l\left\{0.0002167 \mathrm{U} \frac{l}{h}+0.0005363\right\}+h(42 \cdot 12-10.4 \mathrm{U}+15.6 h)
$$

Substituting for the coefficient U in this expression the value assigned to it in 64 and seeking the condition of the minimum of expense, we find that the most advantageous height $h_{1}$ must satisfy the equation: $0 \cdot 23\left(p+p^{\prime}\right) l^{2}=(31510+31200 h) h^{2}$.
127. The loads $p+p^{\prime}$ do not Fary much with the span, because the moving load $p^{\prime}$ decreases, while the dead weight of the girder increases. For spans of 20 and 55 mètres, we may adopt the following loads, according as the girder is to carry one or two rails:-

| 20 mètres span .. |  | Girders carrying |  |
| :---: | :---: | :---: | :---: |
|  |  | One Rail. | Two Rails. |
|  | ( Exterior dead weight . | kilos. 700 | kilos. 1400 |
|  | Weight of the girder $\quad \ddot{\square}$ | 400 | 600 |
|  | Moving load $p^{\prime}$.. .. .. | $2500^{\circ}$ | 5000 |
| 55 mètres span .. | Total load $p+p^{\prime}$ | 3600 | 7000 |
|  | Exterior dead weight W. $^{*}$ | 700 | 1400 |
|  | Weight of the girder Moving load .. . ${ }^{\text {a }}$.. .. | 1300 2000 |  |
|  | 'rotal load A. | 4000 | 7400 |

Substituting these quantities in the equation of the preceding section, we find that, for 20 metres span, the most economical height will be $1^{\mathrm{m}} \cdot 90$ if the girder supports only one rail, and $2^{\mathrm{m} \cdot 45}$ if it supports two rails. For 55 mètres span, we obtain respectively $4^{\mathrm{m}} \cdot 20$ and $5^{\mathrm{m} \cdot 20}$.

Thus, for girders the least loaded, the ratio of the most advantageous height to the span would vary from $0 \cdot 095$ in small spans, to 0.076 in long ones; and for heavy girders from 0.12 to 0.095 . We may alter these heights a little without the weight changing sensibly, and it will often be advantageous to reduce them, either to lessen the vertical cross-bracing and cross-ties if the bridge is loaded on the upper side, or to increase the stability if the girders are loaded on the lower side, or again, to satisfy conditions of limited thickness. In the following Tables we give heights reduced by about $\frac{1}{10}$.
128. It remains to compute U , which, according to 66 , may be expressed by $1 \cdot 06+4 \cdot 80 \frac{\mathrm{R} s_{0} h}{\left(p+p^{\prime}\right) l^{2}}$, or $1 \cdot 06+\frac{1 \cdot 10 \mathrm{R} s_{0}}{h(31510+31200 h)}$ by replacing $\frac{h^{2}}{l^{2}}$ by the value obtained from the equation which ends section 126. From this U may be regarded as independent of the load; it diminishes when $h$ increases, but only by the factor between parentheses, for $\frac{s_{0}}{h}$ will be nearly constant, since it is necessary to adopt a minimum section of flange $s_{0}$ greater in proportion to the dimension of the girder.

For example, if the girder is $1^{\mathrm{m}} \cdot 80 \mathrm{high}$ (bridge of 20 mètres), we may make $s_{0}$ of one plate of $3 / 508$ millimètres and of two angle-irons of $80 / 80 / 10$; for a girder of 4 mètres high ( $l=55$ ), we may take a plate of $550 / 8$ with four angle-irons of $100 / 100 / 12$, two of which are placed next the vertical rib and two at the edges of the flange. In both cases this would give $\frac{1 \cdot 10 \mathrm{R} s_{0}}{h}=$ about 22000 , and consequently $\mathrm{U}=1 \cdot 31$ for $h=1 \cdot 80$, and $1 \cdot 20$ for $h=4$. We shall, therefore, vary this coefficient from $1 \cdot 30$ to $1 \cdot 20$, according as the span varies from 20 to 55 mètres.
129. The several hypotheses made in the preceding sections lead to the following Table for the weight of the lineal mètre of solid girders, the heights adopted being a little under those which would correspond rigorously with the minimum expense.

| Span | $\begin{array}{\|c} \text { Mètres. } \\ 20 \end{array}$ | Mètres. 25 | Mètres. 30 | Mètres. 35 | Mètres. 40 | Mètres. 45 | Mètres. 50 | Mètres. 55 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Girders carrying one rail $\quad$ Height | $1 \cdot 70$ | $2 \cdot 00$ | $2 \cdot 30$ | $2 \cdot 60$ | $2 \cdot 90$ | $3 \cdot 20$ | $3 \cdot 50$ | $3 \cdot 80$ |
| $\left.\begin{array}{l} \text { kilogrammes, accord- } \\ \text { ing to the span) } \\ \text {.. } \end{array}\right\} \text { Weight a metre }$ | Kilos. $370$ | Kilos. 485 | Kilos. <br> 610 | Kilos. 735 | Kilos. $870$ | $\begin{aligned} & \text { Kilos. } \\ & 1000 \end{aligned}$ | Kilos. <br> 1140 | Kilos. 1285 |
| Girders carrying one line \| Height .. :. | $\begin{aligned} & \text { Mè̀tres. } \\ & 2 \cdot 25 \end{aligned}$ | Mètres $2 \cdot 60$ | Mètres. $2 \cdot 95$ | Mètres. $3 \cdot 30$ | Mètres. $3 \cdot 65$ | Mètres. $4 \cdot 00$ | $\begin{aligned} & \text { Mètres. } \\ & 4 \cdot 35 \end{aligned}$ | $\begin{aligned} & \text { Mètres } \\ & 4 \cdot 70 \end{aligned}$ |
| $\left.\begin{array}{l} \text { of rails }\left(p+p^{\prime}=7000\right. \\ \text { to } 7400) \quad . . \quad . . \quad . . \end{array}\right\} \text { Weight a mètre }$ | $\begin{aligned} & \text { Kilos. } \\ & 570 \end{aligned}$ | $\begin{gathered} \text { Kilow. } \\ 750 \end{gathered}$ | $\begin{aligned} & \text { Kilus. } \\ & 940 \end{aligned}$ | Kilos. 1125 | Kilos. 1320 | $\begin{aligned} & \text { Kilos. } \\ & 1525 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 1730 \end{aligned}$ | Kilos. 1940 |

We may compute by interpolation the weight of girders supporting intermediate loads. If, for example, for a bridge of 55 metres span we had $p+p^{\prime}=6400$ kilogrammes, we should find that by adopting a height of $4^{\mathrm{m} \cdot 45}$ the weight of the lineal mètre would be about 1750 kilogrammes. This weight would be substituted for that of 1940 kilogrammes for a girder carrying one line of rails; if, instead of a load of 4000 kilogrammes, which supposes both lines loaded simultaneously, we considered a load of only about 3000 kilogrammes for the ordinary case of the passage of one train, the strain a square millimètre would then exceed 6 kilogrammes in the exceptional case of two heavily-laden trains crossing at the same time.
130. Other Formulce.-In the foregoing discussions we have distinctly stated that the thickness of the vertical rib should not descend below a certain minimum, which may be fixed at $0^{\mathrm{m} \cdot 006 \text {, }}$ but this condition has not been positively expressed in the formula; we have contented ourselves with providing for it by the aid of an approximative extra thickness. This condition is, however, easily expressed, as we shall see; but to do this we must distinguish two cases, that in which this minimum thickness of $0^{\mathrm{m}} \cdot 006$ is sufficient throughout the length of the girder, and the contrary case. The vertical rib being calculated for a strain of 4 kilogrammes a square millimètre, the two case in question are equivalent to those in which the quantity $\left(p+p^{\prime}\right) l$ is less or greater than 48000 h.

First Case: $\left(p+p^{\prime}\right) l<48000 h$.-The vertical rib has a constant thickness of $0^{\mathrm{m} \cdot 006}$, and it requires joint-plates with a single row of rivets on each side of the joint. We shall then have, remarking that the joint-plates are equivalent to an extra thickness of $0^{\mathrm{m}} \cdot 0022$,

$$
\frac{\mathbf{P}}{l}=\left(p+p^{\prime}\right) \frac{l^{2} t \mathrm{U}}{6 h}+h(64+15 \cdot 6 h-8000 t \mathrm{U})
$$

This formula may be applied to girders carrying only one rail, provided the height ne not too small. On the contrary, the formula of section 126 supposed tacitly that the height was not great, in order that the vertical rib should not become too weak. With the new expression of $\frac{\mathrm{P}}{\mathrm{l}}$ the most
economical height will be a little less than hitherto: thus for the bridge of 20 mètres loaded with 3600 kilogrammes a mètre, it will be only $1^{\mathrm{m}} \cdot 75$ instead of $1 \cdot 90$; and for the bridge of 55 mètres loaded with 4000 kilogrammes, it will be 3.95 instead of $4 \cdot 20$. But retaining the same heights, loads, and values of U introduced into the Table in 129, the present formula gives the following weights, which hardly differ from the preceding. This Table ends with 35 mètres span, because beyond this the condition of the present case would not be fulfilled.

|  | 1.75 |  | 2.3 |  |
| :---: | :---: | :---: | :---: | :---: |
| Span .. .. .. .. | $\begin{gathered} \text { Mètres. } \\ 20 \end{gathered}$ | Mètres 25 | $\begin{aligned} & \text { Mềtres. } \\ & 30 \end{aligned}$ | $\begin{aligned} & \text { Mè̀res. } \\ & 35 \end{aligned}$ |
| Weight a mètre of girder carrying only one rail ( $p+p^{\prime}=3600$ kilo- $\}$ grammes for 20 mètres span, to 3770 kilogrammes for 35 mètres) .. | $\begin{gathered} \text { Kilos. } \\ 370 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 480 \end{aligned}$ | $\begin{gathered} \text { Kilos. } \\ 600 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 725 \end{aligned}$ |

131. Second Case: $\left(p+p^{\prime}\right) l>48000 h$.-This will always be the case with girders carrying one line of rails, it will also be the case with girders carrying only one rail, if their height is below a certain value.

Fig. 1752 represents the plane of the theoretical vertical rib, throughout half the bearing, the thicknesses being exaggerated. If the plate had everywhere only $0 \mathrm{~m} \cdot 006$, its volume for the whole bearing would be 0.006 lh ; but we must add to it the triangular prisms one of which is shown in the figure. These prisms should have rigorously a curvilineal face, but we may neglect the influence of $q$, for it is perceptible only in the middle, upon the locus of the stress. The maximum
 stress requires at the ends a thickness $e_{1}=\frac{\left(p+p^{\prime}\right) l}{2 \mathrm{R}_{1} h}$ or $\frac{\left(p+p^{\prime}\right) l}{8000000 h}$ supposing $\mathrm{R}_{1}=4000000$. Consequently, the volume of the two additional prisms will be $\frac{\left[\left(p+p^{\prime}\right) l-48000 h\right]^{2}}{16000000}\left(p+p^{\prime}\right)$.

But as the thickness, instead of varying uniformly, decreases by redans or gradations of $0^{\mathrm{m}} \cdot 002$, we must add a mean thickness of $0^{\mathrm{m}} \cdot 001$ to the part over which the prisms extend; it may be increased to $0^{\mathrm{m}} \cdot 0012$ even, to take into account various circumstances, especially that of having neglected $q$, and it will give a new supplementary volume equal to $\frac{0 \cdot 0012 h}{p+p^{\prime}}\left[\left(p+p^{\prime}\right) l-48000 \mathrm{~h}\right]$.

We shall have besides:-
$\left.\begin{array}{l}\text { Joint-plates and stiffening rods in the part } \\ \text { where the vertical rib is only } 0 \text { m. } 006\end{array}\right\}=\frac{48000 h^{2}}{p+p^{\prime}}\left(\xi_{1}+\xi_{1}^{\prime} h\right)$;
$\left.\begin{array}{c}\text { Joint-plates and stiffening rods in the part } \\ \text { where the vertical rib is strengthened }\end{array}\right\}=\frac{\left(p+p^{\prime}\right) l-48000 h}{p+p^{\prime}}\left(\xi_{2}+\xi_{2}^{\prime} h\right) h$.
132. Collecting the various elements of the vertical portion, as they have been computed, adding to them the flanges according to section 122, then transforming the volumes into weight, and dividing by the bearing, we shall have
$\left(\frac{t \mathrm{U}}{6 h}\left[\left(p+p^{\prime}\right) l^{2}-0.008 \mathrm{R} h^{2}\right]\right.$ (flanges).
$\frac{\mathrm{P}}{l}=\left\{\begin{array}{l}+46 \cdot 8 h+\frac{0 \cdot 0004875}{\left(p+p^{\prime}\right) l}\left[\left(p+p^{\prime}\right) l-48000 h\right]^{2} \text { (vertical rib according to Fig. 1752). } \\ +\frac{9 \cdot 36 h}{\left(p+p^{\prime}\right) l}\left[\left(p+p^{\prime}\right) l-48000 h\right] \text { (redans in the strengthened portions of vertical rib). }\end{array}\right.$ $+\frac{48000 h^{2}}{\left(p+p^{\prime}\right) l}\left(\xi_{1}+\xi_{1}^{\prime} h\right)+\frac{\left(p+p^{\prime}\right) l-48000 h}{\left(p+p^{\prime}\right) l}\left(\xi_{2}+\xi_{2}^{\prime} h\right) h$ (joint-plates, and so on, of Making $\mathrm{R}=6000000 ; t=0.0013$;
Weight of the joint-plates a square mètre of vertical rib in the extreme strengthened portions $\quad . \quad . ..\} \xi_{2}=0^{\mathrm{m} \cdot 0035} \times 7800^{\mathrm{k}}=27^{\mathrm{k}} \cdot 3$,
$\begin{aligned} & \text { Weight of the vertical rods a square mètre in the } \\ & \text { extreme strengthened portions }\end{aligned} . . \quad . \quad$.. $\quad .$.
Weight of the joint-plates a square mètre in the in-
termediate portion .. $\quad . \quad$.. $\quad . . \quad$.. $\quad . . \quad .$.
Weight of the vertical rods a square mètre in the intermediate portion .. .. .. .. .. .. $\} \xi_{1}^{\prime} h=0 \mathrm{~m} \cdot 0016 \times 7800 \mathrm{~h}=12.5 \mathrm{~h}$;
the formula will become

$$
\begin{gathered}
\frac{\mathbf{P}}{l}=\left(p+p^{\prime}\right) l\left(\frac{13 \mathrm{U} l}{60000 h}+0 \cdot 0001875\right)+h(36 \cdot 66-10 \cdot 4 \mathrm{U}+18 \cdot 72 h) \\
-\frac{h^{2}}{\left(p+p^{\prime}\right) l}(298560 h-187200)
\end{gathered}
$$

133. The most economical height will here be given by the following equation of the fourth degree ( U being replaced by the expression in 64):
$0 \cdot 0002297\left(p+p^{\prime}\right)^{2} l^{3}=25 \cdot 6 \pm\left(p+p^{\prime}\right) l h^{2}+37 \cdot 44\left[\left(p+p^{\prime}\right) l+10000\right] h^{3}-895680 h^{4}$, which ought to give values a little greater than hitherto, since we have expressed the fact that, in proportion as the height increases, the lengths of the portions of vertical rib which need to be strengthened are reduced. And, in fact, for $l=20$ mètres, and $p+p^{\prime}=7000$ kilogrammes, we obtain $2^{\mathrm{m} \cdot} 70$, instead of $2^{\mathrm{m} \cdot 45}$, found by the first formula; and for $l=55$, and $p+p^{\prime}=7400$, we find $5 \cdot 45$, instead of $5 \cdot 20$.

By increasing the heights, therefore, a little, we shall form by means of the last formula the following Table, in which $p+p^{\prime}$ varies from 7000 to 7400 , and U from $1 \cdot 30$ to $1 \cdot 20$, in proportion as the bearing increases from 20 to 55 mètres. These loads refer to girders carrying two rails, a test which in general is realized in a complete manner only when both lines are simultaneously loaded.


These weights are a little less than those obtained by the first method.
134. Example of Solid Girder, Figs. 1753 to 1757: $l=35$ mètres, $p+p^{\prime}=7000$ kilogrammes.According to a direct calculation, this girder weighs only 1030 kilogrammes a lineal mètre. The arrangements shown in the plan for the joint-plates of the flanges allows us to reduce the coefficient U to the value $1 \cdot 20$.
135. Girders with Inclined Stays.-When vertical stays or rods are not deemed necessary for the purpose of affixing cross-girders or brackets, they may be replaced by oblique pieces crossing the Hat joint-plates, which would remain vertical and would be interrupted in the line of these pieces. The latter would be inclined to $45^{\circ}$, nearly normally to the wrinkles which would be produced if the plate were not stiffened. This arrangement is alluring, however, the economical advantage of it appears doubtful, because for the same normal distance between the consecutive pieces the diminution of their number would be compensated by the increase in their length; or rather the distance between the oblique bars may be greater, but their section should be increased to an equal degree, for these pieces, concentrating upon themselves by the effect of their rigidity the strains of compression, may be considered as the struts of a lattice which would be completed by the tension of the plate in the direction of one of the diagonals of each panel. Now for one direction given to these diagonal tensions of the plate, we may replace two vertical stays A B, C D, Fig. 1758, by a single stay AD, which would be equivalent to them, for with the inclination of $45^{\circ}$, for example, it would have its strain and its length multiplied by $\sqrt{\overline{2}}$. Thus between the imaginary lattice ABCDEFG, and the lattice A.DE H, there would be this essential difference only, namely, that in the second,
 in the rectangular panels occupied by the oblique bars. But there is small advantage in this, since this plate must resist equal strains in other panels, and besides, these stays were not intended to reduce these strains.

The adoption of oblique pieces will in no degree change the form of the formulæ of weight given, 119 to 135. It would be sufficient to introduce other values for the weight of the stays a square mètre of vertical rib, if it were thought a reduction might be made.
136. Formulce applicable to Bow Bridges.-Bow-girders are metallic frames consisting of two curved flanges more or less distant from each other in the middle, and approaching each other towards the extremities where they are bound together by strong wrought-iron plates. If we consider any vertical section, the action of the flanges, having an inclined direction, will produce a vertical force capable of resisting, in part at least, the stress upon the bridge. The lattice connecting the flanges at certain intervals may thus be greatly relieved.

If the lattice is a simple one, the bars, or at least some of them, must necessarily possess sufficient rigidity to resist compression. This is doubtless a great disadvantage, for the theoretical strains being small we shall obtain the rigidity only at the cost of a great increase of section. This increase, however, is confined within rather narrow limits, even if we do not bring into play the rigidity of the bow itself since the coefficient of stiffness, if it has a high value, is applied to only a small fraction of the total weight. We may, therefore, not hesitate to adopt for this coefficient a value of $2 \cdot 50$, for instance, which raises to 60 the ratio of the length to the reduced thickness (68). But, by doubling the diagonals of the lattice, we may in certain cases entirely suppress the compressions in the vertical portion, or at least render them unworthy of consideration, as we shall see in 144 and 157. As an expenditure of material, this will amount to nearly doubling the theoretical weight of simple diagonals not possessing rigidity; for we arrive at this elimination of the compressions only by supposing, in any cross, that one of the diagonals of which it is composed remains inactive at a given moment. The compression is always thrown upon the vertical rods, where it is destroyed or very much reduced by a permanent tension due to the dead weight of the flooring.

These considerations allow us to study bow bridges as simple articulate systems, which will lead us to formula for the computation of their weight more precise and simple than the method

of introducing the rigidlty of the bows themselves, and, consequently, the form of their section, could do. When a scheme is devised in which this rigidity plays an important part, it is because something is to be gained by it, and then our computations will still be useful if we consider them as forced a little.

It is generally more convenient to make only the upper flange or bow arched, the lower flange or stringer being straight and provided with points for fixing the cross-girders. We shall designate more specially this particular form by the name of arched girder.
137. Parabolical Arched Girder, divided into an odd number of intervals, Fig. 1759.-Let $l=(2 m+1) \delta$ be the bearing and $h$ the height in the middle. The bars, whether vertical or oblique, are num-

bered from the middle. The bow is a polygon, having its summits upon a parabola with a vertical axis, and the sides of which have a constant horizontal projection equal to $\delta$.

The $n$th vertical rod has a length expressed by $h\left[1-\frac{n(n-1)}{m(m+1)}\right]$, and the value of the sum. of the lengths of the $2 m$ rods is $\frac{2 h}{3} '^{\prime} m+1$ ) or $\frac{2}{3} \frac{l h}{\delta}$. The length of the $n$th section $\mathbf{E F}$ of the curved flange is $\frac{1}{m(m+1)} \sqrt{m^{2}(m+1)^{2} \delta^{2}+4(n-1)^{2} h^{2}}$, and makes with the vertical an angle $\gamma$ given by cotan. $\gamma=\frac{2 h(n-1)}{m(m+1) \delta}$.

The length of the $n$th diagonal shown by the full line is

$$
\frac{1}{m(m+1)} \sqrt{ } m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-n(n-1)]^{2} h^{2},
$$

and makes with the horizon an angle $a$ the sine of which is

$$
\text { Sin. } \boldsymbol{\alpha}=\frac{m(m+1)-n(n-1)}{\sqrt{m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-n(n-1)]^{2} h^{2}}} .
$$

The length of the $n$th diagonal shown by the dotted line is

$$
\frac{1}{m(m+1)} \sqrt{m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-n(n-1)(n-2)]^{2} h^{2}}
$$

and the sine of the angle of inclination is

$$
\text { Sin. } \boldsymbol{a}^{\prime}=\frac{m(m+1)-(n-1)(n-2)}{\sqrt{m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-(n-1)(n-2)]^{2} h^{2}}} .
$$

138. 1.-Let us first suppose a simple lattice formed by the vertical rods and the diagonals shown by the full lines.

If the framing is subjected only to a permanent load uniformly distributed according to the horizontal, in such a manner that a weight equal to $p \delta$ is applied to the foot of each vertical rod, this weight will express the constant tension of all the rods, which act as simple suspension-rods to transmit the load to the several summits of the polygonal arc. Their total theoretical weight will be $=\frac{2}{3}(2 m+1) p \delta h t, t$ denoting as before the ratio of the weight of the cubic mètre of iron to the admitted resistance $R$.

The $n$th section of the bow is subject to the following strain of compression:

$$
\frac{p \delta}{2 h} \sqrt{m^{2}(m+1)^{2} \delta^{2}+4(n-1)^{2} h^{2}}
$$

whence we deduce for the total weight of the bow the value $\mathbf{U}(2 m+1) p \delta t\left[\frac{m(m+1) \delta^{2}}{2 h}+\frac{2 h}{3}\right]$.
The lower flange, or stringer, Dears a constant tension equal to $\frac{m(m+1) p \delta^{2}}{2 h}$, which gives it a total weight $=\frac{\mathbf{U} m(m+1)(2 m+1) p \delta^{3} t}{2 h}$. The letter $\mathbf{U}$ still denoting the coefficient for joint$\mu$ lates and other accessories.

It is useless to enlarge upon these formulæ, well known in the theory of suspension bridges.

With the present mode of loading, the parabolic form given to the bow assimilates it to a funicular polygon, that is, its several elements have precisely the directions which they would of themselves assume if they were left free to turn about their summits. It is, indeed, easy to prove that the action of any section has a vertical projection equal to the stress, so that the diagonal is inactive.
139. It will not be the same in the case of a moving load which, although distributed uniformly in the proportion of $p^{\prime} \delta$ a vertical rod, may exist on a certain portion only of the length.

To find the maximum strains upon the several sections of the flanges, the load must be extended over the whole bridge; for these strains depend only upon the moments of rupture about certain summits, and these moments increase always in proportion as we add new loads upon a certain point in the bearing. The strains and weight of the flanges will, therefore, be obtained by the formulæ of the preceding section by substituting $p+p^{\prime}$ for $p$.

But the strains upon the bars are at once functions of the moment of rupture and the stress.
Let T be the strain upon the diagonal E D, Fig. 1759, M the moment of rupture about the point D , the abscissa of which is $(m-n+2) \delta$, and F the stress between C and D . The strain upon the section EI of the bow will be equal to $\frac{M}{\text { DI sin. } \gamma}$, and, consequently, considering the portion of the girder situate to the left of the line GH, the condition: of equilibrium relative to the vertical forces will give, supposing $T$ to be a tension, $T=\frac{F}{\sin . \alpha}-\frac{\mathrm{M} \operatorname{cotan} . \gamma}{\mathrm{DI} \sin \alpha}$.

This expression is of the form $\mathbf{B F}-\mathrm{AM}, \mathrm{A}$ and B being given quantities. Therefore, from section 7, it will become a maximum when the load is applied to the point D and to all those on the right. Taking this maximum value, and substituting the values of cotan. $\gamma, \sin$. $\alpha$ and D I (the length of the $(n-1)$ th vertical rod), we find that the diagonal should be capable of resisting a strain of tension,

$$
\mathrm{T}=\frac{p^{\prime} \delta}{2(2 m+1) h} \sqrt{m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-n(n-1)]^{2} h^{2}} .
$$

We know that if we complete the load by adding weights $p^{\prime} \delta$ at the bottom of the vertical rods situate to the left of GH, the diagonal will be relieved of its strain. Therefore, this complementary load placed alone in its turn should produce a compression equal to the preceding tension. Thus, applying a coefficient of stiffness V , the weight of the diagonal in question will be

$$
\frac{p^{\prime} \delta t \mathrm{~V}}{2 m(m+1)(2 m+1)}\left\{m^{2}(m+1)^{2} \frac{\delta^{2}}{h}+[m(m+1)-n(n-1)]^{2} h\right\}
$$

To find the total weight of the $2 m-1$ diagonal bars, we must scan the preceding expression in which $n$ takes the successive values $1,2, \ldots m$, double the result, and then subtract what the above expression becomes for $n=1$, in order not to reckon twice the middle diagonal. We shall be induced, for the sake of symmetry, to divide this diagonal into two pieces to form a central cross, but the strain will be distributed between the two bars of this cross, and there will be no increase of weight. We obtain:

Total weight of the $(2 m-1)$ diagonals shown by the full lines,

$$
\frac{p^{\prime} \delta t \mathrm{~V}}{2(2 m+1)}\left[m(m+1)(2 m-1) \frac{\delta^{2}}{h}+\frac{h}{15}\left(16 m^{3}+9 m^{2}+m+4\right)\right]
$$

140. Analogous calculations may be applied to the $n$th vertical rod CE. Cutting it by a section $G^{\prime} H^{\prime}$, supposing it to be subject to a compression $T^{\prime}$, and calling $M^{\prime}$ the moment of rupture about a point C , the abscissa of which is $(m-n+1) \delta$, we shall have by the equation of equilibrium relative to the projections of the vertical forces, $\mathrm{T}^{\prime}=\mathrm{F}-\frac{\mathrm{I}^{\prime} \operatorname{cotan} . \gamma^{\prime}}{\mathrm{E}} ; \boldsymbol{\gamma}^{\prime}$ is the angle of the $(n+1)$ th section of the bow with the vertical, and E C the length of the $n$th rod.

This expression is also of the form BF-A MI'. Treating it by the rule in section 7, we obtain for the maxima compression of the rod, $\mathrm{T}^{\prime}=\frac{(m-n)(m+n-1)}{2(2 m+1)} p^{\prime} \delta$.

Here $\mathbf{F}$ was the stress taken beyond the point to which the moment $\mathrm{I}^{\prime}$ is referred. It follows that this maximum strain $T^{\prime}$ is produced when the load is applied to the point $D$, and to those following it as far as B. This compression will be wholly or partly neutralized by the permanent tension due to the dead weight.

When the load is completed, the vertical rod, instead of being subject to a strain of compression $T^{\prime}$, is, on the contrary, subject to a strain of tension $p^{\prime} \delta$. Therefore the complementary load placed alone in its turn should subject the rod to a tension

$$
\mathrm{T}^{\prime \prime}=p^{\prime} \delta+\mathrm{T}^{\prime}=\frac{(m+n+1)(m-n+2)}{2} p^{\prime} \delta
$$

which always greatly exceeds the maxima compression, especially when we add the permanent tension $p \delta$, due to the dead weight.

It may happen that this latter tension is always sufficient to destroy the compressions which the load may produce. We may, therefore, from this point of view dispense with a coefficient of stiffness, and calculate the rods by tension only. Adding the dead to the moving load, and denoting the ratio $\frac{p}{p+p^{\prime}}$ by $q$, the weight of the $n$th rod will be

$$
\left(p+p^{\prime}\right) \delta t h\left[1-\frac{n(n-1)}{m(m+1)}\right]\left[1+q \frac{(m+n-1)(m-n)}{2(2 m+1)}\right]
$$

an expression which being doubled, then summed from $n=1$ to $n=m$ inclusive, leads to

Total weight of the $2 m$ vertical rods $=\frac{2\left(p+p^{\prime}\right) \delta t h}{3}\left[2 m+1+q \cdot \frac{(m-1)(2 m-1)}{5}\right]$.
141. Adding the weight of the flanges, of the diagonals and of the vertical rods, substituting for $\delta$ its value in $l$, and dividing by $l$, we obtain for the mean weight of the lineal metre of girder,

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) t l^{2} \frac{m(m+1)}{(2 m+1)^{2}}\left[\mathrm{U}+q \mathrm{~V} \frac{2 m-1}{2(2 m+1)^{2}}\right] \frac{1}{h}+\frac{\left(p+p^{\prime}\right) t h}{3} \\
\left\{2(\mathrm{U}+1)+\frac{q}{10(2 m+1)^{2}}\left[16 m^{3}(1+\mathrm{V})-m^{2}(16-9 \mathrm{~V})-m(4-\mathrm{V})+4(1+\mathrm{V})\right]\right\}+\Omega
\end{gathered}
$$

The term $\Omega$ includes accessories independent of $h$.
Substituting $N$ for $2 m+1$, we may write also

$$
\begin{gathered}
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) t l^{2} \cdot \frac{\mathrm{~N}^{2}-1}{4 \mathrm{~N}^{2}}\left(\mathrm{U}+q \mathrm{~V} \cdot \frac{\mathrm{~N}-2}{2 \mathrm{~N}^{2}}\right) \frac{1}{h}+\frac{\left(p+p^{\prime}\right) t h}{3} \\
\left\{2(\mathrm{U}+1)+\frac{q}{40}\left[8 \mathrm{~N}^{3}(1+\mathrm{V})-5 \mathrm{~N}^{2}(8+3 \mathrm{~V})+8 \mathrm{~N}(6+\mathrm{V})+15 \mathrm{~V}\right]\right\}+\Omega
\end{gathered}
$$

Such is the formula applicable, N being odd, to a bow-girder with simple rigid diagonals. If the height is great, the vertical rods may with advantage possess rigidity, because they may furnish resting points for fixing the vertical cross-bracing which is needed to maintain the verticality of the girder. In such cases we may either subject them to a coefficient V, or retain the preceding formula, by adding to it a term $\frac{2}{3} \frac{\mathrm{~N} \theta h^{2}}{l}$ for the addition, to each rod, above the theoretical section, of a projecting rib weighing $\theta h$ a lineal mètre.

The first method would have the advantage of simplifying the formula by reducing it to $\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) t\left\{\frac{\mathrm{~N}^{2}-1}{4 \mathrm{~N}^{2}} \cdot \frac{l^{2}}{h}\left(\mathrm{U}+q \mathrm{~V} \cdot \frac{\mathrm{~N}-2}{2 \mathrm{~N}^{2}}\right)+\frac{h}{3}\left[2(\mathrm{U}+1)+\frac{q \mathrm{~V}}{40 \mathrm{~N}^{2}}\left(16 \mathrm{~N}^{3}-55 \mathrm{~N}^{2}+56 \mathrm{~N}+15\right)\right]\right\}+\Omega$.
142. 2.-If the diagonals shown by the full lines of Fig. 1759 are now suppressed and replaced by the diagonals shown by the dotted lines, we shall have another system of simple lattice, which will give occasion to analogous calculations. The value of the strain either by tension or by compression upon the $n$th diagonal is

$$
\frac{p^{\prime} \delta}{2(2 m+1) h} \sqrt{m^{2}(m+1)^{2} \delta^{2}+[m(m+1)-(n-1)(n-2)]^{2} h^{2}},
$$

and consequently we have:
Total weight of the $2 m-1$ diagonals shown by the dotted lines $=$

$$
\frac{p^{\prime} \delta t \mathrm{~V}}{2(2 m+1)}\left\{\frac{m(m+1)(2 m-1) \delta^{2}}{h}+\frac{h}{15(m+1)}\left(16 m^{4}+55 m^{3}+70 m^{2}-85 m+4\right)\right\} .
$$

The vertical rods are subject to the same strains, and make up the same total weight as with the first system of diagonals.
143. 3.-Generally a single system of diagonals will not be sufficient, and a girder of complete crosses will be adopted. The strains are then indeterminate, and we are free to calculate the pieces from one or the other of the two following points of view.

We may, in the first place, consider the complex system as resulting from the superposition of two simple lattices, each comprising a certain portion of the section of the flanges and vertical rods, plus one of the systems of diagonals, to the exclusion of the other. It is natural to attribute a half of the load to each of these two constituent parts; and, consequently, the diagonals will have their strains and their weights given by the same formulæ as if they were simple, by changing $p^{\prime}$ into $\frac{1}{2} p^{\prime}$. The flanges retain the same weight; as to the vertical rods, the present hypothesis divides them into two portions, one of which is subject to a strain of compression and the other to a strain of tension, under the action of a determinate load extending over a part only of the flooring. For example, if the load be applied only to the rod under consideration and to all those to the right, it will produce upon this rod on the one hand a tension $\frac{p^{\prime} \delta}{4(2 m+1)}\left(m^{2}+\overline{3} m-n^{2}+n+2\right)$, and on the other hand a compression $\frac{p^{\prime} \delta}{4(2 m+1)}\left(m^{2}-n^{2}-3 m-n-2\right)$. The resultant is a tension $\frac{3 m+n+2}{2(2 m+1)} p^{\prime} \delta$, the maximum of which is $p^{\prime} \delta$ for $n=m$. Now this maximum is required of all the vertical rods when the bridge is completely loaded. They must, therefore, be all calculated for a tension equal to $\left(p+p^{\prime}\right) \delta$, without taking into account their mode of participating in the resistance of the two component simple lattices.

We therefore give:
Total weight of the $2 m$ vertical rods $=\frac{2}{3}(2 m+1)\left(p+p^{\prime}\right) \delta t h$.
The total weight of the mètre of girder is then

$$
\begin{gathered}
\frac{\mathbf{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{(2 m+1)^{2}}\left\{m(m+1) l^{2}\left(\mathbf{U}+\frac{q \mathrm{~V}}{2(2 m+1)}\right) \frac{1}{h}+\frac{2}{3}\left[(\mathrm{U}+1)(2 m+1)^{2}\right.\right. \\
\left.\left.+q \mathrm{~V} \cdot \frac{4 m^{4}+10 m^{3}+10 m^{2}-10 m+1}{5(m+1)}\right] \hbar\right\}+\Omega,
\end{gathered}
$$

or, if $l$ is expressed by $\mathrm{N} \delta$ ( N being odd),

$$
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) t\left\{\frac{\mathrm{~N}^{2}-1}{4 \mathrm{~N}^{2}} l^{2}\left(\mathrm{U}+\frac{q \mathrm{~V}}{2 \mathrm{~N}}\right) \frac{1}{h} \frac{2 h}{3}\left[\mathrm{U}+1+\frac{q \mathrm{~V}}{10}\left(\mathrm{~N}+\frac{1}{\mathrm{~N}}-\frac{30(\mathrm{~N}-1)}{\mathrm{N}^{2}(\mathrm{~N}+1)}\right)\right]\right\}+\Omega=
$$

a formula to which we shall generally have to add (141) a term $\frac{2}{3 l} \mathrm{~N} \theta h^{2}$, allowing us to enlarge the vertical rods by means of projecting ribs or gussets for the purpose of maintaining the verticality of the girder.
144. But the diagonals being very long and subject to small strains, can be stiffened only by means of a high value of $V$. Now, considering them as very flexible rods, unable to resist any compression, they will of themselves elude this kind of strain by yielding, and the lattice will assume another mode of resistance, in virtue of which the strains of compression will always be thrown upon the vertical rods. Choosing diagonals which at any instant are prepared for a tensile strain, the whole framing will offer a constant resistance. From this point of view, the two systems of diagonals will be calculated each for the whole load, which is equivalent to making $\mathrm{V}=2$, in the formula of the preceding section. It is true that in this case the vertical rods may be required to resist compression, but it will be to a very small coefficient, for the permanent tension $p \delta$ neutralizes in a great measure, if nof altogether, the strains in the opposite direction. The real strain of compression, if such be produced, is limited to $\frac{p^{\prime} \delta}{2(2 m+1)}(m-n)(m+n-1)-p \delta$, whilst under a complete load the maximum tension reaches $\left(p+p^{\prime}\right) \delta$. If, therefore, $p$ be not less than $\frac{m(m-1)}{2(2 m+1)} p^{\prime}$ the vertical rods can never be subject to compression, in which case we obtain this remarkable result, that all the lattice-bars have to resist tensile strains only, which obviates the necessity for rigidity. This allows us to adopt great heights for girders, and to make the arched form more advantageous than the straight. These favourable conditions still exist when the maxima compression of the vertical rods, without being always nul, remains at least within certain limits; and if there is occasion for applying to these rods a coefficient V, as we are about to do, it is much less on account of the possible compression, than for the purpose of maintaining the vertical position of the girder by means of suitable arrangements; such, for instance, as the vertical cross-bracing fixed to the rods in Fig. 1762, which rods must furnish sufficiently firm points. By adding this coefficient $V$ applied to the vertical rods (but no longer affecting the diagonal bars), the addition of a term $\frac{2}{3} \frac{\mathrm{~N} \theta h^{2}}{l}$ as in the preceding case, is rendered superfluous. It is the same addition expressed under another form.

It is evident that there will be an advantage in bringing the pieces of the vertical portion of the girder into accordance with the second hypothesis rather than with the first, whenever the latter would require for the rigidity of the diagonals the application of a coefficient V greater than 2. But it is scarcely necessary to remark that, although freed from all compression according to the second hypothesis, the diagonals will offer greater security if we give them all the rigidity which, with a convenient form, the area of their theoretical section admits, for we shall thus render them capable of resisting at pleasure in two different ways; the mode of effective resistance will be a kind of mean which will limit the real strains to a figure smaller than the admitted limits.
145. From the considerations in the preceding section, the weight of the lineal metre of arched girder with complete crosses may be represented by

$$
\begin{gathered}
\frac{\mathbf{P}}{l}=\frac{\left(p+p^{\prime}\right) t}{(2 m+1)^{2}}\left\{m(m+1) l^{2}\left(\mathrm{U}+\frac{q}{2 m+1}\right) \frac{1}{h}+\frac{2 h}{3}\right. \\
\left.\left[(\mathrm{U}+\mathrm{V})(2 m+1)^{2}+\frac{2 q}{5(m+1)}\left(4 m^{4}+10 m^{3}+10 m^{2}-10 m+1\right)\right]\right\}+\Omega
\end{gathered}
$$

or, if $\iota$ is expressed by $\mathrm{N} \delta(\mathrm{N}$ being odd),

$$
\frac{\mathrm{P}}{l}=\left(p+p^{\prime}\right) t\left\{\frac{\mathrm{~N}^{2}-1}{4 \mathrm{~N}^{2}} l^{2}\left(\mathrm{U}+\frac{q}{\mathrm{~N}}\right) \frac{1}{h}+\frac{2 h}{3}\left[\mathrm{U}+\mathrm{V}+\frac{q}{5}\left(\mathrm{~N}+\frac{1}{\mathrm{~N}}-\frac{30 \mathrm{~N}-1}{\mathrm{~N}^{2}(\mathrm{~N}+1)}\right)\right]\right\}+\Omega .
$$

146. Parabolic Bow-girder divided into an even number of intervals. - We will now suppose that the bearing $l$ is expressed by $2 m \delta$, and we shall consider the general case of a bow both flanges of which are curved, as in Fig. 1760.

The total height in the middle is the sum of the versed sines $h$ and $h$ ' of the curve above and below the horizontal A B, drawn through the points of meeting at the ends. The various pieces are, as before, numbered from the middle.

If the flooring were placed according to the tangent to the lower flange, we should have to lengthen the vertical rods to this tangent and to add a horizontal longitudinal girder. But nothing prevents us from placing the flooring at the height of the line AB, and saving, if not the longitudinal girder, at least the lengthening of the vertical rods; unless the upper bow be, as in the bridge at Saltash, formed of a single tubular section projecting over the space occupied by the flooring, an arrangement which requires the extremities $\mathbf{A}$ and $\mathbf{B}$ to be kept at a sufficient height above the rails. But two distinct framings well tied and braced together, leaving, of course, a free passage for the trains, may constitute a sufficiently rigid whole; and we shall, consequently, consider only the pieces represented in the figure, except the casual addition of a longitudinal beam tying together the vertical rods at the height of the flooring, this piece being regarded as included in the term $\Omega$ independent of the height.


We have as geometrical data:
Length of the $n$th vertical rod $=\frac{m^{2}-(n-1)^{2}}{m^{2}}\left(h+h^{\prime}\right)$;
Sum of the lengths of the $2 m-1$ vertical rods $=\left(h+h^{\prime}\right) \frac{\left(4 m^{2}-1\right)}{3 m}$;
Length of the $n$th section of upper flange $=\frac{1}{m^{2}} \sqrt{m^{4} \delta^{2}+(2 n-1)^{2} h^{2}}$;
Length of the $n$th diagonal shown by a full line $=\frac{1}{m^{2}} \sqrt{m^{4} \delta^{2}+\left[\left(m^{2}-n^{2}\right)\left(h+h^{\prime}\right)+(2 n-1) h^{\prime}\right]^{2}}$.
This diagonal is inclined upon the horizon at an angle $\alpha$ given by

$$
\operatorname{Sin} . \alpha=\frac{\left(m^{2}-n^{2}\right)\left(h+h^{\prime}\right)+(2 n-1) h^{\prime}}{\sqrt{m^{4} \delta^{2}+\left[\left(m^{2}-n^{2}\right)\left(h+h^{\prime}\right)+(2 n-1) h^{\prime}\right]^{2}}} .
$$

The $n$th section of upper flange makes with the vertical an angle $\gamma$ such that cotan. $\gamma=\frac{(2 n-1) h}{m^{2} \delta}$.
We shall obtain the analogous quantities, relative to the lower flange and to the diagonals shown by the dotted lines, by simply changing $h$ into $h^{\prime}$, and $h^{\prime}$ into $h$.
147. The flanges should always be calculated under a full load, whatever the diagonals adopted may be. The value of the strain upon the $n$th section of the upper flange is

$$
\frac{\left(p+p^{\prime}\right) \delta}{2\left(h+h^{\prime}\right)} \sqrt{m^{4} \delta^{2}+(2 n-1)^{2} h^{2}}
$$

and that upon the $n$th lower section is found by the same formula modified by interchanging $h$ and $h^{\prime}$, which only alters the second term under the radical. If we cut these two correspanding sections according to the radical, their action will furnish a vertical resultant projection equal to $\frac{\left(p+p^{\prime}\right) \delta}{2}(2 n-1)$, which is exactly in equilibrio with the stress; thus, when the load is complete throughout the length of the bridge, the diagonals do not resist.

The weight of the $n$th section of the upper flange being expressed by

$$
\frac{\left(p+p^{\prime}\right) \delta t}{2 m^{2}\left(h+h^{\prime}\right)}\left[m^{4} \delta^{2}+(2 n-1)^{2} h^{2}\right]
$$

we conclude that the total weight of this flange will be $\frac{\left(p+p^{\prime}\right) \delta t}{h+h^{\prime}}\left[m^{3} \delta^{2}+\left(4 m^{2}-1\right) \frac{h^{2}}{3 m}\right]$, and adding the coefficient U , the two flanges together will weigh

$$
\frac{\left(p+p^{\prime}\right) \delta t}{h+h^{\prime}}-\left[\check{U}\left[m^{3} \delta^{2}+\left(4 m^{2}-1\right) \frac{\left(l^{2}+h^{\prime 2}\right)}{3 m}\right]\right.
$$

148. The load still extending over the whole bearing, the strain upon the vertical rods will be found by remarking that, for the equilibrium of the point I, the rod D I must destroy the resultant of the strains from the two sections L I and E I, abutting at this point, the diagonals being inactive, as we have just seen. We have:
Tension of the $n$th $\operatorname{rod}=\left(p+p^{\prime}\right) \delta \frac{h}{h+h^{\prime}}$, a constant value whatever $n$ may be.
In the case in which the maximum strain is always limited to this varue, the total weight of the $2 m-1$ rods will be independent of $h^{\prime}$, and will have as its expression $\left(p+p^{\prime}\right) \delta t h \frac{4 m^{2}-1}{3 m}$.

This supposes that the load is applied to the bottom of the vertical rods. If, on the contrary; the flooring is on a level with the line AB, the upper portion of each rod will have the same
tensile strain as before, but the lower portion will bear a compression $\left(p+p^{\prime}\right) \delta \frac{h^{\prime}}{h+h^{\prime}}$, and if we give different sections to the two portions, the total weight will be expressed by

$$
\left(p+p^{\prime}\right) \delta t \cdot \frac{4 m^{2}-1}{3 m} \cdot \frac{h^{2}+\mathrm{V} h^{\prime 2}}{h+h^{\prime}}
$$

V being a coefficient of stiffness applied to the compressed portions, and which may be extended to the upper portions for the purpose of adding rigidity to the bridge considered as a whole.
149. If the diagonals shown by the full lines alone existed, we should find the strain $T$ upon the $n$th diagonal E D, Fig. 1760, by cutting it by a plane G H, and expressing the nullity of the sum of the vertical projections of the forces applied to the portion of the framing comprised between $\mathbf{A}$ and the section GH.

But the $n$th section of the upper flange, cut by the line GH, is subject to a strain of compression $=\frac{\mathrm{M}}{\mathrm{ID} \times \sin \cdot \gamma}, \mathrm{M}$ being the moment of rupture about the point D . The section CD is subject, on the contrary, to a tensile strain $=\frac{\mathbf{M}^{\prime}}{\mathrm{EC} \times \sin . \gamma^{\prime}}, \mathrm{MI}^{\prime}$ being the moment of rupture at the point E or C . Consequently, if $\mathrm{F}=$ the stress in the interval CD , we shall have

$$
\mathrm{T} \sin . \alpha=\mathrm{F}-\frac{\mathrm{M} \operatorname{cotan} \cdot \gamma}{\mathrm{ID}}-\frac{\mathrm{M} \mathrm{I}^{\prime} \operatorname{cotan} \cdot \boldsymbol{\gamma}^{\prime}}{\mathrm{EC}}
$$

This expression being of the form $\mathrm{F}-\mathrm{AMI}-\mathrm{A}^{\prime} \mathrm{IN}^{\prime}$, will be a maximum when the load goes from D (inclusive) to the abutment B , according to the observation made in section 7. Applying the general rule, and replacing cotan. $\gamma, \operatorname{cotan} . \gamma^{\prime}$, and the lengths ID, EC, of the $n$th and $(n+1)$ th vertical rods by their values, we arrive at the maximum tensile strain upon the diagonal, namely, $\mathrm{T}=\frac{p^{\prime} \delta}{4 m\left(h+h^{\prime}\right)} \sqrt{ } \frac{m^{4} \delta^{2}+\left[\left(m^{2}-n^{2}\right)\left(h+h^{\prime}\right)+(2 n-1) h^{\prime}\right]^{2}}{}$. The weight of this bar is $=\frac{p^{\prime} \delta t}{4 m^{3}\left(h \frac{h^{\prime}}{}\right)}\left\{m^{4} \delta^{2}+\left[\left(m^{2}-n^{2}\right)\left(h+h^{\prime}\right)+(2 n-1) h^{\prime}\right]^{2}\right\}$.

Summing this expression, in which $n$ takes the successive values $1,2, \ldots m-1$, then doubling that the result may include the whole bearing, we obtain as the total weight of the $2(m-1)$ diagonals in full lines,

$$
\begin{gathered}
\frac{p^{\prime} \delta t}{60 m^{2}\left(h+h^{\prime}\right)}\left\{30 m^{3}(m-1) \delta^{2}+\left(16 m^{4}-15 m^{3}-1\right) h^{2}\right. \\
\left.+\left(16 m^{4}+15 m^{3}-120 m+119-\frac{30}{m}\right) h^{\prime 2}+8\left(4 m^{4}-5 m^{2}+1\right) h h^{\prime}\right\} .
\end{gathered}
$$

In a simple lattice, we must multiply this weight by a coefficient of stiffness; for the bars are subject, under a partial complementary load, to compressions as great as the tensions T.

A load extending from one abutment to any point in the flooring, causes a strain of compression upon all the diagonals (full lines) in the half-bay contiguous to this abutment, and one of tension upon all those of the opposite half-bay.
150. Having still only the diagonals shown by the full lines, a partial load upon the right side will cause upon the $n$th vertical rod a strain of compression, the value of which will be found by considering a section such as $\mathrm{G}^{\prime} \mathrm{H}^{\prime}$, Fig. 1760, and writing again the equation of the vertical projections of the forces. Into this equation will enter the stress after the point $D$, and the moment of rupture about this point, a moment upon which depend the strains upon the divisions EI, DS, crossed by the section GH. We shall find as the maximum of the compression sought, due to the load $p^{\prime}, \frac{p^{\prime} \delta}{4} \frac{(m+n-2)}{\left(h+h^{\prime}\right)}\left[(m-n)\left(h+h^{\prime}\right)+2 h^{\prime}\right]$, a formula which does not apply to the middle bar, for which $n=1$.

The effective compression which may be produced will be only the difference between the above value and that of the permanent tension $\frac{p \delta h}{h+h^{\prime}}$.

The maxima tension will exceed the compression by the whole value $\frac{\left(p+p^{\prime}\right) \delta h}{h+h^{\prime}}$; for the special effect of a partial load upon the left side, complementary of the preceding, should be to annul, in the first place, the compression produced by the load on the right, and then besides this to raise the tension to the value which it possesses when the weights $p^{\prime} \delta$ are applied to all the vertical rods. Thus this complementary load, acting above, will produce, adding the permanent strain, a tension $\frac{\left(p+p^{\prime}\right) \delta}{h+h^{\prime}}\left\{h+q \frac{(m+n-2)}{4 m}\left[(m-n)\left(h+h^{\prime}\right)+2 h^{\prime}\right]\right\}$, according to which the weight of the vertical rods should becalculated, if the diagonals shown by the full lines exist alone, and the load be applied to the bottom of the rods.
151. But if the load be placed at the height of the line A B, passing through the points of junction of the flanges, the lower portion of the vertical rod will be subject to less tension and to greater compression, for the stress will not be the same when the line $\mathrm{G}^{\prime} \mathrm{H}^{\prime}$ cuts the rod above or below the points where the flooring is affixed. When the weights $p^{\prime} \delta$ are applied only to the $n$th vertical rods and to all those on the right, the lower portion of this $n$th rod will be subject to a strain of compression $=p^{\prime} \delta \frac{m+n}{4 m} \cdot \frac{(m-n)\left(h+h^{\prime}\right)+2 h^{\prime}}{h+h^{\prime}}+\frac{p \delta h^{\prime}}{h+h^{\prime}}$,

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152. The middle vertical bar is an exception to the preceding formulæ, because a secant plans cannot cross it without cutting a diagonal also. But the strain upon this rod is determined by the condition of being in equilibrio with the resultant, always vertical, of the compressions borne by the two sections of the upper flange. When the bridge is completely loaded, the rod supports a maxima tension of $\frac{\left(p+p^{\prime}\right) \delta h}{h+h^{\prime}}$ in its upper portion; and below the point where it receives the cross-girder, it is at the same time subject to a strain of compression of $\frac{\left(p+p^{\prime}\right) \delta h^{\prime}}{h+h^{\prime}}$; but the compression will be greater if we unload the bridge entirely, with the exception of the middle rod under consideration, and in this case it reaches the value $\frac{\left(p+p^{\prime}\right) \delta h^{\prime}}{h+h^{\prime}}+p^{\prime} \delta \frac{(m-1) h}{m\left(h+h^{\prime}\right)}$ or $\frac{\left(p+p^{\prime}\right) \delta h^{\prime}}{h+h^{\prime}}\left[1+\frac{(m-1) h}{m h} q\right]$. If, on the contrary, the load were over the whole of the bridge, except upon the middle rod, the lower portion of this rod would support only $\frac{p \delta h^{\prime}}{h+h^{\prime}}-\frac{p^{\prime}(m-1) \delta h}{m\left(h+h^{\prime}\right)}$ which may be changed into tension if the sign becomes negative.
153. If now we exclude the diagonals shown by the full lines and adopt those shown by the dotted ones, it is easy to see that we shall obtain the strains and the weights of the latter by the formulæ found for the former in 149, by merely changing $h$ into $h^{\prime}$ and $h^{\prime}$ into $h$, as if the girder were turned upside down.

As to the vertical rods, the $n$th one will bear, at least in its upper portion, a maxima tension $=\frac{p \delta h}{h+h^{\prime}}+p^{\prime} \delta \frac{(m+n)(m-n)\left(h+h^{\prime}\right)+2 h}{4 m\left(h+h^{\prime}\right)}$; and if the load be applied to a certain point in the height of the rod, the portion below this point may be compressed by a force equal to $\mu^{\prime} \delta^{m-n+2} \frac{(m+n)\left(h+h^{\prime}\right)-2 h}{4 n}+\frac{p \delta h^{\prime}}{h+h^{\prime}}$, which will be produced when the rod in question and those on the left support alone weights $p^{\prime} \delta$.

It is easy to see a priori that this last strain may be deduced from the last formula of 150 by simply changing $h$ into $h^{\prime}$ and $h^{\prime}$ into $h$. The maxima compression of 151 is also connected with the maxima tension of the present section by virtue of the same symmetry. Indeed, if we turn the bow upside down, the lower portion of the vertical rod will become the upper, the system of diagonals will have changed, $h$ and $h^{\prime}$ will have interchanged, and the signs of all the forces will also have changed. The middle rod is subject to the same considerations.
154. The preceding formulm enable us to find easily the net cost of a simple lattice, but we shall stop to consider only the case in which the diagonals are doubled and form complete crosses.

On account of the indeterminateness of the strains in a multiple lattice, we may consider the resistance in either manner explained in 143 and 144 (rigid or flexible crosses). But, without omitting to give to the diagonals the rigidity which their theoretical section allows, by way of an additional precaution we shall follow the second method which consists in throwing the strains of compression upon the vertical rods. They will, indeed, need to be stiffened on account of their theoretical strains being more considerable than those of the diagonals, and the necessity which will nearly always exist of giving them a surplus section for the sake of the general effect of the flooring. It is obvious that if we suppose the flooring placed towards the middle of the vertical rods, the latter must possess rigidity ; thus they will be able to resist compression, whether they be required to do so throughout their length or not. The diagonals, on the contrary, will require a coefficient of stiffness greater than 2; it is not too much, therefore, to double them, reckoning at each instant only upon those which resist by tension; and as additional security, those subject to compression will not remain quite useless, for they may be stiffened.
155. Before establishing formulæ of the total weight, we must point out exactly the strains which the vertical rods have to resist.

We must remark, in the first place, that under the action of a partial load moving from one of the abutments to a given point, the diagonals subjected to a tensile strain, which alone are considered as acting, will all be inclined in the same direction, from one end of the framing to the other, they will be those the feet of which are inclined towards the abutment affected by the partial load. It follows from this that, under such a mode of loading, any vertical rod may be cut by a plane which does not cross any active diagonals, and its strain will be determined as if the lattice were a simple one. The middle bar itself will come into the general formulæ, for there will no longer be in this point the forced inversion of resisting diagonals which was produced in the simple lattices with the forms adopted.

The upper portion of the rods will be calculated generally for a constant $\operatorname{strain}\left(p+p^{\prime}\right) \frac{\delta h}{h+h^{\prime}}$, which is no other than the tension produced under a complete load. A partial load might produce a compression equal to $\frac{p^{\prime} \delta(m+n-2)}{4 m\left(h+h^{\prime}\right)}\left[(m-n)\left(h+h^{\prime}\right)+2 h^{\prime}\right]-\frac{p \delta h}{h+h^{\prime}}$ (see 150); but in order that this strain may exceed the preceding, $h^{\prime}$ must become great and $h$ small. For $h=h^{\prime}$, the first formula, is sufficient so long as $m$ does not exceed the limit $\frac{1}{2}\left(3+4 \frac{p}{p^{\prime}}+\sqrt{\left(3+4 \frac{p}{p^{\prime}}\right)^{2}+1}\right)$; for example, for $q=0.60$ or $\frac{p}{p^{\prime}}=\frac{2}{3}, m$ may reach the value 5 . For $h^{\prime}$ nul, it is sufficient if $m$ be less than $6+8 \frac{p}{p^{\prime}}$, which is always the case, and even then it may happen that no compression is

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manifested, if.the permanent tension always predominates over the compressions which the load mey produce. Usually $m$ is less than 5 ; if it exceeded this slightly, a few of the rods would be subject to a strain of compression a little greater than the tensile strain, but this small difference might be neglected on account of the surplus section which will be given in every case. We shall aff ct the weight of the vertical rod with a coefficient V, designed to give the rigidity necessary to resist compression and to add to the transverse stability of the structure.

If now we pass on to the lower portion of the rods, situate below the point where the load is applied to them, the strain of compression would be $\frac{\left(p+p^{\prime}\right) \delta h^{\prime}}{h+h^{\prime}}$ considering only the complete load. But under a partial load, this strain will reach the greater value (153)

$$
\frac{p \delta h^{\prime}}{h+h^{\prime}}+p^{\prime} \delta \cdot \frac{m-n+2}{4 m} \cdot \frac{(m+n)\left(h+h^{\prime}\right)-2 h}{h+h^{\prime}},
$$

according to which the piece should be calculated. We obtain the weight of the lower portion of the $n$th rod, by multiplying by the length and by the quantity $t \mathrm{~V}$; and as the formula subsists for the middle rod, it will be sufficient to sum, supposing $n$ to take the successive values $1,2, \ldots m$, then to double and subtract the value relative to $n=1$, in order not to reckon the middle bar twice. We thus find the value given in the following section for the total weight of the lower portions of the $2 m-1$ rods.
156. Adding to the rods thus calculated the total weight of the flanges (147) and that of the two systems of diagonals (149 and 153), we shall have the total weight of the bow-girder
Weight of the flanges $=\frac{\left(p+p^{\prime}\right) \delta t \mathrm{U}}{h+h^{\prime}}\left[2 m^{3} \delta^{2}+(4 m-1) \frac{h^{2}+h^{\prime 2}}{3 m}\right] ;$
Weight of the diagonals

$$
\begin{aligned}
& =\frac{p^{\prime} \delta t}{30 m^{2}\left(h+h^{\prime}\right)}\left[30 m^{3}(m-1) \delta^{2}+\left(h^{2}+h^{\prime 2}\right)\left(16 m^{4}-60 m+59-\frac{15}{m}\right)+8 \hbar h^{\prime}\left(4 m^{4}-5 m^{2}+1\right)\right] ; \\
& \text { Weight of the vertical rods }\left\{\begin{array}{l}
\text { Upper portion }=\frac{\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \delta t h^{2} \mathrm{~V}}{3 m\left(h+h^{\prime}\right)} ; \\
\text { Lower portion }=\frac{\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \delta t h^{\prime 2} \mathrm{~V}}{3 m\left(h+h^{\prime}\right)}+
\end{array}\right. \\
& \frac{p^{\prime} \delta t h^{\prime} \mathrm{V}}{60 m^{2}\left(h+h^{\prime}\right)}\left[h\left(16 m^{4}+15 m^{3}-20 m^{2}-15 m+4\right)+h^{\prime}\left(16 m^{4}-40 m^{3}+20 m^{2}+10 m-6\right)\right] .
\end{aligned}
$$

Mean weight of the bow-girder a lineal mètre

$$
\begin{aligned}
= & \frac{\left(p+p^{\prime}\right) t}{12 m^{2}\left(h+h^{\prime}\right)}\left\{3 m^{2} \mathrm{U} l^{2}+2(\mathrm{U}+\mathrm{V})\left(4 m^{2}-1\right)\left(h^{2}+h^{\prime 2}\right)+\frac{q}{10 m^{2}}\left[15 m^{2}(m-1) l^{2}\right.\right. \\
& +2\left(h^{2}+h^{\prime 2}\right)\left(16 m^{5}-60 m^{2}+59 m-15\right)+16 m h h^{\prime}\left(4 m^{4}-5 m^{2}+1\right)+m \mathrm{~V} h h^{\prime}\left(16 m^{4}\right. \\
& \left.\left.\left.+15 m^{3}-20 m^{2}-15 m+4\right)+2 m \mathrm{~V} h^{\prime 2}\left(8 m^{4}-20 m^{3}+10 m^{2}+5 m-3\right)\right]\right\}+\Omega
\end{aligned}
$$

For the case in which $h=h^{\prime}$, this formula becomes, representing the total height by H :
Mean weight a mètre of bow-girder with flanges of an equal curve

$$
=\frac{\left(p+p^{\prime}\right) t}{12 m^{2}}\left\{\frac{3 m^{2} \mathrm{U} l^{2}}{\mathrm{H}}+(\mathrm{U}+\mathrm{V})\left(4 m^{2}-1\right) \mathrm{H}+\frac{q}{10 m^{2}}\right.
$$

$\left.\left[\frac{15 m^{2}(m-1) l^{2}}{\mathrm{H}}+\mathrm{H}\left(32 m^{5}-20 m^{3}-60 m^{2}+63 m-15\right)+\frac{m \mathrm{VH}}{4}\left(32 m^{4}-25 m^{3}-5 m-2\right)\right]\right\}+\Omega$; or making $2 m=\mathrm{N}$ : Weight a mètre

$$
=\left(p+p^{\prime}\right) t\left\{\frac{\mathrm{U} l^{2}}{4 \mathrm{H}}+\frac{(\mathrm{U}+\mathrm{V})\left(\mathrm{N}^{2}-1\right) \mathrm{H}}{3 \mathrm{~N}^{2}}+\frac{q}{60 \mathrm{~N}^{4}}\right.
$$

$\left.\left[15 \mathrm{~N}^{2}(\mathrm{~N}-2) \frac{l^{2}}{\mathrm{H}}+4 \mathrm{H}\left(2 \mathrm{~N}^{5}-5 \mathrm{~N}^{3}-30 \mathrm{~N}^{2}+63 \mathrm{~N}-30\right)+\frac{\mathrm{NVH}}{8}\left(16 \mathrm{~N}^{4}-25 \mathrm{~N}^{3}-20 \mathrm{~N}-16\right)\right]\right\}+\Omega$.
The important part played by the vertical rods, which serve in several ways, will require a high value of V .
157. Parabolic Arched Girder, divided into an even number of intervals.-The particular case in which $h^{\prime}=0$ gives:
Weight a mètre of arched girder
$=\left(p+p^{\prime}\right) t\left\{\frac{\mathrm{U} l^{2}}{4 \mathrm{H}}+\frac{(\mathrm{U}+\mathrm{V})\left(4 m^{2}-1\right) h}{6 m^{2}}+\frac{q}{m^{2}}\left[\frac{(m-1) l^{2}}{8 h}+\frac{h}{60 m^{2}}\left(16 m^{5}-60 m^{2}+59 m-15\right)\right]\right\}+\Omega$;
or, N being even : Weight a mètre
$=\left(p+p^{\prime}\right) t\left\{\frac{l^{2}}{4 h}\left(\mathrm{U}+\frac{\mathrm{N}-2}{\mathrm{~N}^{2}} q\right)+\frac{2 h}{3 \mathrm{~N}^{2}}\left[(\mathrm{U}+\mathrm{V})\left(\mathrm{N}^{2}-1\right)+\frac{q}{5 \mathrm{~N}^{2}}\left(\mathrm{~N}^{5}-30 \mathrm{~N}^{2}+59 \mathrm{~N}-30\right)\right]\right\}+\Omega$.
The observations in 144 apply here. No rod will be subject to compression if we have $p>\frac{(m-1)^{2}}{4 m} p^{\prime}$, and even when this condition is not fulfilled, the compressions may be neglected, for they are always much less than the tensions. The coefficient V is, therefore, really retained to give to the rods a rigidity which may enable us to utilize them in connecting the girders; but its value may be considerably less than in the case in which $h=h^{\prime}$, of the preceding section.
158. The strains upon the pieces are easily deduced from the general formulæ. We have::

Constant maximum strain upon the lower flange $=\frac{m^{2}\left(p+p^{\prime}\right) \delta^{2}}{2 h}$;
Maximum strain upon the $n$th section of the upper flange $=\frac{\left(p+p^{\prime}\right) \delta}{2 h} \sqrt{m^{4} \delta^{2}+(2 n-1)^{2} h^{2}}$;
Maximum strain upon the $n$th diagonal (full lines) $=\frac{p^{\prime} \delta}{4 m h} \sqrt{m^{4} \delta^{2}+\left(m^{2}-n^{2}\right)^{2} h^{2}}$;
Maximum strain upon the $n$th diagonal (dotted lines) $=\frac{p^{\prime} \delta}{4 m h} \sqrt{m^{4} \delta^{2}+\left[m^{2}\right.}-\left(\overline{\left.n-1)^{2}\right]^{2} h^{2}}\right.$;
Maximum strain upon the $n$th vertical rod $\left\{\begin{array}{l}\text { tension }+\left(p+p^{\prime}\right) \delta ; \\ \text { compression (if any) }=\frac{(m-n)(m+n-2) p^{\prime} \delta}{4 m}-p \delta . ~\end{array}\right.$
159. Bow Bridges continued.-Examples.-The great height which bow bridges attain renders the bow subject to compression liable to distortion in the middle of the bearing; but this height will generally enable us to use ties on the upper part without impeding the passage of the trains. The lower and upper ties, with the vertical rods, will constitute a rigid frame, binding the whole together. If the height is very great, simple gussets will be insufficient at the corners of these frames, and recourse should be had to vertical St. Andrew crosses, similar to those shown in Fig. 1770. This is the case in which it is especially necessary to give the vertical rods an excess of section, to prevent their yielding at the points to which the vertical cross-bracing is fixed.

At the end, the decrease of height allows the suppression of the upper ties without compromising the stability. In the intermediate parts, it will be prudent to place vertical rods with broad projecting ribs, firmly fixed to the horizontal portion of the bridge, as in the case of girders loaded on the lower side.

The rigidity of the whole is completed by the lower horizontal cross-bracing, and a cylindrical upper cross-bracing extending over that portion of the bridge where the height is sufficient. In this way distortion, or the strain of torsion, will be better prevented or overcome than in girders loaded on the lower side, and it may be done at a less expense than in a straight tubular bridge, in which the upper tying must extend throughout the length.

The framing, or bow-girder, should terminate at its two extremities in solid wrought-iron plates binding the two flanges together; great strength is here required, but it may be obtained without much expense, because the height is inconsiderable.
160. Rigidity of the Bows.- If the form of a bow-girder with a straight lower flange, or stringer, be preferred, we saw (144 and 157) that it is easy, whatever the height may be, to elude completely the compressions upon the diagonals, and almost wholly also upon the vertical rods, which, besides, are always provided with considerable rigidity. It is, therefore, useless in this case to make the bow rigid; perhaps it is wise to allow the flanges a certain flexibility, in order that the spandrils may have to maintain the form of the bow; for if the rigidity of the latter usurped this duty in too marked a manner, there would result moments of rupture in virtue of which the distribution of the pressures would be made in an unequal manner among the several sections.

In a certain degree this will always be the case, and it should be taken into account by slightly increasing the coefficient U. This remark applies also to straight girders which are not strictly articulate or jointed pieces, though calculated as such. It is advantageous to regard the two flanges as solid with each other, and furnishing a moment of resistance in virtue of which the pressure is greater upon certain fibres than upon others. The cross and the vertical bars will benefit by this circumstance, and will bear strains a little less than those given in the calculations.

It is when the moving load is great with respect to the dead weight that it is especially necessary to consider the bow-girder as an articulate system. On the contrary, if we place the resistance to distortion in the rigidity of the bow, we are obliged to increase the permanent load by means of a thick layer of ballast. The bow may then be calculated by the formulæ of the flexion of the curved pieces; the diagonals are, however, retained, but they are reduced to very slender rods; the flanges, on the contrary, are strengthened, and their section should have a considerable height. Under these conditions, the principal part will be played by rigidity; for when several pieces act together, the strain will concentrate itself upon those which are best able to resist it.
161. Comparison between Straight and Arched Girders.-When the height allowed is unlimited, a bridge of straight girders loaded on the upper side will have, over bow-girders, the advantage of diminishing the cube of masonry by requiring a lower bed. But if the height is limited, we have only, in making choice of a system, to compare the metallic portions.

If the bridge had to carry only the permanent load $p$, uniformly distributed throughout the length, the comparison would be exceedingly simple. Indeed, an arched girder may, in this case, be deprived of diagonals, and by giving it the most advantageous height, its weight a lineal mètre will be expressed by the following value, supposing $l=2 \mathrm{~m} \delta$ :

$$
\frac{\mathrm{P}_{1}}{l}=\frac{p l t}{m} \sqrt{\frac{4 m^{2}-1}{3}}\left(\text { for height }=\frac{m l}{2} \sqrt{\frac{3}{4 m^{2}-1}}\right)
$$

Neglecting all coefficients, a girder of triangles, Fig. 1657, would give for the theoretical minimum expense ( $\alpha$ being equal to $\beta$ ),

$$
\frac{\mathrm{P}_{2}}{l}=\frac{p l t}{2 m} \sqrt{m\left[m+\frac{2}{3}\left(4 m^{2}-1\right)\right]} \text { with height }=\frac{l}{4 m} \sqrt{1+\frac{2}{3 m}\left(4 m^{2}-1\right)} .
$$

| Or for $m=.$. |
| :---: |
| We have $\frac{P_{2}}{P_{1}}=$ |

Thus for $m=8$, the straight girder would weigh more than twice as much as the arched girder; but this supposes that the rise of the latter is not less than $0 \cdot 43$ of the bearing, whilst the height of the former would not exceed $0 \cdot 148$.

The effect of a moving load will be to render necessary the addition of diagonals to this latter girder; but for the straight girder also the strains upon the bars will be greater than with a permanent load. Rigidity requires another increase, but more especially upon the struts of the straight girder. The arched girder will generally have a great advantage in respect to the lattice, even with heights which are not the most economical; but in order that there may be nothing to lose upon the flanges, it must have a rise $\kappa l$ greater than the height $\kappa^{\prime} l$ which would be given to the straight girder. The weights of the flanges will be equal in the two systems when we have between $\kappa$ and $\kappa^{\prime}$ the following relation, the coefficient U being supposed the same in both systems: $\kappa^{\prime}=\frac{2 \kappa\left(\mathrm{~N}^{2}-1\right)}{3\left(\mathrm{~N}^{2}-1\right)+8 \mathrm{~N}^{2} \kappa^{2}}$, in the case of N odd; or

$$
\kappa^{\prime}=\frac{2 \kappa\left(\mathrm{~N}^{2}-1\right)}{3 \mathrm{~N}^{2}+8\left(\mathrm{~N}^{2}-1\right) \kappa^{2}}, \text { if } \mathrm{N} \text { is even. }
$$

N has, however, but a small influence; if for the straight girder we take $\kappa^{\prime}=0.10$, we shall have $\kappa^{\prime}=$ about $0 \cdot 16$; for if $\kappa^{\prime}=\frac{1}{8}$, the rise would be 0.21 of the bearing.

In general, with the rise allowed in practice, we ought not to reckon upon saring in the flanges, but the advantage of the arched form will be essentially in relieving the vertical portion (see 168).
162. First Example.-Arched Girder with Simple Rigid Diagonals, Figs. 1761 to 1765.Data: $l=36^{\mathrm{m}}, p=2000^{\mathrm{k}}, p^{\prime}=3300^{\mathrm{k}}, p+p^{\prime}=5300^{\mathrm{k}}, q=0.623 ; \delta=4^{\mathrm{m}}, \mathrm{N}=9$, or $m=4$, $h=6^{\mathrm{m}} \cdot 50$.

The lower flange supports, under a load, a constant strain of $130 \frac{1}{2}$ tons. It is composed of three angle-irons of $\frac{100 / 100}{12}$ millimètres, a vertical rib of $550 / 10$, to whick the vertical bars aro fixed by twelve rivets of $0^{\mathrm{m}} \cdot 025$, and two horizontal plates of $600 / 10$ each.

The upper flange supports strains expressed by $\frac{\left(p+p^{\prime}\right) \delta}{2 h} \sqrt{m^{2}(m+1)^{2} \delta^{2}+4(n-1)^{2} h^{2}}$, which vary from $130 \frac{1}{2}$ tons in the middle ( $n=1$ ), to $155 \cdot 6$ tons in the extreme section $(n=5$ ).

The vertical rods have a maximum tension which varies from 30 to $21 \cdot 2$ tons. The first may, besides, be compressed by a force equal to $\frac{p^{\prime} \delta}{2(2 m+1)}(m-n)(m+n-1)-p \delta=800^{\mathrm{k}}$ only, and for the others the strain is always one of tension. The vertical rods are therefore strengthened only with a vier to the stability of the whole flooring. The two girders, or framings, are bound firmly together in tro places torrards the middle of the bearing by tie-beams so arranged as to form a rectangular figure enclosing the whole bridge.

The strain upon the diagonals (139) raries from 17.2 tons for the first (middle) to 10.8 tons for the last $(n=4)$. The first is divided into two portions to form a cross, each arm of which supports only 8.6 tons. The sections adopted have sufficient rigidity, regard being had to their strain a unit of section, which strain is very small.

Calculation gives:
Flanges $\left\{\right.$ Angle-irons and rertical ribs, with their joint-plates and solid ends $10500^{\mathrm{k}}$
$18800^{\text {b }}$
Vertical rods .. .. .. .. .. .. .. .. .. .. .. .. .. .. 2130
Diagonals $. . . \quad . . \quad$....$\quad$....$\quad$....$\quad$.. .. ..
Two horizontal pieces of railing and central gusset .. .. .. .. .. 270
Tntal .. .. .. .. .. .. .. .. .. $\overline{23000^{k}}$
Say 612 kilogrammes a lineal mètre (of the whole length $37^{\mathrm{m}} \cdot 60$ ).
The formula of 141 becomes, with the data of the present example, and adding a special term for the addition to the vertical rods,

$$
\frac{\mathrm{P}}{l}=369 \mathrm{U}+22 \cdot 6 \mathrm{~V}+38 \cdot 5+7 \cdot 04 \theta+\Omega .
$$

The flanges weighing $\frac{18800^{\mathrm{k}}}{37^{\mathrm{m} \cdot 60}}=500^{\mathrm{k}}$ a mètre, we shall have $\mathrm{U}=\frac{500}{369}=1 \cdot 35$, including the plates at the ends. These latter having an excess of strength, the value of $\mathbb{U}$ may be reduced a little.

For the diagonals, $\mathrm{V}=\frac{50}{22 \cdot 6}=2 \cdot 21$.

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The vertical rods weigh, according to the calculation, about 60 kilogrammes a lineal mètre of girder; the theoretical weight being only $38 \cdot 5$, we have to cover $21^{\mathrm{k}} 5 \mathrm{by}$ means of a term $\theta$ requiring $\theta=3$; in other words, the excess of section given to the rods is equivalent to the addition of a projecting piece weighing $\theta h=19^{\mathrm{k}} \cdot 5 \mathrm{a}$ mètre (of its own length).
$\Omega$ is here only $\frac{270}{36}=8$ kilogrammes (say 0.013 of the total weight).
163. If double diagonals were adopted, we might leave the vertical rods unchanged, since they are determined by other conditions than the theoretical strain; and the double diagonals, calculated for tension only, would give a weight of 58 kilogrammes a mètre. Thus, in this example, the single diagonals, though V is a little greater than 2, give a small advantage, which is due to the fact of their having been chosen short, and exposed to the least strain. The saving is, however, so trifling that it would have been quite as advantageous, and perhaps more so, to adopt the cross. This will be done in the following example.
164. Second Example.-Arched Girder of Tension-crosses, Figs. 1770 to 1787.-Data: $l=66^{\mathrm{m}}, p=2500^{\mathrm{k}}, p^{\prime}$ $=3000^{\mathrm{k}}, p+p^{\prime}=5500^{\mathrm{k}}, q=0 \cdot 545$, $\delta=6^{\mathrm{m}}$, ' $\mathrm{N}=11$, or $m=5, h=11$.

As in the preceding example, we shall adopt a moderate load, for the value $p^{\prime}=4000$ corresponds with the purely fortuitous case of two heavilyladen trains crossing the bridge at the same time.

The lower flange bears, under a complete load, a constant tension of 270 tons; the upper, 270 in the middle to $316 \cdot 5$ tons at the end; or more correctly, this last strain will be less, for towards the ends the bow ceases to be parabolic and terminates by tangents, for the purpose of lengthening sufficiently the bearing upon the rollers.

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Regarding the diagonals as resisting only by tension, so that one arm only of each cross acts under a given load, the strains will be:

| Short diagonals | . | . | No. 1 (middle) | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Long diagonals | .. | . | Nos. 1 and 2 |  |  |  |  |
| $27 \cdot 860$ | $26 \cdot 510$ | $23 \cdot 764$ | $19 \cdot 90 t$ | $15 \cdot 690$ |  |  |  |

The sections adopted have a form possessing considerable rigidity. If they had been made to resist compression for the half of the load, the maximum strains would have been the half of the above, so that the sections adopted would give at least $V=2$, which would warrant us in increasing to 50 the ratio of the length to the reduced thickness, a condition which is not far from being fulfilled. The spandrils are thus susceptible of two modes of resistance; the one which will be produced will be an intermediate mode, in virtue of which the strains will be less than those provided for.

The vertical rods bear 33 tons under a full load. They always resist by tension, except the first towards the middle, which may be subject to a strain of compression of $1 \cdot 345$ ton only. Their sections are, however, increased to give rigidity to the rectangular figure formed by the tie-beams already alluded to, to which the vertical cross-bracing is fixed.

Direct calculation gives the following results:-
$\left\{\begin{array}{l}\text { Angle-irons with vertical ribs with their joint-plates and the end plates } 27550^{\mathrm{k}} \\ \text { Horizontal }\end{array}\right.$
Flanges Horizontal plates with their joint-plates .. .. .. .. .. .. .. 37400
Gussets between the vertical rib and the horizontal plate, angle-iron on
the edge of upper flange ..
Vertical rods (including $330^{k}$ for joint-plates and other accessories required in fixing) .. 8100
Diagonals (including $1100^{\mathrm{k}}$ for joint-plates, filling-plates at the points of crossing).. .. 7250

Total
$83000^{k}$
Say 1203 kilogrammes a mètre of the total length 69 mètres.
The formula of 145 becomes in the present case $\frac{\mathrm{P}}{l}=754 \cdot 5 \mathrm{U}+52.5 \mathrm{~V}+97+\Omega$. The flanges weigh $\frac{65000^{\mathrm{k}}}{69^{\mathrm{m}}}=942$ kilogrammes a lineal mètre of girder, which gives $\mathrm{U}=1 \cdot 25$. For the vertical rods we have $\mathrm{V}=$ about $2 \cdot 25$. These quantities do not include the accessories independent of $h$, the details of which are as follows:-

which gives $\Omega=57$ kilogrammes a mètre of the total length, or 0.0475 of the total weight.
To discuss the height $h$, suppose that with the present data we have to adopt $\mathrm{V}=0.205 h$ (for a height above 5 mètres), and that the value $U$ is regarded as nearly constant. The weight of the girder (145) may be put in the form $\frac{\mathrm{P}}{l}=7 \cdot 15\left(\frac{1392}{h}+1 \cdot 624 h+0 \cdot 137 h^{2}\right)+57^{\mathrm{k}}$. It becomes then a minimum for $h=15^{\mathrm{m}} \cdot 50$, which brings it down to 1115 kilogrammes, instead of 1208 kilogrammes, which the same formula gives for the height adopted of 11 mètres.
165. To compare the arched girder with the bow properly so called, having an equal height in the middle, let us take for example the data: $l=60^{\mathrm{m}}, \mathbf{N}=10, p+p^{\prime}=5500^{\mathrm{k}}, q=0 \cdot 55$, total maximum height $=10^{\mathrm{m}}, \mathrm{U}=1 \cdot 25, \mathrm{~V}=2$ for the arched girder.

We must first ascertain what value of V will be required for the bow-girder having flanges of an equal curve. Now the upper portion of the vertical rods has as a constant maximum strain $16 \frac{1}{2}$ tons, whilst the lower portion may be subject to strains of compression varying from 30 to $21 \cdot 9$ tons. The vertical rods of the arched girder will have to resist loads less than 33 tons, but in the present case the rigidity to resist compression is necessarily in the lower portion; but it is not this condition even which fixes the limit, the rods must be stronger than theory requires on account of the duty they have to perform as mediums between the girder and the flooring. We shall, therefore, give them a section at least equal to that adopted for the arched girder, which leads to a coefficient $\mathrm{V}^{\prime}$ equal to about 3.

The term $\Omega$ has also a more disadvantageous value in the case in which both flanges are curved, on account of the longitudinal girder which will always be required on a level with the flooring, though it is not strictly indispensable. Let us take, for example, $\Omega=55$ for the arched girder, and $\Omega=120$ for the girder with both flanges curved.

Then the formulæ of 156 (for $\left.h=h^{\prime}\right)$ and $157(h=0)$ give respectively for the example under consideration:

Weight of the mètre of bow-girder with both flanges curved $=1137$ kilogrammes.
Weight of the mètre of arched girder with a straight lower flange $=1092$
Thus, with an equal total height, the simple arched form, which is the more convenient, is to be preferred. Yet it is possible that the double curve form may become the more advantageous in certain cases in which it may be regarded as warranting a greater total height, on account of the
triple tying on three different levels, provided this additional tying do not consume all the saving effected in the girder itself.
166. The last plan, Fig. 1770, gives us U smaller and $\Omega$ greater than the first example. But we mqy, to make the matter more simple, bring them to about the same values: to do this we may include in U the accessories forming part of the fianges, so that the latter will amount to 67000 kilogrammes (164); consequently, U will be equal to $1 \cdot 29$, instead of $1 \cdot 25$, and $\Omega$ to $0 \cdot 022$ of the total weight, instead of 0.0475 . Already, in section 102, we have similarly attributed to U the angleirons upon the edge of the flanges and the plates at the ends of the cross-girders, although strictly speaking these accessories do not depend upon the height, but rather upon the particular form of the flanges.
167. Admitting generally $h=0 \cdot 18 l, \mathrm{U}=1 \cdot 30, \mathrm{~V}=2 \cdot 50$, and $\Omega=0.025$ of the weight, we may form Table $\mathbf{X}$. of the quantities $\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}$ for various values of N and $q$, and for arched girders with crosses. The numbers of this Table, being multiplied by $\left(p+p^{\prime}\right) l$, will give the mean weight of the lineal mètre. They have been calculated by means of the formulæ in sections 145 and 157 , which, with the data admitted, become respectively

$$
\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}=0.0030135-\frac{0.0024}{\mathrm{~N}^{2}}+\left[0.00185 \frac{\mathrm{~N}^{2}-1}{\mathrm{~N}^{3}}+0.000032\left(\mathrm{~N}+\frac{1}{\mathrm{~N}} \frac{30(\mathrm{~N}-1)}{\mathrm{N}^{2}(\mathrm{~N}+1)}\right)\right] q
$$

for N odd; and
$\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}=0.0030135-\frac{0.0006}{\mathrm{~N}^{2}}+\left[0.00185 \frac{\mathrm{~N}-2}{\mathrm{~N}^{2}}+0.000032\left(\mathrm{~N}-\frac{30\left(\mathrm{~N}^{2}+1\right)-59 \mathrm{~N}}{\mathrm{~N}^{4}}\right)\right] q$,
for N even. $\Omega$ is included in the numerical quantities, which have been multiplied by 1.025 .
168. For girders carrying a whole line of rails, we may allow :-

|  |  | Mềres. | Mềres. | Mètres. |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  | .. | 20 | 60 | 80 |
|  |  | Kilos. | Kilos. | Kilos. |  |
|  | $p+p^{\prime}=\ldots$ | 6900 | 6700 | 7300 |  |
| And $q=.$. | 0.72 | 0.60 | 0.55 |  |  |
|  |  |  |  |  |  |

Supposing besides, as in $92, \mathrm{~N}=7$ for bridges of 20 mètres, 8 for those of 30 to 60 mètres, 9 for 70 mètres, and 10 for 80 mètres, we may draw up the following Table :-

| $l=$ | Mètres. 20 | Mètres. 30 | Mètres. 40 | Mètres. $50$ | Mètres. $60$ | Mètres. $70$ | Mètres. $80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight of the mètre of girder carrying a whole line of rails .. | $\begin{gathered} \text { Kilos. } \\ 460 \end{gathered}$ | $\begin{aligned} & \text { Kilos. } \\ & 680 \end{aligned}$ | Kilos. $890$ | $\begin{aligned} & \text { Kilos. } \\ & 1100 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 1310 \end{aligned}$ | $\begin{aligned} & \text { Kilos. } \\ & 1600 \end{aligned}$ | $\begin{aligned} & \text { Kilus } \\ & 1900 \end{aligned}$ |

These weights differ but little from those of girders of simple triangles (92), or of crosses without vertical rods (105); so that the advantage of the arched form is not in the girder itself, but consists merely in diminishing the length over which the upper tying and cross-bracing extends. Over other systems of girders the advantage in weight will be more sensible; but it may happen that the builder asks a higher price the kilogramme for structures with curved flanges than for those with straight girders. In this case the price must be taken into consideration in comparing the relative merits of the several systems.

As the arched form allows the rigidity of the vertical portion to be relegated to the rank of secondary questions (144 and 157), an arched girder carrying only one rail will weigh but little more than half of the weights given above for girders loaded with a whole line. Therefore, in the case of a small load, the advantage of the arched form becomes more decided.

We will repeat once more a remark already made several times: it is that, in general, in order that a girder may support wholly the load of one line of rails, we must consider the case of two heavy trains crossing at the same time; if, therefore, for this fortuitous case we are willing to raise slightly the value of R , we may reduce the weights given above.
181. Weight of the Lineal Mètre of Railway Bridges carrying Two Lines of Rails.-Case of Limited Height.-When the depth between the level of the rails and the lower side of the girders is limited, the system of loading the girders upon the lower side is resorted to, which for long spans is developed into the tubular system.
A.-Three-girder Bridges loaded on the lower side.-If the depth allowed, though limited, is yet sufficient to enable us to give to the cross-girders a height of $0^{\mathrm{m}} \cdot 70$ or $0^{\mathrm{m} \cdot 80}$; these, added to the minor longitudinal girders will make up a weight of about 640 kilogrammes the lineal mètre of flooring under two lines of rails, the distance apart being included between 3 and 5 mètres. In detail :-
With a distance apart of 3 mètres and Cross-girders $=375^{\mathrm{k}}$; minor longitudinal girders $=265^{\mathrm{k}}$. a height of $0^{\mathrm{m}} \cdot 70$, we should have .. $\}$ Total $=640^{\mathrm{k}}$.
And with a distance apart of 5 mètres Cross-girders $=250^{\mathrm{k}}$; minor longitudinal girders $=385^{\mathrm{k}}$. and a height of $0^{\mathrm{m}} \cdot 80$, we should have $\}$ Total $=635^{\mathrm{k}}$.
For spans of 30 mètres and above, a horizontal cross-bracing is added, weighing from 80 to 100 kilogrammes the mètre of flooring; and in that case the total of the pieces and the chief girders will increase to 720 or 740 kilogrammes.

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If the height of the girders exceeds 5 mètres, we shall adopt the tubular system by adding tiebeams at the top reckoned at 90 or 100 kilogrammes, Fig. 1719, and an upper cross-bracing reckoned at 70 or 90 kilogrammes. The total will be from 880 to 930 , according to the span. The upper tie-beams increase but little with the span; for if, in a large bridge, they are increased in size, they may be placed farther spart.

The outside girders carry only one rail, and should never be subjected to a strain greater than 6 kilogrammes the square millimètre; but the middle girder may support a whole line in the case of two trains crossing the bridge at the same time, an exceptional circumstance when the strain may without injury be raised to $7 \mathrm{k} \cdot 50$, especially as the loads supposed (34) are for very heavy trains. In the ordinary case of the passage of one train, the middle girder will not be subject to a strain of 6 kilogrammes. In virtue of this increase of $R$, the weight of the girder, or at least the part which is a function of $t$, will be reduced to $\frac{4}{5}$; or, which amounts to the same thing, $t$ will have the value 0.00104 , instead of 0.0013 . If the height be not changed, something will yet be gained by the fact that the lightening of the girder diminishes to an equal amount the load which it bears.

Girders loaded on the lower side are badly supported if their height is great without being sufficient to allow an upper tying. The hèights between $3 \mathrm{~m} \cdot 50$ and 5 mètres should, therefore, be avoided. For example, if the height theoretically the most favourable were 4 metres or under, it should be reduced to about $3^{\mathrm{m}} \cdot 30$; if it were above 4 mètres, it should be increased to $5^{\mathrm{m} \cdot 30 \text {, in }}$ order to adopt the tubular system. However, if the depth allowed will permit, we may sometimes raise the cross-girders as shown in Fig. 1718, in a two-girder bridge, and in that case we may without inconvenience retain the height calculated.
182. Table XII. a gives the weights of three-girder bridges calculated according to the preceding indications. An addition of $\frac{1}{10}$ has been made to include unforeseen necessities. Multiplying the weight of the lineal mètre of total length by 1.045 , we obtain the weight a lineal mètre of bearing (37), given in the last column. Multiplying this last by $1 \cdot 05$, or immediately those of the preceding column by 1.0973 , we obtain the weight a mètre of clear span between the abutments.

In the case of multiple lattices, we suppose that the outside girders have single vertical rods placed upon the inside of the girder, and that they may be computed by a mean between the weights of 117 and 118, at least when the height does not differ too widely from that of $0.11 l$, which was supposed in the sections referred to. The middle girder will have double vertical rods and a flat lattice if there is no upper tying; if the bridge is tubular, it will, on the contrary, have a rigid lattice and no vertical rods.

In arched girders, the upper tying extends only over those portions where the height is sufficient; so that its mean weight is less than in straight bridges. But in long spans this economy diminishes, and is compensated by the necessity of placing struts or St. Andrew crosses forming a vertical cross-bracing. The weights given in the Table include these circumstances.
183. If the depth allorved be very limited, we shall not be able to give the cross-girders the heirht supposed in 181. If, for example, we are obliged to restrict this height to $0^{\mathrm{m}} \cdot 35$, as shown in Fig. 1592, the weight of tho lower cross-girders and longitudinal girders will be increased to 960 kilogrammes the mètre of flooring, instead of 640 . We must, therefore, increase by 320 kilogrammes the numbers of the fifth column of Table XII. A, by 350 those of the following, and by 370 those of the seventh or last column.

The limit in height may be still less, and may require more expensive arrangements, such as those of Fig. 1591. In these cases we must of necessity adopt the three-girder system; for that of two girders, which we are about to consider, does not allow the height of the cross-girders to be decreased to such an extent.
184. B.-Two-girder Bridges loaded on the lower side.-When the cross-girders can be of a gond height we may reckon as a mean for their weight, joined to that of the longitudinal girders, upon 750 kilogrammes the mètre of flooring. In detail :-
With a distance apart of 3 mètres and Cross-girders $=530^{\mathrm{k}}$; minor longitudinal girders $=240^{\mathrm{k}}$. a height of 1 mètre, we have .. ..) Total $=770^{\mathrm{s}}$.
With a distance apart of 4 mètres and Cross-girders $=4.10^{\mathbf{k}}$; minor longitudinal girders $=320^{\mathbf{k}}$. a height of 1 mètre, we have .. ..) Total $=730^{\text {k }}$. Fig. 1600 .
With a distance apart of 4 mètres and Cross-girders $=425^{\mathrm{k}}$; minor longitudinal girders $=320^{\mathrm{k}}$. a height of $0^{\mathrm{m}} \cdot 80$, we have .. .. $\}$ Total $=745^{\text {k }}$. Fig. 1708.
With a distance apart of 5 mètres and Cross-girders $=310^{\mathrm{k}}$; minor longitudinal girders $=360^{\mathrm{k}}$. a height of 1 mètre, we have .. .. $\}$ Total $=700^{5}$.
Above 30 mètres span, a horizontal cross-bracing must be added, estimated at 60 or 80 kilogrammes. Total 810 to 830 , according to the span.

When the bridge becomes tubular, 170 kilogrammes must be added for upper tie-beams, and 70 kilogrammes for upper horizontal cross-bracing, in long spans these pieces are strengthened, and, in a case of necessity, vertical cross-bracings would be added; but, at the same time, the distance of the cross-pieces apart may be increased. We shall reckon, for all the pieces, as well as for the principal girders, 1050 to 1100 kilogrammes, according as the span varies from 40 to 80 mètres.

In arched girders the upper tying extends only over a fraction of the length, but this fraction increases with the span, while the vertical cross-bracing acquires at the same time more importance: therefore in long spans the secondary pieces weigh as much as in straight girders.

When there is but one train upoz the bridge, the girder subjected to the greatest strain will sup-
port only about $\frac{3}{4}$ of the load of a whole line, and under these circumstances we may subject it to a strain of 6 kilogrammes, since on the exceptional hypothesis of two heavy trains being on the bridge at the same time, the maximum strain will still be less than $7^{\mathrm{k}} \cdot 50$. The weight of girders may, in general, be rapidly calculated by interpolation. For example, for a solid girder of 20 mètres span, we may reckon $p=1800$ kilogrammes (including 500 kilogrammes for the girder itself) ; $p^{\prime}=\frac{3}{4}$ of a line of rails $=3750$; total, $p+p^{\prime}=5550$ kilogrammes. Now the Table of 129 gives 370 kilogrammes for a girder carrying one rail, and the load of which was valued at 3600 kilogrammes, and 570 kilogrammes as the weight of a girder carrying two rails, and for which we had supposed $p+p^{\prime}=7000$. Interpolation will give for the present case

$$
370+200 \cdot \frac{5550-3600}{7000-3600}=485 \text { kilogrammes. }
$$

The neight, interpolated between $1^{\mathrm{m}} \cdot 70$ and $2^{\mathrm{m}} \cdot 25$, would be about 2 mètres. In these cases, in which the height thus found would have to be modified considerably to prevent its falling
 the formulæ, with suitable values of U and V .

Table XII. в gives approximative weights for two-girder bridges constructed according to the hypotheses which have just been made. The multiple lattices are supposed to be with single vertical rods, which allows one of the systems of bars at $45^{\circ}$ to be constructed of pieces with projecting ribs.
185. In the case in which it is required to decrease to $0^{\mathrm{m}} \cdot 60$, for example, the height of the cross-girders, in consequence of the limited depth, we shall have to increase by a constant equal to about 300 kilogrammes the numbers of the third column of Table XII.b. The following column, including an addition of $\frac{1}{10}$, will increase by 330 kilogrammes; and the weights of the last column will be augmented by 345 kilogrammes.
186. Case of Unlimited Height.-When the height allowed is unlimited, we are none the less free to adopt one of the preceding systems, but we have the advantage of placing the girders entirely below the level of the rails, which will give rise to other systems: these systems we are about to consider. They may be preferred, if they are found to be more economical, and it may be at once remarked that they possess an advantage in requiring a lower bed of masonry. The fact of the bed being far removed from the plane of vibration may appear detrimental to the stability of the structure; but this may be remedied by means of vertical cross-bracing, especially that which is placed above the supports.
187. C.-Four-girder Bridges under Rails.-According to Figs. 1612, 1614, supposing a distance of 3 mètres between the cross-girders, we may reckon from 260 to 345 kilogrammes a mètre of flooring for the pieces other than the principal girders, and the horizontal cross-bracing, according as the height of the girders varies from 1.50 to 4 mètres; say 300 to 400 kilogrammes, including the horizontal cross-bracing.

The girders carry each one rail, and their weight is found ready calculated, according to their mode of construction, in the Tables of sections $92,105,106,117,118,129,133$.

Table XII. c gives the weight a mètre of four-girder bridges for spans of 20 to 50 mètres, for this system will never be adopted for very long spans.
188. D.-Three-girder Bridges under Rails.-According to Figs. 1605 and 1618, 1619, the pieces other than the principal girders and the horizontal cross-bracing will weigh from 460 to 620 kilogrammes, according as the height of the girders varies from 2 to 5 metres. As in the preceding case, the resisting breadth of the flooring hardly exceeding 5 mètres, it is necessary to have recourse to horizont $l$ cross-bracing beyond 20 mètres span; reckoning it at 40 to 60 kilogrammes, we shall have a total of 500 to 680 kilogrammes for heights of 2 to 5 mètres.

Beyond 50 mètres span, it is preferable to increase the distance between the outside girders by adopting the second arrangement, Fig. 1619: we may then reckon upon $830+70=900$ kilogrammes for a span of 50 mètres to 1100 kilogrammes for a span of 80 mètres.

Each girder will be calculated for a permanent load $p$ equal to its own weight a mètre, increased by 700 to 900 kilogrammes. As to $p^{\prime}$, it may be estimated at $\frac{7}{5}$ of a rail, for an outside girder, if the distance apart is 5 mètres, and at $\frac{7}{6}$ of a rail if the girders are arranged as in Fig. 1619 (second arrangement). For the middle girder, $p^{\prime}$ will be, in the same cases, respectively equal to $\frac{6}{5}$ or to $\frac{5}{3}$ of a rail, both lines being loaded simultaneously by heavy trains. For this exceptional case we 3 may raise R to $7^{\mathrm{k}} \cdot 50$, or reduce $t$ to $0 \cdot 00101$, for then the maximum strain will always be less, than 6 kilogrammes a square millimetre in the ordinary case of one line loaded, in which $p^{\prime}$ would be reduced by one-half.

It is possible to give different heights to the middle and to the outside girders, but the advantage of this complication is too small to render it worthy of adoption.

Table XII. D gives approximative weights for bridges constructed according to this system.
189. E.-Two-girder Bridges under Rails.-Fig. $161 \check{0}$ would give 560 kilogrammes a mètre, for the pieces other than the principal girders, including 60 kilogrammes for the horizontal crossbracing, the height of the girders being $2^{\text {m }} \cdot 50$. With a height of 3 mètres, and taking the
arrangement of Fig. 1606, we may reckon 600 kilogrammes: Fig. 1742 would give 660 kilogrammes, the height of the girders being $4^{\mathrm{m}} \cdot 50$.

With the first kind, in which the distance between the girders is 5 mètres, we shall take from 550 to 700 kilogrammes, according as the height varies from 2 to 5 mètres.

Beyond a span of 40 mètres, it is better to widen the flooring by adopting the arrangement shown in Fig. 1580. We may then reckon, for the pieces other than the principal girders, from 800 to 1000 kilogrammes, including the horizontal cross-bracing, according as the height varies from 4 to 9 mètres.

Each girder is calculated for a strain $\mathrm{R}=6$ kilogrammes a square millimètre, taking for $p$ the weight of the girder itself increased by 1000 to 1200 kilogrammes, according to the span, and for $p^{\prime}$ the load of $1 \cdot 70$ or $1 \cdot 55$ of a rail, according as the distance between the girders is 5 or 6.40 mètres. These values relate to the case of only one line loaded; but in the case of two heavy trains passing each other upon the bridge, R will not exceed 7 kilogrammes.

Table XII. e gives weights for two-girder bridges, a mètre, increased by $\frac{1}{10}$.
In all the systems of girders loaded on the upper side, vertical rods will be quite unnecessary.
190. Comparison of the several Systems.-On comparing Tables XII. A, XII. b, XII. c, XII. D, and XII.e, we see immediately the advantage of reducing the number of girders to two, especially in the case of long spans. But when the depth allowed is limited to the absolute minimum, the system of three girders (Table XII. a) becomes the only possible one. When the height is unlimited, the two-girder bridge loaded on the upper side (Table XII. e) is preferable to the two-girder bridge loaded on the lower side (Table XII. b) ; but in long spans, this latter system will be generally more advantageous than those of four or of three girders (Tables XII. c and XII. D).

As to the mode of constructing girders considered by themselves, the solid form is the most expensive: then come multiple, triangular, and cross lattice-girders, and last, cross-lattice arched girders. However, in two-girder bridges loaded on the lower side (Table XII. B), the arched form offers but a trifling advantage in weight over the straight form with cross-lattice, and it is besides more complicated in construction. Compared with a straight girder with multiple lattice, it has again a certain advantage : for example, for a two-girder bridge of 60 mètres span, it would offer a total saving of about 44 tons; but if there were a difference in price, and the arched-girder bridge were paid for at the rate of 0.70 franc, for example, a kilogramme, whilst the straight-girder bridge cost $0 \cdot 60$ franc, the saving would be reduced to 4000 francs.

To make the comparisons more evident and palpable, we might represent graphically the weights given by the Tables.

191 a. Observation.-In using the Tables, care must be taken not to lose sight of the hypotheses on which the calculations are based. Thus, there will be no ground for surprise that the bridge represented by Fig. 1708 weighs 4335 kilogrammes a mètre, whilst Table XII. would give a less weight: this apparent anomaly proceeds from the fact of our having calculated that structure for a heavier load, on the supposition that R should never exceed 6 kilogrammes, even in the exceptional case of both lines loaded simultaneously. So in the two-girder tubular bridge, Fig. 1690, calculated for $p+p^{\prime}=7000$, the span being 56 mètres, the weight of a girder reached 1300 kilogrammes a mètre, whilst by interpolating in Table XII. B we should compute it at 1080 kilogrammes only ; but this is because, according to the hypotheses of this Table, the load considered for a strain of 6 kilogrammes is only $\frac{3}{4}$ of a line of rails, and, besides this, the permanent load, reduced to its just value by taking into account the lightening of the girder, is, by 400 or 500 kilogrammes, smaller than the value liberally computed in section 89 . And, indeed, if we multiply 1300 kilogrammes by the ratio of the loads supposed, say about $\frac{5550}{7000}$, we obtain 1030 kilogrammes, which is not quite so much as the Table gives. In the same bridge, calculation had given also a slightly greater weight for the flooring, because the cross-girders had been reduced to a height of $0^{\operatorname{mon}} \cdot 70$.
$191 \beta$. Bridges of 10 mètres span and less are always of solid girders, this form being simpler and more advantageous for an inconsiderable height. The best arrangements in these small structures are, in general, easily discovered, and ready-made plans exist in abundance. In the case of very limited depth, we may adopt the hollow form of girder, if the bearing does not exceed 5 or 6 mètres. Beyond that, the three-girder system, with minor longitudinal girders of the hollow or of the double $T$ form, will be the best. And when the height permits, we may take the bridge of four girders, one under each rail, with very light tie-beams.

If the depth is unlimited, it is sufficient to reckon from 600 to 1200 kilogrammes (including parapet) as the weight of a flooring under two lines of rails, a mètre of bearing, according as the latter varies from 3 to 10 mètres. In the case of very limited depth, these figures must respectively be increased to 1000 and 2000 kilogrammes, if a general notion, only, of the expense is required.
192. Weight of Road Bridges.-When there are inclined struts, as in Figs. 1607 to 1611, and 1625, care must be had to take into account the increase of weight which they occasion when the height increases. For instance, in the last of these three figures, we may suppose that the weight of the pieces other than the principal girders is represented by $720+50 h, h$ being the height of the girders.

For the kinds shown by Figs. 1625 and 1788, 1789, which are applicable to small spans, solid girders from 0.70 to 1 mètre high should be adopted; the joints in the vertical rib should be far apart, indeed there should be only one in spans of 10 metres; the vertical stays or stiffening rods should consist of single angle-irons, serving to fix the cross-girders and brackets. For other kinds, it will be advantageous to employ multiple lattice-girders, or, better still, cross lattice-girders; these girders should in general be provided with vertical rods for the purpose of affixing the cross-girders and brackets, but often it will be sufficient to give them a small sectional area : it was seen in

108 to 118, that in multiple lattice-girders it is advantageous to use single vertical rods, that is, rods upon one side only of the lattice, in order that one at least of the systems of oblique bars may have projecting ribs.

In three-girder bridges, the middle girder is more heavily loaded than the outside ones, and, when nothing prevents, it will be well to give it greater height ; this is easily done in the case of girders loaded on the lower side; when the height is unlimited, as in Fig. 1609 for example, the bed-plate of the middle girder may be placed at a lower level than those of the outside girders. It is in the kind of bridge represented in Fig. 1626 that the difference in the load upon the girders is especially great; when the bridge is tubular, the upper tie-beams may be arched, and the middle girder raised.

1789.


Table XIII. gives some general values of weights for roads of 6,10 , and 14 mètres broad, and for spans of 10 to 50 mètres.

In long spans a considerable saving may be effected by suppressing the ballasting, which will diminish the permanent load. This may be done without danger, because vibration is much less to be feared than in railway bridges, where the moving load is driven at greater velocities, and the ratio $q$ usually possesses a value near $0 \cdot 60$; whilst in a road bridge where the ballast would
 retained, it is evident that the cross-bracing will be much less important in a road bridge than in a railway bridge.
193. Bridges of several Bays.- When a continuous girder rests upon several supports, and carries a load uniformly distributed throughout a bay, the locus) of the moments of rupture, in any bay in which the load is $p_{n}$ a mètre, will always be a parabola with a vertical axis, having as a parameter $\frac{1}{p_{n}}$, and the position of which will be determined by the condition of passing through the summits of the ordinates which, raised upon the supports limiting the bay under consideration, will represent the moments of rupture in a line with these
 supports. Thus for the bay A B of Fig. 1790, the length being $l_{n}$, the load a metre $p_{x}$, the moments of rupture upon the supports A and B

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being $\mu_{n-1}$ and $\mu_{n}$, we shall have as the variable moment in the point M, the abscissa of which is $x$ reckoning from A , the expression

$$
\text { Sfin } \quad \mu=-\frac{p_{n} x^{2}}{2}+\left(\frac{p_{n} l_{n}^{2}}{2}+\mu_{n}-\mu_{n-1}\right) \frac{x}{l_{n}}+\mu_{n-1}
$$

and the stress in the same point will be the derivative

$$
\mathbf{F}=-p_{n} x+\frac{p_{n} l_{n}}{2}+\frac{\mu_{n}-\mu_{n-1}}{l_{n}} .
$$

194. The moments of rupture upon the supports depend upon the circumstances in which the other bays are placed, and are calculated by Clapeyron's formula, the demonstration of which is found in the works of MM. Belanger and Bresse, and to which we can merely allude: it is the following relation connecting the moments of rupture $\mu_{n-1}, \mu_{n}, \mu_{n}+1$, upon three consecutive supports, Fig. 1790,
 being the number of bays in the bridge) between the $m-1$ moments of rupture upon the piers, those upon the abutments being nul. These equations may be immediately written under a numerical form with the given bearings and loads, and the resolution effected; then the moments upon the supports being all known and represented by ordinates to a given scale, we have only to apply a parabolic model corresponding to each bay, keeping their axes vertical. We thus construct the geometrical locus of the moments of rupture over all the bridge, not only for a particular mode of loading, but for all which are capable of producing a maximum strain upon certain portions of the girder. For a determinate plan, two parabolic models are sufficient, one for the dead weight only, which is supposed to be constant throughout the bridge, at least when the length of the bays do not differ too much; and the other for the maximum total load, including the dead and the rolling weights.

The several loci thus traced cut each other in certain points, and we make use only of the envelope, that is, of the portions of the curves giving the maxima ordinates in absolute value. In calculations for bridges of several bays, the cases in which the load would affect only portions of a bay are not considered.

Clapeyron's formula is established on the hypothesis that the moment of inertia of the girder remains constant throughout the length. This condition is hardly ever fulfilled, because the thickness of the flanges is made to vary for the purpose of economizing material. Yet we generally regard calculations made on the hypothesis of a constant section as applying with sufficient approximation to a case in which the section varies. It is then prudent not to reduce too much the maximum thickness of the flanges, which, in the computation of weight, will amount to forcing a little the coefficient U.
195. In the particular case of two bays each equal to $l$, and loaded respectively with $p_{1}$ and $p_{2}$ a mètre, the moment of rupture upon the pier will be $-\left(p_{1}+p_{2}\right) l^{2}$.

In the case of three bays, $l_{1}, l_{2}, l_{3}$, the respective loads of which are $p_{1}, p_{2}, p_{3}$, we have for the moments $\mu_{1}$ and $\mu_{2}$ upon the first and second pier

$$
\mu_{1}=-\frac{2 p_{1} l_{1}{ }^{3}\left(l_{2}+l_{3}\right)+p_{2} l_{2}^{3}\left(l_{2}+2 l_{3}\right)-p_{3} l_{2} l_{3}{ }^{3}}{16\left[l_{1}\left(l_{2}+l_{3}\right)+l_{2}\left(\frac{3}{4} l_{2}+l_{3}\right)\right]}, \text { and } \mu_{3}=-\frac{p_{2} l_{2}{ }^{3}+p_{3} l_{3}{ }^{3}-4 l_{2} \mu_{1}}{8\left(l_{2}+l_{3}\right)} .
$$

In a bridge in which the bays are numerous, the load upon one of them has a considerable effect upon those which are contiguous to it, but very little upon the others. Thus we may, with sufficient approximation, study successively each bay by taking into account the two next and rejecting the others; the calculations will in this way be made similar to those for bridges of two bays.
196. According to the remark made in section 84, the weight of the flanges of a girder is proportional to the mean ordinate $G$ of the locus of the maximum moments of rupture in absolute value, and the vertical portion depends generally upon the mean ordinate $\mathrm{G}^{\prime}$ of the locus of the stress. We will, therefore, give the values of $G$ and $G^{\prime}$, applicable to bridges of several bays, and by the aid of which the computation of the weight of the girders may be effected without difficulty, as in the case of a single bay.

Let us consider, in the first place, a bridge of unlimited length, consisting of an infinite number of equal bays. Let $l$ be the length of each bay, $p$ the permanent load a mètre, and $p^{\prime}$ the moving load.

The load $p$ will by itself produce, in a given bay, moments of rupture represented by the parabola $\mu=\frac{-p}{2}\left(\frac{l^{2}}{6}-l x+x^{2}\right)$; $\mathcal{A} t$ the supports $(x=0$ and $x=l)$ the moment is $\frac{-p l^{2}}{12}$, it is nul at the abscissa $0.211 l$, and takes with the contrary sign the value $\frac{p l^{2}}{24}$ in the middle of the bearing. This parabola is identical in form with that which we should have for a bridge of a single bay, but it is raised in such a manner that the segment which remains below the axis of the $x$ 's is equivalent to the sum of the two segments situate above this axis, and having the from of triangles, one side of which is curvilineal. This circumstance, due to the continuity of the girder beyond the supports, affects only the flanges, the weight of which is diminished by it, but it has no influence whatever upon the vertical portion.

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As to the load $p^{\prime}$, if, in the first place, it be applied exclusively to the bay under consideration, A B, Fig. 1791, it will give upon the supports $\mathbf{A}$ and $\mathbf{B}$ moments of rupture equal to $-0.0528 p^{\prime} l^{2}$,

which we carry above the axis of the $x^{\prime} s$, and which will determine the position of the parabola of writ the parameter $\frac{1}{p^{\prime}}$, having for equation $\mu^{\prime}=\frac{p^{\prime} x}{2}(l-x)-0.0528 p^{\prime} l^{2}$.

If now we apply the load $p^{\prime}$, not upon AB, but only upon the contiguous bay to the left of A, the moment upon $\mathbf{A}$ will retain the same value, that in a line with the support $\mathbf{B}$ will take the value $+0.0141 p^{\prime} l^{2}$, and a right line joining the summits of the ordinates to the supports, and having for equation $\mu^{\prime \prime}=0.0669 p^{\prime} l x-0.0528 p^{\prime} l^{2}$, will represent the moments of rupture produced in the bay under consideration. If we consider again the case in which the bay AB and the preceding one are simultaneously loaded, to the exclusion of all the others, the moments upon A and B will be respectively $-0.1056 p^{\prime} l^{2}$ and $-0.0387 p^{\prime} l^{2}$, and the locus of the intermediate moments due to $p^{\prime}$ will be the parabola $\mu^{\prime \prime \prime}=-\frac{p^{\prime} x^{2}}{2}+0 \cdot 5669 p^{\prime} l x-0 \cdot 1056 p^{\prime} l^{2}$. The maximum moments in absolute value will be given by the portions of the curves traced in full lines upon the figure; that is, by $\mu^{\prime \prime \prime}$ between the limits $x=0$ and $x=0 \cdot 12 l$, by $\mu^{\prime \prime}$ between $x=0 \cdot 12 l$ and $0 \cdot 235 i$, and beyond as far as the middle by $\mu^{\prime}$. Upon the other half of the bay the same branches of the curves may be symmetrically reproduced.

If we add the moment due to the dead weight $p$, as it changes its sign at the abscissa $0.211 l$, it will be in a point comprised between $x=0 \cdot 211 l$ and $x=0 \cdot 235 l$, values which differ but little, so that the resulting positive moment will begin to exceed the negative, and when $\mu^{\prime \prime}$ should be abandoned for $\mu^{\prime}$.

The area comprised between the axis of the $x$ 's and the locus of the absolute maximum moments is equal to $l^{3}\left(0.0160 p+0.0296 p^{\prime}\right)$ for the half-bay, and dividing by $\frac{l}{2}$ we obtain the reduced ordinate $\mathbf{G}=l^{2}\left(0.032 p+0.059 p^{\prime}\right)=0.032\left(p+p^{\prime}\right) l^{2}(1+0.844 q)$.

The maximum stress corresponds throughout the bay to $\mu^{\prime \prime \prime}$, which gives by differentiation $\mathbf{F}^{\prime \prime \prime}=\frac{d \mu^{\prime \prime \prime}}{d x}=-p^{\prime} x+0 \cdot 5669 p^{\prime} l$. Adding the stress due to the permanent load $p$, we find the value of the reduced ordinate of the limiting stress to be

$$
\mathrm{G}^{\prime}=l\left(0.25 p+0.3169 p^{\prime}\right)=0.25\left(p+p^{\prime}\right) l(1+0.268 q)
$$

197. In the ordinary cases in which the mean weight of the lineal mètre of girder maybe expressed by formulæ of the form $A G+B G^{\prime}+C$, the values found above will be substituted for $G$ and $G^{\prime}$, and for $\mathrm{A}, \mathrm{B}, \mathrm{C}$, the quantities applicable to the system of girder adopted and determined as in bridges of a single bay.

But for simple approximative computations it may be more convenient to profit by the readyfound results which we have given for bridges of a single bay; and to do this, we must know the span of a single-bay bridge the mean weight of which per lineal mètre would be equivalent to that of the bridge of several bays under consideration.

Now, in the case of a single bay of a span $l^{\prime}$, the mean ordinate of the limiting moments of rupture is $\mathrm{G}_{1}=\frac{1}{12}\left(p+p^{\prime}\right) l^{\prime 2}$, and that of the stress $\mathrm{G}^{\prime}{ }_{1}=\frac{1}{4}\left(p+p^{\prime}\right) l^{\prime}\left(1+\frac{1}{6} q\right)$. Thus for the bridge of unlimited length considered in the preceding section, we may make, supposing that the

$$
\begin{aligned}
& \text { term C, which is always small, remains constant, } \\
& \qquad \begin{array}{l}
0.032 \mathrm{~A} l^{2}(1+0.844 q)+0.25 \mathrm{~B} l(1+0.268 q)=\frac{1}{12} \mathrm{~A}^{\prime} l^{\prime 2}+\frac{1}{4} \mathrm{~B}^{\prime} l^{\prime}\left(1+\frac{1}{6} q\right) \\
\text { or } l^{\prime 2}+3\left(1+\frac{1}{6} q\right) \frac{\mathrm{B}^{\prime}}{\mathbf{A}^{\prime}} \cdot l^{\prime}-0.384(1+0.844 q) \frac{\mathrm{A} l^{2}}{\mathbf{A}^{\prime}}-3(1+0.268 q) \frac{\mathrm{B} l}{\mathbf{A}^{\prime}}=0
\end{array}
\end{aligned}
$$

an equation whence we may deduce the span $l^{\prime}$ of the auxiliary bridge.
198. Suppose the case of multiple lattice-girders at $45^{\circ}$, stiffened by special vertical rods. We shall have $\mathbf{A}=\frac{2 \mathrm{U} t}{}$ and $\mathrm{B}=\mathrm{B}^{\prime}=2 t$. If we take $h=\frac{l}{10}$ and $\mathrm{U}=1 \cdot 25$, and retain these values
for the auxiliary bridge, so that we have $\mathrm{A}=\mathrm{A}^{\prime}$; and if we take $q=0.60$, a value which will be generally near enough, we shall obtain $l^{\prime}=0 \cdot 803 l$. Thus, under these conditions, the weight a metre of a bridge of numerous bays will be evidently the same as that of a single bay equal in length to 0.8 of one of the bays of the bridge under consideration, and having girders of the same height.

For example. if the bridge of unlimited length consists of bays of 40 mètres, wo shall have $l^{\prime}=32$ mètres. Now, by interpolating in the Tables given for the case of one bay, we may estimate at 2300 kilogrammes the weight a mètre of a bridge of 32 mètres, composed of two girders of $3 \mathrm{~m} \cdot 85$ high and placed beneath the rails, if we suppose the depth allowed unlimited (Table XII. e). As this height of 3.85 differs but little from that of 4 mètres, supposed by the preceding calculation, we may admit that the weight of 2300 kilogrammes a mètre of bearing is applicable to the unlimited bridge under consideration.
199. In a bridge of two bays each equas to $t$, if we go from one abutment taken as the beginning of the $x$ 's, the maximum moment in absolute value will be first given by the hypothesis that the bay under consideration bears alone the full load $p+p^{\prime}$ a mètre, whilst the second is reduced to the permanent load $p$. This moment is then expressed by the equation

$$
\mu=\frac{1}{16}\left(6 p+7 p^{\prime}\right) l x-\frac{1}{2}\left(p+p^{\prime}\right) x^{2} .
$$

From $x=\frac{3}{4} l$, we must, on the contrary, load only the second bay, and we have the equation $\mu=\frac{1}{16}\left(-6 p+p^{\prime}\right) l x+\frac{1}{2} p x^{2}$. And from $x=\frac{7}{8} l$ to $x=l$ we have, both bays being loaded, $\mu=\frac{1}{8}\left(p+p^{\prime}\right)(4 x-3 l) x$. Squaring the area of the locus of the limiting moments and dividing the result by $l$, we obtain the reduced ordinate of the moments,

$$
\mathrm{G}=l^{2}\left(0.0495 p+0.0702 p^{\prime}\right)=0.0195\left(p+p^{\prime}\right) l^{2}(1+0.418 . q)
$$

For the stress, the reduced ordinate will be

$$
\mathrm{G}^{\prime}=l\left(0 \cdot 2656 p+0 \cdot 2910 p^{\prime}\right)=0 \cdot 2656\left(p+p^{\prime}\right) l(1+0 \cdot 096 . q) .
$$

With the hypotheses of the preceding section, we should find as the bearing of the auxiliary bridge of the same weight a mètre $l^{\prime}=0.883 l$, a value which should be increased to $0.90 l$ for example, because in a bridge of two bays we should increase $\mathrm{G}^{\prime}$ if, instead of confining ourselves to hypotheses of complete loads for each bay, we considered besides loads extending over any portion of the girder. This consideration loses its importance as the number of the bays increases, because then the several hypothetical loads becoming more numerous offer more varied circumstances. In certain cases two independent bays of bow-girders may be more advantageous than two continuous bays of straight girders.
200. In a bridge of three equal bays, if $l$ is the length of bay, haif the length of the bridge will be $\mathrm{L}=\frac{3}{2} l$, and we shall have
$\mathrm{G}=\mathrm{L}^{2}\left(0.0201 p+0.0319 p^{\prime}\right)=0.0201\left(p+p^{\prime}\right) \mathrm{L}^{2}(1+0.589 . q)=0.452\left(p+p^{\prime}\right) l^{2}\left(1+0^{\circ} 589 . q\right) ;$ and $\quad \mathrm{G}^{\prime}=0 \cdot 1711\left(p+p^{\prime}\right) \mathrm{L}(1+0 \cdot 188 \cdot q)=0 \cdot 2567\left(p+p^{\prime}\right) l(1+0 \cdot 188 \cdot q)$.
With the same data as for the preceding cases, we find $l^{\prime}=0 \cdot 881 . l$.
201. In a bridge of three bays, if we suppose the middle one equal to $\frac{5}{4}$ of one of the end ones, a small saving will be effected over the arrangement of three equal bays, for, $L$ still denoting half the length of the bridge ( $=1 \cdot 625$ times an end bay), we shall have

$$
\mathrm{G}=\mathrm{L}^{2}\left(0.0188 p+0.0308 p^{\prime}\right) \text { and } \mathrm{G}^{\prime}=0.1749\left(p+p^{\prime}\right) \mathrm{L}(1+0 \cdot 180 . q) .
$$

202. For a bridge of four equal bays, $l$ being the length of one bay, we shall have

$$
\mathrm{G}=l^{2}\left(0.0415 p+0.0693 p^{\prime}\right)=0.0415\left(p+p^{\prime}\right) l^{2}(1+0.67 . q) ;
$$

$$
\text { and } \quad \mathrm{G}^{\prime}=l\left(0 \cdot 256 p+0 \cdot 31 p^{\prime}\right)=0 \cdot 256\left(p+p^{\prime}\right) l(1+0 \cdot 22 . q) \text {. }
$$

with the data already adopted (198) for multiple lattice-girders, we should have here $l^{\prime}=0.865 l$.

203 a. For bridges of more than four bays, the weight a mètre will be the same as for a bridge of a single bay having a bearing between the values of $l^{\prime}$ calculated for the case of four bays, and for that of an unlimited number of bays. For example, for five bays, with the same hypotheses as before, we might take $l^{\prime}=0.85 l$.
$203 \mathrm{\beta}$. In computations which are required to be as exact as possible, the weight of the girders should be calculated directly by formule in which $G$ and $G^{\prime}$ have the values given above according to the number of bays. In general, girders resting upon several supports will be either of multiple lattice or of crosses or cross-lattice. In the former case, we shall have as the mean weight of the mètre of girder without vertical rods,

$$
\frac{\mathrm{P}}{l}=\frac{2 t \mathrm{U}}{h} \mathrm{G}+(1+\mathrm{V}) t \mathrm{G}^{\prime}+\frac{1}{2}\left(p+p^{\prime}\right) t h+\frac{60 h^{2}}{l}+\Omega_{\mathrm{e}}
$$

If there are vertical rods placed at a distance $\delta$ from each other, a term $\frac{15 h^{2}}{\delta}$ must be added, supposing the sections of Fig. 1728 to be adopted.

For a girder of crosses or cross-lattice, without vertical rods, we shall have the following approximative formula, more general than those of sections 97 and 99 ;

$$
\frac{\mathbb{P}}{l}=\frac{2 t \mathrm{U}}{h} \mathrm{G}+\frac{1}{2}(1+\mathrm{V})\left(\frac{\mathrm{N} h}{l}+\frac{}{\mathrm{N} h}\right) t \mathrm{G}^{\prime}+\frac{1}{4}(1+\mathrm{V})\left(p+p^{\prime}\right) t h+\Omega
$$

N denoting the number of crosses in a bay $l$. If there are vertical rods, they must be taken into account by a term $\frac{r \mathrm{~N} h}{l}$ as in section $100, r$ being the weight of the rod a mètre.

The continuity of the girders from one bay to another having the effect of lightening the flanges, whilst it tends to increase slightly the vertical portion, a less height should be adopted than in bridges of a single bay. In general the ratio of the height of the girders to the length of a bay will not exceed $\frac{1}{10}$.

Figs. 1792 to 1801 are of the principal details of the Chelsea Suspension Bridge,
The abutments of this bridge consist of a mass of brickwork and concrete, measuring at the base 112 ft . in length by 56 ft . broad, and at the top 100 ft . by 46 ft ., and 40 ft . deep.

The face of the abutment adjoining the river is composed of cast-iron piles and plates, somewhat similar to those of the pier, with the exception that the ironwork is not brought above the level of low water.


The portion of the abutment on which the 1796. land saddles and cradles bear, for changing the direction of the chains, rests upon timber piles, 14 in . square, driven deep into the bed of the river, and are from 3 ft .2 in . to 4 ft . from centre to centre: these piles are cut off at the level of low water, 16 ft . below Trinity high-water mark, and the spaces between filled up with hydraulic concrete; the cast-iron and timber piles are tied together with wrought-iron ties $3 \mathrm{in} . \times \frac{3}{4} \mathrm{in}$. On the top is bedded a series of landings forming a table at the level of low water 53 ft .6 in . $\times 27 \mathrm{ft} . \times 6$ in., upon which a mass of brickwork is erected, up to a mean level of 3 ft . below the level of the road way; upon this, 12 in . landings are bedded for the reception of the cradles which carry the saddles on rollers : the cradles are bedded in asphalted felt, and firmly secured by wrought-iron holding-down bolts, brought up through the masonry from below. An invert, springing from beneath each saddle, is built in the brickwork below, so as to distribute equally the pressure from the cradles over the whole area of the foundation.

The mooring chains are carried down tunnels to the moorings, the tunnels forming an angle of $155^{\circ}$ with a horizontal line. The chains are secured to massive cast-iron mooring plates, resting against three courses of 12 in. landings, respectively $12 \mathrm{ft} . \times 8 \mathrm{ft}$. $9 \mathrm{in} ., 16 \mathrm{ft} . \times 12 \mathrm{ft} .6 \mathrm{in}$. , and $20 \mathrm{ft} . \times 16 \mathrm{ft}$. 3 in . The tunnels are contracted at the bottom by elliptical brick domes, thus afford-

ing a complete bearing for that portion of the landings at the end of the tunnel. These landings rest against a mass of brickwork, with inverts, to distribute the pressure over the whole area of the abutment.

This mass of brickwork rests on a series of timber piles, driven at an angle of $65^{\circ}$ with a horizontal line; the tops of the piles coming up above the level of the concrete, and having a good bond with the brickwork, by which means the tendency to slide is greatly diminished, the whole spaces between the masses of brickwork being filled up solid with concrete.

The Pier Foundations, Figs. 1792, 1793.-The construction of the foundations of the piers combines all the advantages of foundations on bearing-piles, made by means of cofferdams, without the expense and obstruction to the waterway which they involve, and which would have rendered their use at Westminster Bridge all but impracticable.


The foundations of the piers consist of timber bearing-piles 14 in . square, driven deep into the bed of the river at intervals of 3 ft . over the whole area of the pier, varying in depth from 40 ft .6 in . to 25 ft . below the level of low water, according to the resistance offered by the bed of the river.

The face or external surface of the piers consists of a cast-iron casing of piles and plates driven alternately; the main piles are 12 in . in diameter and 27 ft . long, with longitudinal grooves on each side for the reception of the plates. These piles are driven to a uniform depth of 25 ft . below the level of low water, and between them are driven cast-iron plates or sheeting 7 ft .2 in . wide, so that the pier is entirely cased from the foundations to the top, which is 7 ft .6 in . above Trinity datum. The space enclosed by this casing is then dredged to the hard gravel above the clay, and filled in solid with concrete up to the level of the top of the timber piles. On this foundation a flooring of stone landings is bedded, and on this the cast-iron plates, frames, \&c., forming the base of the towers, are placed.

The portion of the caisson situated above low water is hollow, being so formed to avoid throwing useless weight on the foundation, and is merely lined with brickwork, strengthened by cross-walls and iron ties.

The whole of the ironwork below the water was covered when hot with a protecting coating of tar. The thickness of metal in the caisson is 1 in .

The towers, Figs. 1800, 1801, which support the chains are entirely independent of the ornamental cast-iron casing surrounding them, and consist of a cast-iron columnar framing strongly braced both horizontally and vertically, carried to a height of 57 ft . above high water.

The columns are cast in pairs and have a diameter of 10 in ., and thickness of metal of 1 in . They are arranged in clusters of fours, and the whole are connected with six horizontal frames, occurring at intervals. The columns are not vertical, but incline towards each other upwards from either side of the pier, the columnar framing being 13 ft .6 in . at the base, and 9 ft .9 in . at the top. In the direction of the piers the columns are 4 ft .3 in . apart, and rise parallel to each other. There are two towers on each pier, 32 ft . from centre to centre. The pressure from the chains coming directly from their centre, each tower carries therefore one-fourth of the whole weight of the bridge, or about 375 tons, or about 670 tons when the bridge is completely loaded; the sectional area of the columns is 284 sq. in., and there is, therefore, a pressure upon them, when the bridge is loaded, of $2 \cdot 36$ tons per sq. in. of section. The weight of the towers, exclusive of the ornamental cast-iron casing, is 350 tons.

On the towers are fixed massive cast-iron cradles upon which the saddles rest.
Triangular Lattice-girders loaded on the Lower Side.

| Case in which $\alpha=\beta=45^{\circ}$. | Case in which $\beta=90^{\circ}$. | Case in which $\alpha=90^{\circ}$. |
| :---: | :---: | :---: |
| $h \sqrt{2}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right]$ | $\begin{gathered} \frac{\delta \sqrt{h^{2}+\delta^{2}}}{2 h}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right] \\ \frac{\delta}{2}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right] \end{gathered}$ | $\begin{gathered} \frac{\delta}{2}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right] \\ \frac{\delta \sqrt{h^{2}+\delta^{2}}}{2 h}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right] \end{gathered}$ |
| $2(m-n+1)(m+n-1)\left(p+p^{\prime}\right) h$ | $(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ | $(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ |
| $2\left(m-n+\frac{1}{2}\right)\left(m+n-\frac{1}{2}\right)\left(p+p^{\prime}\right) h$ |  | $\left(m^{2}-n^{2}\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ |
| $\left(p+p^{\prime}\right) \frac{l^{2}}{8 h}$ | $\left(p+p^{\prime}\right) \frac{l^{2}}{8 h}$ | $\left(p+p^{\prime}\right) \frac{l^{2}}{8 h}$ |
| $\left(2 m^{2}-1\right)\left(p+p^{\prime}\right) h$ |  | $\left(m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ |
| $2 h^{2} t \mathrm{~V}\left(2 m^{2} p+\frac{7 m^{2}-1}{3} p^{\prime}\right)$ | $\frac{\delta\left(h^{2}+\delta^{2}\right) t \mathbf{V}}{h}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ | $\delta h t \mathrm{~V}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ |
| $2 h^{2} t\left(2 m^{2} p+\frac{7 m^{2}-1}{3} p^{\prime}\right)$ | $\delta h t\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ | $\frac{\delta\left(h^{2}+\delta^{2}\right) t}{h}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ |
| $4 m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{h^{2} t \mathrm{U}}{3}$ | $m(4 m+1)(m-1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathbf{U}}{6 h}$ | $m(4 m-1)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h}$ |
| $\frac{8 m}{3}\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) h^{2} t \mathrm{U}$ | $\begin{gathered} m(4 m-1)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathbf{U}}{6 h} \\ m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t}{3 h} \frac{\mathbf{U}}{} \end{gathered}$ | $\begin{gathered} m(4 m+1)(m-1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h} \\ m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{3 h} \end{gathered}$ |
| $h \sqrt{2}\left[2 n p+\frac{(m+n)(m+n+1)}{2 m+} p^{\prime}\right]$ | $\begin{gathered} \frac{\delta \sqrt{\tilde{n}^{2}+\delta^{2}}}{h}\left[n p+\frac{(m+n)}{2(2} \frac{(m+n+1)}{m+1)} p^{\prime}\right] \\ \delta\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right] \end{gathered}$ | $\begin{gathered} \delta\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right] \\ \frac{\delta \sqrt{h^{2}+\delta^{2}}}{h}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right] \end{gathered}$ |
| $2(m+n)(m-n+1)\left(p+p^{\prime}\right) h$ | $(m+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ | $(m+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 \check{h}}$ |

Triangular Lattice－girdirs loaded ov the Lower Side－continuca．

| Case in which $\alpha=\beta=45^{\circ}$ ． | Case in which $\beta=90^{\circ}$ ． | Case in which $\alpha=90^{\circ}$ ． |
| :---: | :---: | :---: |
| $\begin{gathered} 2\left(m^{2}-n^{2}+m+2 n-1\right)\left(p+p^{0}\right) h \\ 2 m(m+1)\left(p+p^{\prime}\right) h \\ 4 m(m+1) h^{2} t \mathrm{~V}\left(p+\frac{7}{6} p^{\prime}\right) \\ 4 m(m+1) h^{2} t\left(p+\frac{7}{6} p^{\prime}\right) \\ \frac{8}{3} m(m+1)(2 m+1)\left(p+p^{\prime}\right) h^{2} t \mathrm{U} \\ \frac{16 m}{3}(m+1)(2 m+1)\left(p+p^{\prime}\right) h^{2} t \mathrm{U} \end{gathered}$ | $\begin{gathered} \left(m^{2}-n^{2}+m+3 n-2\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h} \\ m(m+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h} \\ \frac{m(m+1) \delta\left(h^{2}+\delta^{2}\right) t \mathrm{~V}}{h}\left(p+\frac{7}{6} p^{\prime}\right) \\ m(m+1) \delta h t\left(p+\frac{7}{6} p^{\prime}\right) \\ m(4 m-1)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h} \\ m(4 m+5)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h} \\ \frac{2}{3} m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{h} \end{gathered}$ | $\begin{gathered} (m+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h} \\ m(m+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h} \\ m(m+1) \delta h t \mathrm{~V}\left(p+\frac{7}{6} p^{\prime}\right) \\ \frac{m(m+1) \delta\left(h^{2}+\delta^{2}\right) t}{h}\left(p+\frac{7}{6} p^{\prime}\right) \\ m(4 m+5)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h} \\ m(4 m-1)(m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h} \\ \frac{2}{2} m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{h} \end{gathered}$ |

Moments of Resistance（13）．Table I．－Double I Iron．

|  |  |
| :---: | :---: |
|  |  స⿻丅⿵冂⿰⿱丶丶⿱丶丶⿰⿱口⿻上丨⿱夂口犬心． |
| 5 － |  |
|  |  |
|  |  |
| $\begin{aligned} & \text { +0. } \\ & \text { 荡 } \end{aligned}$ |  |
|  |  |
|  |  <br>  |
|  | 菌 |
|  |  |
|  | 苗运： |

Table II.-Double-flanged Wrought-iron Girders with Angle-iron.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Height. \& Vertical Rib. \& Four Angleirons of \& Two Flanges of \& Moment of Resistance for $\mathrm{R}=$ 6000000. \& $$
\begin{aligned}
& \text { Weight } \\
& \text { the } \\
& \text { Lineal } \\
& \text { Mètre. }
\end{aligned}
$$ \& Height. \& Vertical Rib. \& Four Angleirons of \& Two Flanges of \& Moment of Resistance for $\mathrm{R}=$ 6000000 . \& Weight the Lineal Mètre. <br>
\hline \& mill. \& mill. \& mill. \& \& kilos. \& \& mill. \& 100/100 \& \& \& kilos. <br>
\hline 0.200 \& 200/6 \& 60/60/9 \& \& 1890 \& 40 \& $0 \cdot 350$ \& 290/10 \& 100/100/12 \& $$
300 / 30
$$ \& 20964 \& 234 <br>
\hline , , \& 200/6 \& 70/70/10 \& \& 2320 \& 50 \& \& \& \& \& \& <br>
\hline , , \& 200/10 \& 70/70/10 \& \& 2478 \& 56 \& $0 \cdot 400$ \& 400/6 \& 70/70/10 \& \& 6020 \& 59 <br>
\hline ,, \& 200/8 \& 80/80/10 \& \& 2630 \& 60 \& , , \& 400/6 \& 80/80/10 \& \& 6663 \& 66 <br>
\hline , , \& 200/8 \& 80/80/11 \& \& 2820 \& 64 \& , , \& $400 / 8$ \& 110/70/10 \& \& 8165 \& 78 <br>
\hline , \& 200/10 \& 110/70/10 \& \& 3146 \& 69 \& , , \& 400/10 \& 100/100/12 \& \& 9730 \& 102 <br>
\hline , , \& 200/10 \& 110/70/11 \& \& 3593 \& 74 \& , , \& 400/10 \& 100/65/14 \& \& 10085 \& 97 <br>
\hline , , \& 180/6 \& 70/70/10 \& 160/10 \& 3528 \& 74 \& \& 380/10 \& 70/70/10 \& 200/10 \& 10453 \& 102 <br>
\hline , , \& 180/10 \& 70/70/10 \& 200/10 \& 4078 \& 86 \& , , \& $380 / 10$ \& 80/80/10 \& 200/10 \& 11022 \& 107 <br>
\hline , , \& 180/10 \& 110/70/11 \& 230/10 \& 5288 \& 108 \& \& 380/10 \& 70/70/10 \& 250/10 \& 11594 \& 109 <br>
\hline , \& 160/10 \& 100/70/10 \& 250/20 \& 6710 \& 135 \& , , \& $380 / 10$ \& 80/80/10 \& 300/10 \& 13305 \& 123 <br>
\hline 0.220 \& 200/8 \& 60/60/10 \& 150/10 \& 3763 \& 70 \& , \& $380 / 10$ \& 100/100/12 \& 250/10 \& 14311 \& 139 <br>
\hline 0 \& 160/12 \& 110/70/11 \& 300/30 \& 10888 \& 210 \& , \& 380/10 \& 100/100/12 \& 300/10 \& 15452 \& 147 <br>
\hline $0 \cdot 250$ \& 250/6 \& 60/60/10 \& \& 2830 \& 46 \& \& 360/10 \& 100/100/12 \& 250/20 \& 18398 \& 177 <br>
\hline , , \& 250/6 \& 70/70/10 \& \& 3175 \& 52 \& ,, \& 360/10 \& 80/80/10 \& 300/20 \& 18680 \& 169 <br>
\hline , , \& 250/6 \& 80/80/10 \& - \& 3500 \& 59 \& , , \& 340/10 \& 80/80/10 \& 250/30 \& 20378 \& 190 <br>
\hline , , \& 250/10 \& 110/70/11 \& \& 4870 \& 78 \& , , \& 360/10 \& 100/100/12 \& 300/20 \& 20566 \& 192 <br>
\hline , , \& 230/8 \& 70/70/10 \& 200/10 \& 5455 \& 86 \& , , \& 340/10 \& 100/100/12 \& 250/30 \& 22021 \& 214 <br>
\hline , 9 \& 230/10 \& 80/80/10 \& 200/10 \& 5826 \& 96 \& , , \& 360/10 \& 100/100/12 \& $350 / 20$ \& 22734 \& 203 <br>
\hline , \& 210/10 \& 70/70/10 \& 150/20 \& 6055 \& 104 \& , , \& 340/10 \& 80/80/10 \& $300 / 30$ \& 23460 \& $21 \pm$ <br>
\hline , , \& 230/10 \& 70/70/10 \& 200/30 \& 8754 \& 149 \& , , \& $340 / 10$ \& 100/100/12 \& 300/30 \& 25108 \& 237 <br>
\hline , ${ }^{\text {, }}$ \& $250 / 10$
$210 / 10$ \& $140 / 70 / 16$
$110 / 70 / 11$ \& \& 7677
10864 \& 116 \& , , \& 340/10 \& 100/100/12 \& 350/30 \& 28195 \& 261 <br>
\hline $\because 9$ \& $210 / 10$
$190 / 10$ \& $110 / 70 / 11$
$110 / 70 / 11$ \& $$
\begin{aligned}
& 300 / 20 \\
& 300 / 30
\end{aligned}
$$ \& 10864
13063 \& $$
\begin{aligned}
& 168 \\
& 213
\end{aligned}
$$ \& 0:45 \& \& \& \& 7039 \& 62 <br>
\hline $\stackrel{\square}{ }$ \& \& 11 \& \& 13063 \& 213 \& 0 \& 450/6 \& $$
\begin{aligned}
& 70 / 70 / 10 \\
& 80 / 80 / 10
\end{aligned}
$$ \& \& 7795 \& 68 <br>
\hline 0 300 \& 300/6 \& 70/70/7 \& . \& 3126 \& 43 \& , , \& 450/10 \& $80 / 80 / 10$ \& - \& 8605 \& 82 <br>
\hline , , \& 300/6 \& 60/60/9 \& . \& 3365 \& 45 \& , , \& 450/10 \& 110/70/10 \& \& 9125 \& 88 <br>
\hline \%, \& 300/6 \& 60/60/10 \& \& 3630 \& 48 \& , , \& 450/10 \& 100/100/12 \& -00/10 \& 11455 \& 106 <br>
\hline , , \& 300/6 \& 70/70/10 \& . \& 4036 \& 55 \& , , \& 430/10 \& 70/70/10 \& 200/10 \& 12198 \& 105 <br>
\hline - 9 \& 300/6 \& 80/80/10 \& . \& 4513 \& 61 \& , , \& 430/10 \& 80/80/10 \& 200/10 \& 12882 \& 112 <br>
\hline , , \& 300/6 \& 110/70/11 \& \& 5860 \& 72 \& , , \& 430/10 \& 80/80/10 \& $300 / 10$ \& 15464 \& 127 <br>
\hline O, \& 300/6 \& 100/100/12 \& \& 6125 \& 85 \& , , \& 430/10 \& 100/100/12 \& 250/10 \& 16734 \& 143 <br>
\hline , , \& 280/10 \& 70/70/10 \& 150/10 \& 6282 \& 86 \& , , \& 430/10 \& 100/100/12 \& 350/10 \& 19316 \& 159 <br>
\hline , 9 \& 280/10 \& 70/70/10 \& 250/10 \& 7970 \& 102 \& , , \& 390/10 \& 70/70/10 \& 200/30 \& 19704 \& 175 <br>
\hline , , \& 280/10 \& 80/80/10 \& 250/10 \& 8328 \& 108 \& , , \& 390/10 \& 80/80/10 \& 250/30 \& 23778 \& 194 <br>
\hline , \& 280/10 \& $80 / 80 / 10$ \& 300/10 \& 9170 \& 115 \& , , \& 390/10 \& 80/80/10 \& 300/30 \& 27312 \& 218 <br>
\hline , , \& 260/10 \& 80/80/10 \& 200/20 \& 9724 \& 130 \& , , \& 390/10 \& 100/100/15 \& 350/30 \& 34338 \& 281 <br>
\hline , , \& 280/10 \& $100 / 100 / 12$
$80 / 80 / 10$ \& $300 / 10$
$250 / 20$ \& 10530 \& 140 \& $0 \cdot 480$ \& 480/8 \& 80/80/10 \& .. \& 8952 \& 77 <br>
\hline ", \& $260 / 10$
$240 / 10$ \& $80 / 80 / 10$
$70 / 70 / 10$ \& $250 / 20$
$200 / 30$ \& 11295 \& 145 \& $0 \cdot 500$ \& 500/6 \& \& \& 6889 \& 57 <br>
\hline ,', \& 260/10 \& 80/80/10 \& 300/20 \& 12866 \& 161 \& $0 \cdot 500$ \& $500 / 6$
$500 / 6$ \& 70/70/10 \& - \& 8088 \& 64 <br>
\hline , \& 260/10 \& 100/100/12 \& $300 / 20$ \& 14000 \& 185 \& \& 500/8 \& 70/70/10 \& \& 8592 \& 72 <br>
\hline , \& 240/10 \& 80/80/10 \& 300/30 \& 16006 \& 206 \& , , \& $500 / 10$ \& 80/80/10 \& - \& 9962 \& 86 <br>
\hline $0 \cdot 330$ \& 330/8 \& 90/90/11 \& \& 6258 \& 79 \& , \& 500/10 \& 110/70/10 \& \& 11397 \& 92
110 <br>
\hline $0 \cdot 350$ \& 350/6 \& $50 / 50 / 7$ \& . \& 3240 \& 36 \& , \& 500/10 \& $100 / 100 / 12$
$70 / 70 / 10$ \& \& 13240

7 \& 110 <br>
\hline , , \& 350/6 \& 60/60/10 \& \& 4470 \& 51 \& \& 480/10 \& 80/80/10 \& 200/10 \& - 4800 \& 115 <br>
\hline , , \& 350/6 \& 70/70/8 \& - \& 4261 \& 50 \& , , \& 480/8 \& 80/80/10 \& 250/10 \& 15798 \& 116 <br>
\hline , , \& 350/6 \& 70/70/10 \& .. \& 5034 \& 57 \& , \& 480/8 \& 80/80/10 \& 300/10 \& 17681 \& 131 <br>
\hline \& 350/6 \& 80/80/10 \& . \& 5568 \& 63 \& \& 480/8 \& 100/100/12 \& 250/10 \& 19222 \& 140 <br>
\hline , \& 350/6 \& 110/70/10 \& \& 6618 \& 70 \& \& 480/8 \& 100/100/12 \& 300/10 \& 20663 \& 147 <br>
\hline , , \& $330 / 10$ \& 70/70/10 \& 150/10 \& 7764 \& 90 \& , , \& 460/10 \& 80/80/10 \& 250/20 \& 21995 \& 161 <br>
\hline , , \& $350 / 10$ \& 100/100/12 \& \& 8071 \& 98 \& , , \& 480/8 \& 100/100/12 \& 350/10 \& 22104 \& 155 <br>
\hline , \& $330 / 10$ \& 70/70/10 \& 200/10 \& 8760 \& 98 \& , \& 440/10 \& 70/70/10 \& 200/30 \& 22620 \& 168 <br>
\hline , , \& $330 / 10$ \& 80/80/10 \& 200/10 \& 9222 \& 104 \& , , \& 460/10 \& 100/100/12 \& $250 / 20$ \& 24695 \& 185 <br>
\hline , , \& $330 / 10$ \& 80/80/10 \& 300/10 \& 11205 \& 119 \& , , \& 460/10 \& 80/80/10 \& $300 / 20$ \& 24761 \& 176 <br>
\hline , \& 320/6 \& 70/70/10 \& 250/15 \& 11298 \& 114 \& , \& $440 / 10$ \& 80/80/10 \& $250 / 30$ \& 27248 \& 198 <br>
\hline , , \& 310/8 \& $70 / 70 / 10$
$100 / 100 / 12$ \& $250 / 20$ \& 12743 \& 138 \& , , \& 460/10 \& 100/100/12 \& $300 / 20$ \& 27461 \& 200 <br>
\hline , , \& $330 / 10$ \& $100 / 100 / 12$
$80 / 80 / 10$ \& $300 / 10$ \& 12950 \& 143 \& , , \& 460/10 \& 100/100/12 \& $350 / 20$ \& 30228 \& 216 <br>
\hline , , \& 310/10 \& 80/80/10 \& $250 / 20$ \& 13867 \& 149 \& , , \& 440/10 \& 80/80/10 \& 300/30 \& 31229 \& 222 <br>
\hline , , \& $310 / 8$ \& 100/100/12 \& 250/20 \& 15365 \& 168 \& , , \& 440/10 \& 100/100/12 \& 300/30 \& 33662 \& 24.5 <br>
\hline , \& $310 / 8$ \& 70/70/10 \& $300 / 20$ \& 15716 \& 154 \& , \& 420/10 \& 100/100/12 \& 250/40 \& 34203 \& 259 <br>
\hline , , \& 310/8 \& 100/100/12 \& $300 / 20$ \& 17234 \& 183 \& ,, \& $440 / 10$ \& 100/100/12 \& $350 / 30$ \& 37644 \& 268 <br>
\hline ,' \& 310/8 \& 100/100/12 \& $350 / 20$ \& 19103 \& 199 \& , \& $440 / 10$ \& 100/100/15 \& $350 / 30$ \& 39379 \& 285 <br>
\hline , \& 290/10 \& 80/80/10 \& 300/30 \& 19692 \& 210 \& ,' \& 440/10 \& 100/100/15 \& 400/30 \& 43363 \& 308 <br>
\hline
\end{tabular}

Table II.-Double-flanged Wrotght-iron Girders with Angle-iron-continued.

| Height. | $\begin{aligned} & \text { Vertical } \\ & \text { Rib. } \end{aligned}$ | Four Angle- irons of | $\begin{gathered} \text { Two } \\ \text { Flanges of } \end{gathered}$ | Moment of Resistance for 600000 | $\begin{aligned} & \text { Weigh } \\ & \text { the } \\ & \text { Lineal } \\ & \text { Metre. } \end{aligned}$ | Height. | $\underset{\text { Vib }}{ } \mathrm{V}$ | Four Angle- irons of | $\begin{gathered} \text { Two } \\ \text { Flanges of } \end{gathered}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.500 | ${ }_{420 / 10}^{\text {mill. }}$ | $\frac{\text { mill. }}{100 / 100 / 12}$ | $3 \text { mill. }$ | 44385 | $\begin{gathered} \overline{\text { kilos. }} \\ 322 \end{gathered}$ | 0800 | $\begin{aligned} & \text { mill } \\ & 740 / 10 \end{aligned}$ | $70 / 70 / 10$ | $200 / 30$ | 41286 | $\begin{aligned} & \text { kiling. } \\ & 19 \end{aligned}$ |
| $0 \cdot 550$ | 550 |  |  |  |  |  | 780/10 | 100/100/15 | 400/10 | 46326 | 210 |
| ,, |  |  |  | 107 |  |  | 740/10 | 80/80/10 | 250/30 | 49278 | 226 |
| ,', | 530/10 | 70/70/10 | 200/10 | 15858 | 113 |  | $740 / 10$ | 80/80/10 | 300/30 | 55956 | 245 |
| $\because$ | 490/10 | 70/70/10 | 200/30 | 25590 | 172 |  | 740/10 | 100/100/15 | \&00/30 | 77796 | 332 |
| , | 530/10 | 80/80/10 | 250/10 | 18360 | 127 | $0 \cdot 850$ | 850/8 | 70/70/10 |  | 17784 | 94 |
| , | 490/10 | 80/80/10 | 250/30 | 30774 | 202 |  | 850/8 | 80/80/10 |  | 19470 | 100 |
| $\because$ | 490/10 | $80 / 80 / 10$ | 300/30 | 35202 | 225 |  | 830/10 | 70/70/10 | 200/10 | 28104 | 137 |
| , , | 530/10 | 100/100/15 | 350 '10 | 27378 | 183 | ,, | 830/10 | 80/80/10 | 250/10 | 33000 | 151 |
| , | 490/10 | 100/100/15 | 400/30 | 48936 | 312 | ", | $830 / 10$ | 80/80/10 | 300/10 | 34692 | 158 |
| - 600 | 600/6 | 70/70/10 |  | 1029 | 69 | ,' | $830 / 10$ | 100/100/15 | 400/10 | 49962 | 214 |
| , , | 600/6 | 80/80/10 |  | 11393 | 75 | :' | 790/10 | 80/80/10 | 250/30 | 53142 | 225 |
| ,, | 600/6 | 90/90/10 |  | 1246 | 81 | ', | 790/10 | 100/100/15 | 400/30 | 83742 | 335 |
| , | 600/10 | 80/80/10 |  | 12834 | 94 | $0 \cdot 900$ | $900 /$ | 70/70/10 |  | 19260 | 97 |
| , , | 600/10 | 100/100/12 |  | 16983 | 117 |  | $900 / 8$ | 80/80/10 |  | 21066 | 103 |
| , | $580 / 6$ | 80/80/10 | 200/10 | 17497 | 105 |  | 850/10 | 70/70/1) | 200/10 | 30324 | 140 |
| , | 580/10 | 70/70/10 | 200/10 | 17778 | 117 |  | 880/10 | 80/80/10 | 250/10 | $3+686$ | 154 |
| ,, | 580/10 | 80/80/10 | 250/10 | 20538 | 131 | , | 880/10 | 80/80/10 | 300/10 | 37326 | 162 |
| " | 580/10 | 80/80/10 | 300/10 | 22279 | 139 | ,' | 840/10 | 70/70/10 | 200/30 | 47934 | 200 |
| ,, | 580/10 | 100/100/12 | 250/10 | 24380 | 155 | , 0 | 880/10 | 100/100/15 | 400/10 | 53646 | 218 |
| , | 580/10 | 100/100/12 | 350/10 | 27861 | 170 | , | $8 \pm 0 / 10$ | 100/100/15 | 400/30 | 89748 | 340 |
| , , | $540 / 10$ | 70/70/10 | 200/30 | 28620 | 176 | 0950 | 950/8 | 70/70/10 |  | 20778 | 100 |
| , , | 580/10 | 100/100/15 | $350 / 10$ | 30588 | 187 | 0950 | 9500 | 80/80/10 |  | $2270 \pm$ | 106 |
| , | 540/10 | $80 / 80 / 10$ | 250/30 | 34368 | 206 | ',', | 930 -10 | 70/70/10 | 200/10 | 32592 | 144 |
| , | $540 / 10$ $540 / 10$ | $80 / 80 / 10$ $100 / 100 / 15$ | $300 / 30$ $350 / 30$ | 39246 | 229 | ',', | $890 / 10$ | 70/70/10 | 200/30 | 51330 | 204 |
|  | $540 / 10$ $540 / 10$ | $100 / 100 / 15$ $100 / 100 / 15$ | 350/30 | 49704 | 293 | ,', | $930 / 10$ | 100/100/15 | 400/10 | 57390 | 222 |
| $0 \cdot 650$ | 510/10 | 100/100/15 | 400/30 |  | 316 |  | 890/10 | 100/100/15 | 400/30 | 95808 | 343 |
|  | 650/6 | 80 |  | 122859 |  | 1.000 | 1000 |  |  | 22338 | 103 |
| ,, | 650/10 | 80/80/10 |  | 14348 | 98 | , , | 1000/8 | 80/80/10 |  | 26382 | 125 |
| , | $650 / 10$ | 100/100/12 |  | 18937 | 121 | , , | 1000/7 | 100/100/12 |  | 31082 | 125 |
| , | 630/10 | 70/70/10 | 200/10 | 19740 | 121 |  | 980/10 | 70/70/10 | 200/10 | 34914 | 148 |
| , | 630/10 | 80/80/10 | 250/10 | 22764 | 135 |  | 980/10 | 89/80/10 | 250/10 | 39810 | 162 |
| ,, | 630/10 | 80/80/10 | 300/10 | 24654 | 143 |  | 980/10 | 80/80/10 | 300/10 | 750 | 170 |
| ,, | 590/10 | 70/70/10 | 200/30 | 31704 | 180 |  | 940/10 | 70/70/10 | 200/30 | 1786 | 208 |
| ,, | 630/10 | 100/100/15 | 350/10 | 33858 | 190 |  | 940,10 | 100/100/15 | 400/30 | 101922 | 347 |
| ', | 590/10 | 80/80/10 | 300/30 | 43338 | 233 | $1 \cdot 100$ | 1080/10 | /80/10 | 300/10 | 48384 | 178 |
| , | 590/10 | 100/100/15 | 400/30 | $6029 \pm$ | 320 |  | 1080/10 | 100/100/15 | 350/10 | 65676 | 226 |
| 0700 | 700 | 70/70/10 |  | 1358 | 84 | ,' | 1040/10 | 100/100/15 | 400/30 | 114312 | 355 |
| ,, | 700/8 | 80/80/10 |  | 14934 | 90 | $1 \cdot 200$ | 1200/10 | 100/100/12 |  | 43854 | 164 |
| , | 680/10 | 70/70/10 | 200/10 | 21756 | 125 |  | 1180/10 | 100/100/12 | 300/10 | 64860 | 210 |
| - | 700/10 | 100/100/12 |  | 21924 | 125 |  | 1180/10 | 100/100/15 | 400/10 | 76866 | 241 |
| , | 680/10 | $80 / 80 / 10$ | 250/10 | 25044 | 139 |  | 1140/10 | 100/100/15 | 400/30 | 126906 | 363 |
| $\because$ | 680/10 | $80 / 80 / 10$ $100 / 100 / 15$ | 300/10 | 27084 | 147 | 1.250 | 1230/10 | 80/80/10 | 300/10 | 57204 | 190 |
| ", | 640/10 | $100 / 10 / 15$ $80 / 80 / 10$ | 400/10 $300 / 30$ | 39216 47478 | 202 | ,, | 1230/10 | 100/100/15 | 350/10 | 77220 | 237 |
| ,', | 640/10 | 100/100/15 | 400/30 | 66072 | 324 | ,', | 1230/10 | 100/100/15 | 400/10 | 80910 | 245 |
| 0750 | 750/8 | 70/70/10 |  | 14952 | 88 |  |  | 100/15 | 400/30 | 133281 | 367 |
|  | 750/8 | 80/80/10 |  | 16404 | 94 | 1.300 | 1300/10 | 100/100/12 |  | 49059 | 172 |
|  | 730/10 | 70/70/10 | 200/10 | 23820 | 129 | , | 1280/10 | 100/100/12 | 300/10 | 72460 | 219 |
|  | 730/10 | 80/80/10 | 250/10 | 27378 | 143 | ', | 1280/10 | 100/100/15 | 350/10 | 81168 | 241 |
|  | 730/10 | 80/80/10 | 300/10 | 29568 | 150 | , , | 1280/10 | 100/100/15 | 400/10 | 85008 | 249 |
|  | 690/10 | 70/70/10 | 200/30 | 38040 | 183 | ,, | 1180/10 | 100/100/12 | 300/60 | 189 | 452 |
|  | 730/10 | 100/100/15 | 400/10 | 42744 | 206 | $1 \cdot 400$ | 1380/10 | 100/100/12 | 300/10 | 79660 | 226 |
|  | 690/10 | 80/80/10 | 250/30 | 45468 | 218 | , , | 1380/10 | 80/80/10 | 300/10 | 66480 | 201 |
| ", | 690/10 | 80/80/10 | 300/30 | 51690 | 241 | , | 1380/10 | 100/100/15 | 350/10 | 89220 | . 249 |
| ,' | 690/10 | 100/100/15 | 400/30 | $7190 \pm$ | 328 |  | 1380/10 | 100/100/15 | 400/10 | 93360 | 257 |
| 0800 | 800/8 | 10 |  | 16350 | 91 | ,' | 1280/10 | 100/100/12 | 300/60 | 205660 | 460 |
| ,, | 800/8 | 80/80/10 |  | 17922 | 97 | 1-500 | 1500/10 | 100/100/12 |  | 60100 | $18 \%$ |
| ,, | 800/10 | 100/100/12 |  | 25116 | 133 |  | 1430/10 | 100/100/12 | 300/10 | 87100 | 242 |
| , | 780/10 | 70/70/10 | 200/10 | 25938 | 133 |  | 1480/10 | 100/100/12 | 400/10 | 96100 | 250 |
| $\because$ | 780/10 | 80/80/10 | 250/10 | 29766 | 147 |  | 1489/10 | 100/100/15 | 400/10 | 101916 | 264 |
| , | 780/10 | 80/80/10 | 300/10 | 32106 | 154 |  | 1380/10 | 100/100/12 | 400/60 | 276100 | 467 |

Table III. (14).-Moment of Resistance of Four Angle-irons (Two to each Flange) for $R=6000000$.

| Height. | Angle-iron of | Angle-iron of | Height. | Angle-iron of | Angle-iron of |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.400 | 5703 | 8131 | $0{ }^{\text {m. }} 960$ | 15662 | 23007 |
| $0 \cdot 410$ | 5757 | 8389 | $0 \cdot 970$ | 15841 | 23276 |
| $0 \cdot 420$ | 6053 | 8649 | $0 \cdot 980$ | 16020 | 23544 |
| $0 \cdot 430$ | 6229 | 8909 | $0 \cdot 990$ | 16200 | 23813 |
| $0 \cdot 440$ | 6405 | 9169 | 1.000 | 16380 | 24082 |
| $0 \cdot 450$ | 6580 | 9430 | 1.010 | 16559 | 24350 |
| $0 \cdot 460$ | 6756 | 9692 | 1.020 | 16738 | 24620 |
| $0 \cdot 470$ | 6933 | 9953 | 1.030 | 16917 | 24888 |
| $0 \cdot 480$ | 7109 | 10216 | 1.010 | 17096 | 25157 |
| $0 \cdot 490$ | 7285 | 10479 | 1.050 | 17275 | 25426 |
| $0 \cdot 500$ | 7462 | 10741 | 1.060 | 17454 | 25695 |
| $0 \cdot 510$ | 7639 | 11004 | $1 \cdot 070$ | 17633 | 25965 |
| 0. 520 | 7816 | 11267 | 1.080 | 17813 | 26234 |
| 0.530 | 7993 | 11531 | $1 \cdot 090$ | 17992 | 26503 |
| 0.540 | 8170 | 11795 | $1 \cdot 100$ | 18171 | 26772 |
| 0.550 | 8347 | 12059 | 1110 | 18351 | 27041 |
| $0 \cdot 560$ | 8524 | 12323 | 1.120 | 18530 | 27310 |
| $0 \cdot 570$ | 8702 | 12588 | $1 \cdot 130$ | 18710 | 27579 |
| $0 \cdot 580$ | 8879 | 12853 | 1140 | 18889 | 27848 |
| $0 \cdot 590$ | 9057 | 13118 | $1 \cdot 150$ | 19068 | 28117 |
| 0.600 | 9235 | 13383 | 1.160 | 19247 | 28386 |
| $0 \cdot 610$ | 9412 | 13648 | 1170 | 19427 | 28655 |
| $0 \cdot 620$ | 9590 | 13914 | $1 \cdot 180$ | 19606 | 28925 |
| $0 \cdot 630$ | 9768 | 14180 | 1-190 | 19785 | 29195 |
| $0 \cdot 640$ | 9946 | 14446 | 1.200 | 19965 | 29464 |
| $0 \cdot 650$ | 10124 | 14712 | $1 \cdot 210$ | 20144 | 29733 |
| $0 \cdot 660$ | 10302 | 14978 | 1-220 | 20324 | 30003 |
| $0 \cdot 670$ | 10480 | 15244 | $1 \cdot 230$ | 20503 | 30272 |
| 0.680 | 10658 | 15510 | $1 \cdot 240$ | 20683 | 30542 |
| 0.690 | 10836 | 15777 | 1.250 | 20862 | 30811 |
| 0.700 | 11014 | 16044 | 1-260 | 21041 | 31081 |
| $0 \cdot 710$ | 11192 | 16311 | 1-270 | 21220 | 31350 |
| 0.720 | 11370 | 16578 | 1. 280 | 21400 | 31620 |
| $0 \cdot 730$ | 11549 | 16845 | 1-290 | 21580 | 31889 |
| $0 \cdot 740$ | 11727 | 17112 | $1 \cdot 300$ | 21760 | 32159 |
| $0 \cdot 750$ | 11906 | 17379 | $1 \cdot 310$ | .. | 32428 |
| 0.760 | 12085 | 17646 | $1 \cdot 320$ | .. | 32698 |
| $0 \cdot 770$ | 12264 | 17914 | $1 \cdot 330$ | . | 32968 |
| $0 \cdot 780$ | 12443 | 18182 | $1 \cdot 340$ | - | 33237 |
| $0 \cdot 790$ | 12622 | 18450 | $1 \cdot 350$ | . | 33507 |
| 0.800 | 12800 | 18717 | $1 \cdot 360$ | .. | 33776 |
| 0.810 | 12973 | 18985 | $1 \cdot 370$ | . | 34046 |
| 0.820 | 13157 | 19253 | 1.380 | - | 34315 |
| 0.830 | 13336 | 19520 | 1.390 | . | 34585 |
| $0 \cdot 840$ | 13515 | 19788 | 1.400 | -• | 31855 |
| $0 \cdot 850$ | 13694 | 20056 | 1.410 | $\stackrel{.}{\text {. }}$ | 35125 |
| 0.860 | 13873 | 20324 | 1.420 . | .. | 35395 |
| $0 \cdot 870$ | 14052 | 20592 | $1 \cdot 430$ | .. | 35665 |
| $0 \cdot 880$ | 14230 | 20860 | 1.440 | .. | 35935 |
| $0 \cdot 890$ | 14409 | 21128 | 1.450 | .. | 36205 |
| $0 \cdot 900$ | 14588 | 21397 | $1 \cdot 460$ | .. | 36475 |
| $0 \cdot 910$ | 14767 | 21665 | $1 \cdot 470$ | - | 36745 |
| $0 \cdot 920$ | 14946 | 21933 | $1 \cdot 480$ | .. | 37015 |
| $0 \cdot 930$ | 15125 | 22202 | $1 \cdot 490$ | - | 37285 |
| C. 910 | 15304 | 22470 | $1 \cdot 500$ | - | 37555 |
| 0.950 | 15433 | 22738 |  | - |  |
|  |  |  |  |  |  |

Table IV. (20).-Table of Sections for

|  | $h=0 \mathrm{~m} \cdot 200$. | $h=0 \mathrm{~m} \cdot 250$. | $h=0 \mathrm{~m} \cdot 300$. | $h=0 \mathrm{~m} \cdot 350$. |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 200 / 10 \\ 80 / 80 / 10 \\ 57^{\mathrm{k}} \end{gathered}$ | $\begin{gathered} 250 / 8 \\ 70 / 70 / 8 \\ 45 \end{gathered}$ | $\stackrel{.}{ }$ | . |
| $l=1^{\mathrm{m} \cdot 750}\left(\begin{array}{l}\text { Vertical rib } . . \\ \text { Angle-irons.. } \\ \mathrm{M}=3255)\end{array}\right.$.. | $\begin{gathered} 200 / 12 \\ 110 / 70 / 10 \\ 66 \end{gathered}$ | $\begin{gathered} 250 / 8 \\ 70 / 70 / 10 \\ 52 \end{gathered}$ | .. <br> . | . . . |
|  | $\begin{gathered} 200 / 10 \\ 110 / 70 / 11 \\ 70 \end{gathered}$ | $\begin{gathered} 250 / 10 \\ 80 / 80 / 10 \\ 62 \end{gathered}$ | $\begin{gathered} 300 / 7 \\ 70 / 70 / 8 \\ 46 \end{gathered}$ | .. |
|  | $\begin{gathered} 180 / 10 \\ 60 / 60 / 10 \\ 230 / 10 \\ 79 \end{gathered}$ | $\begin{gathered} 250 / 8 \\ 110 / 70 / 10 \\ \ddot{65} \end{gathered}$ | $\begin{gathered} \hline 300 / 8 \\ 70 / 70 / 10 \\ \ddot{57} \end{gathered}$ | $\stackrel{.}{\square}$ |
|  | $\begin{gathered} 176 / 10 \\ 60 / 60 / 10 \\ 250 / 12 \\ 90 \end{gathered}$ | $\begin{gathered} 250 / 12 \\ 110 / 70 / 11 \\ \ddot{78} \end{gathered}$ | $\begin{gathered} 300 / 10 \\ 30 / 30 / 10 \\ \ddot{6} \end{gathered}$ | $\begin{gathered} 350 / 6 \\ 70 / 70 / 10 \\ \ddot{5} \end{gathered}$ |
|  | .. | $\begin{gathered} 230 / 10 \\ 80 / 80 / 10 \\ 200 / 10 \\ 90 \end{gathered}$ | $\begin{gathered} 300 / 6 \\ 110 / 70 / 11 \\ \ddot{6} \end{gathered}$ | $\begin{gathered} 350 / 8 \\ 80 / 80 / 10 \\ \ddot{6} \end{gathered}$ |
| $l=3^{\mathrm{m} \cdot 000}\left(\begin{array}{l}\text { Vertical rib.. } \\ \text { Angle-irons } \\ \text { A }\end{array}\right.$. $\quad . .$. | -. <br> . | $\begin{gathered} 226 / 8 \\ 70 / 70 / 10 \\ 250 / 12 \\ 97 \end{gathered}$ | $\begin{gathered} 280 / 7 \\ 70 / 70 / 10 \\ 200 / 10 \\ 82 \end{gathered}$ | $\begin{gathered} 350 / 8 \\ 110 / 70 / 10 \\ \ddot{70} \end{gathered}$ |
| $l=3^{\mathrm{m} \cdot 250}(\mathrm{M}=7670)\left\{\begin{array}{l} \text { Vertical rib .. } \\ \text { Angle-irons.. } \\ \text { Flanges }\left\{\begin{array}{l} \text { 1st plate } \\ \text { 2nd } \end{array}\right. \\ \mathrm{P} . . . \\ \text { plate } \end{array}\right\}$ | $\because$ <br>  <br>  | $\begin{gathered} 220 / 10 \\ 70 / 70 / 10 \\ 250 / 7 \\ 250 / 8 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}} \cdot \mathrm{C0} \\ 104 \end{gathered}$ | $\begin{gathered} 234 / 10 \\ 80 / 80 / 10 \\ 250 / 8 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m} \cdot 60} \\ \ddot{91} \end{gathered}$ | $\begin{gathered} 334 / 7 \\ 70 / 70 / 10 \\ 200 / 8 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m} \cdot 50} \\ \ddot{75} \end{gathered}$ |
|  | $\stackrel{.}{ }$ | .. | $\begin{gathered} 280 / 8 \\ 70 / 70 / 10 \\ 300 / 10 \\ 100 \end{gathered}$ | $\begin{gathered} 334 / 6 \\ 70 / 70 / 10 \\ 250 / 8 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m} \cdot 80} \\ 80 \end{gathered}$ |
| $l=3^{\mathrm{m} \cdot 750}(\mathrm{M}=9570)\left\{\begin{array}{l} \text { Vertical rib } . . \\ \text { Angle-irons. } \\ \text { 1s. } \\ \text { Flanges }\left\{\begin{array}{lll} \text { 1st plate } \\ \text { 2nd } & \text { plate } \end{array}\right. \\ \mathrm{P} . . . \\ . . \\ . \end{array}\right.$ | $\because$ <br>  <br>  | $\because$ <br>  <br> . | $\begin{gathered} 260 / 8 \\ 70 / 70 / 10 \\ 210 / 10 \\ 210 / 10 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}}-70 \\ 109 \end{gathered}$ | $\begin{gathered} 324 / 8 \\ 70 / 70 / 10 \\ 200 / 13 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 10} \\ \ddot{9} \end{gathered}$ |
|  | -. | .. | $\begin{gathered} 280 / 10 \\ 100 / 100 / 12 \\ 300 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 10 \\ 126 \end{gathered}$ | $\begin{gathered} 330 / 8 \\ 80 / 80 / 10 \\ 270 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 10 \\ 98 \end{gathered}$ |
| $l=4^{\mathrm{m} \cdot 250}\left(\begin{array} { l }  { \mathrm { M } = 1 1 5 1 5 ) } \end{array} \left\{\begin{array} { l }  { \text { Vertical rib } . . } \\ { \text { Angle-irons. } } \\ { \text { 1st } } \\ { \text { Flanges } } \end{array} \left\{\left.\begin{array}{l} \text { 1st plate } \\ \text { 2nd plate } \end{array} \right\rvert\, \begin{array}{lll} \text { P.. } & . . & . . \end{array}\right.\right.\right.$ | $\because$ <br> 0 <br> . | $\because$ <br> $\because$ <br> $\because$ <br> $\square$ | $\begin{gathered} 272 / 6 \\ 80 / 80 / 10 \\ 300 / 14 \\ \ddot{120} \end{gathered}$ | $\begin{gathered} 320 / 7 \\ 80 / 80 / 10 \\ 250 / 7 \\ 250 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 00} \\ 111 \end{gathered}$ |
| $l=4^{\mathrm{m} \cdot 500}(\mathrm{M}=18500)\left\{\begin{array}{l} \text { Vertical rib .. } \\ \text { Angle-irons.. } \\ \text { Flanges }\left\{\begin{array}{l} \text { 1st plate } \\ \text { 2nd plate } \end{array}\right. \\ \mathrm{P} . . \quad . . \end{array}\right.$ | $\because$ <br> $\because$ <br> $\because$ <br> $\because$ <br> . | . <br> . <br> . <br> . <br> . | $\begin{gathered} 250 / 10 \\ 80 / 80 / 10 \\ 250 / 12 \\ 250 /^{\prime} 13 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot} 30 \\ 147 \end{gathered}$ | $\begin{gathered} 310 / 8 \\ 70 / 70 / 10 \\ 250 / 10 \\ 250 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 20 \\ 124 \end{gathered}$ |
| $l=5^{\mathrm{m} \cdot 000}(\mathrm{M}=14500)\left\{\begin{array}{l} \text { Vertical rib.. } \\ \text { Angle-irons... } \\ \text { Flanges }\left\{\begin{array}{l} \text { 1st plate } \\ \text { 2nd plate } \end{array}\right. \\ \mathrm{P} . . \\ . . \end{array}\right.$ | $\because 0$ <br>  | $\because$ $\because$ $\square$ | $\because$ $\because$ $\because$ $\because$ | $\begin{gathered} 314 / 7 \\ 80 / 80 / 10 \\ 300 / 9 \\ 300 / 9 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 40} \\ 131 \end{gathered}$ |

Double-flanged Minor Longitudinal Girders.

| $3=0 \mathrm{~m} \cdot 400$. | $h=0 \mathrm{~m} \cdot 450$. | $h=0 \mathrm{~m} \cdot 500$. | $h=0 \mathrm{~m} \cdot 550$. | $h=0 \mathrm{~m} \cdot 600$. | $h=0 \mathrm{~m} \cdot 650$. | $h=0 \mathrm{~m} \cdot 700$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $\cdots$ | - | . | - | - | - |
| .. | .. | .. | .. | .. | .. | .. |
| - | - | .. | .. | .. | .. | . |
| . | . | . | .. | .. | . | .. |
| . | .. | . | .. | .. | .. | . |
| - | $\cdots$ | . | . | . | - | . |
| -. | . | $\cdots$ | .. | .. | .. | .. |
| - | $\cdots$ | - | -• | -• | - | - |
| - | . | $\cdots$ | - | - | - | $\cdots$ |
| .. | .. | $\cdots$ | .. | $\ldots$ | .. | .. |
| - | $\cdots$ | $\cdots$ | - | - | - | - |
| - | . | $\cdots$ | $\cdots$ | .. | - | - |
| . | .. | .. | .. | .. | .. | .. |
| 400/6 | . | -• | - | -• | $\cdots$ | - |
| 70/70/10 | .. | $\cdots$ | .. | .. | . | .. |
| $\ddot{56}$ | .. | .. | .. | .. | .. | .. |
| $\begin{gathered} 400 / 7 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 450 / 6 \\ 70 / 70 / 10 \end{gathered}$ | .. | .. | .. | . | .. |
| 80/80 |  | $\stackrel{.}{.}$ | ." | $\because$ | . ${ }^{\text {. }}$ | .. |
| 66 | 60 | .. | .. | . | .. | .. |
| $\begin{gathered} 400 / 10 \\ 80 / 80 / 11 \end{gathered}$ | $\begin{gathered} 450 / 6 \\ 80 / 80 / 10 \end{gathered}$ | .. | .. | $\because$ | .. | - |
| .. | .. | - | .. | -. | .. | .. |
| $\ddot{80}$ | $\ddot{65}$ | . | .. | . 0 | .. | -. |
| $\begin{gathered} 400 / 8 \\ 110 / 70 / 11 \end{gathered}$ | $\begin{gathered} 450 / 10 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 500 / 8 \\ 70 / 70 / 10 \end{gathered}$ | - | $\because$ | ". | .. |
| $\ddot{80}$ | $\ddot{80}$ | 70 | - | .. | . | . $\quad$. |
| $\begin{gathered} 400 / 8 \\ 100 / 100 / 12 \end{gathered}$ | $\begin{gathered} 450 / 12 \\ 80 / 80 / 11 \end{gathered}$ | $\begin{gathered} 500 / 8 \\ 80 / 80 / 10 \end{gathered}$ | - | $\because$ | .. | $\because$ |
| .. | - | .. | . | . | . | . |
| $\ddot{9} 3$ | $\ddot{92}$ | $\ddot{76}$ | - | $\cdots$ | .. | .. |
| $\begin{gathered} 380 / 10 \\ 70 / 70 / 10 \end{gathered}$ | $\begin{gathered} 450 / 6 \\ 100 / 100 / 12 \end{gathered}$ | $\begin{gathered} 500 / 8 \\ 110 / 70 / 10 \end{gathered}$ | $\begin{gathered} 550 / 8 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 600 / 7 \\ 70 / 70 / 10 \end{gathered}$ | .. | .. |
| $\begin{gathered} 200 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 00} \\ 92 \end{gathered}$ | $\ddot{89}$ | $\ddot{8}$ | $\ddot{80}$ | $\ddot{72}$ | .. | .. |
| $\begin{gathered} 384 / 8 \\ 80 / 80 / 10 \\ 280 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot} 20 \end{gathered}$ | $\begin{gathered} 432 / 8 \\ 70 / 70 / 10 \\ 200 / 9 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 10 \end{gathered}$ | $\begin{gathered} 500 / 8 \\ 110 / 70 / 11 \end{gathered}$ | $\begin{gathered} 550 / 10 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 600 / 7 \\ 80 / 80 / 10 \end{gathered}$ | $\because$ | .. |
| $280 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 20}$ | $200 / 9 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 10}$ | , |  |  | .. | .. |
| $\ddot{95}$ |  | $\ddot{90}$ |  | 73 | .. | .. |
| $\begin{gathered} 380 / 7 \\ 80 / 80 / 10 \\ 300 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 70 \end{gathered}$ | $\begin{gathered} 434 / 8 \\ 80 / 80 / 10 \\ 250 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} 20 \end{gathered}$ | $\begin{array}{\|c\|} \hline 484 / 8 \\ 70 / 70 / 10 \\ 200 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 10 \end{array}$ | $\begin{array}{c\|} \hline 538 / 8 \\ 70 / 70 / 8 \\ 180 / 6 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m} \cdot} 70 \end{array}$ | $\begin{gathered} 600 / 10 \\ 80 / 80 / 10 \\ . . \end{gathered}$ | $\begin{gathered} 650 / 6 \\ 80 / 80 / 10 \end{gathered}$ | $\cdots$ |
| 105 |  |  |  | $\ddot{92}$ | $\ddot{76}$ | . ${ }^{\text {. }}$ |
| $\begin{gathered} 374 / 8 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 426 / 8 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 484 / 8 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 532 / 8 \\ 70 / 70 / 10 \end{gathered}$ | $\begin{gathered} 600 / 6 \\ 100 / 100 / 12 \end{gathered}$ | $\begin{gathered} 650 / 10 \\ 80 / 80 / 10 \end{gathered}$ | $\begin{gathered} 700 / 8 \\ 80 / 80 / 10 \end{gathered}$ |
| $300 / 13 \mathrm{~s}^{\mathrm{s}} 4^{\mathrm{m}} \cdot 20$ | $\begin{aligned} & 240 / 6 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m} \cdot 00} \\ & 246 / 6 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}} \cdot 80 \end{aligned}$ | $250 / 8 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 50}$ | $200 / 9 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}-50}$ |  | .. | - |
| 120 |  | $\ddot{97}$ | $\ddot{92}$ | $\ddot{96}$ | $\ddot{96}$ | $\ddot{90}$ |

Table V. (23).-Cross-girders for Three-Girder Bridges.

| Distance between the Cross-girders, and Moments of Rupture. | Height of the girder. | Section. |  |  |  |  | Weight |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Vertical Rib. | Angle-irons. | Flanges. |  |  |  |  |
|  |  |  |  | First Plate 4 of long. | Second Plate. | Third Plate. |  |  |
| $\begin{gathered} 1^{\mathrm{m} \cdot 500} \\ (\mathrm{M}=12090) \end{gathered}$ | \% $0 \cdot 300$ | $\operatorname{mill}_{260 / 10}$ | $\begin{aligned} & \text { mill. } \\ & 80 / 80 / 10 \end{aligned}$ | 280/10 | $280 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 000$ | . | kilos. <br> 670 | $\begin{aligned} & \text { kilos. }_{895} \end{aligned}$ |
|  | $0 \cdot 350$ | 310/8 | 70/70/10 | 240/10 | $240 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 000}$ | - | 600 | 800 |
|  | $0 \cdot 400$ | 380/10 | 80/80/10 | 250/10 | .. | .. | $\begin{aligned} & 570 \\ & 495 \end{aligned}$ | $\begin{aligned} & 760 \\ & 660 \end{aligned}$ |
|  | $0 \cdot 500$ | 500/6 | 100/100/12 | .. | .. |  |  |  |
| $\begin{gathered} 2^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=14950) \end{gathered}$ | 0.300 | $\begin{aligned} & 248 / 10 \\ & 310 / 10 \\ & 376 / 10 \\ & 480 / 10 \\ & 600 / 6 \end{aligned}$ | $\begin{array}{r} 80 / 80 / 10 \\ 80 / 80 / 10 \\ 80 / 80 / 10 \\ 80 / 80 / 10 \\ 100 / 100 / 12 \end{array}$ | $\begin{gathered} 300 / 13 \\ 300 / 10 \\ 290 / 12 \\ 200 / 10 \\ . . \end{gathered}$ | $\begin{aligned} & 300 / 13 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 000} \\ & 300 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 000} \end{aligned}$ | -• | 780 | 780 |
|  | $0 \cdot 350$ |  |  |  |  | .. | 680 | 680 |
|  | $0 \cdot 400$ |  |  |  | $300 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 000}$ | .. | 630 | 630 |
|  | $0 \cdot 500$ |  |  |  | .. | . | $\begin{aligned} & 580 \\ & 530 \end{aligned}$ | $\begin{aligned} & 580 \\ & 530 \end{aligned}$ |
|  | $0 \cdot 600$ |  |  |  | .. |  |  |  |
| $\begin{gathered} 2^{\mathrm{m} \cdot 500} \\ (\mathrm{M}=16500) \end{gathered}$ | 0.300 | 240/12 | 80/80/10 | $\begin{aligned} & 300 / 10 \\ & 250 / 10 \end{aligned}$ | $\begin{aligned} & 300 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m} \cdot 400} \\ & 250 / 10 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}} \cdot 800 \end{aligned}$ | $300 / 10 \mathrm{~s}^{\text {r }} 2^{\mathrm{m}} \cdot 600$ | 840 | 670 |
|  | $0 \cdot 400$ | $360 / 10$ | 80/80/10 |  |  | .. | 670 | 540 |
|  | $0 \cdot 500$ | 480/10 | 80/80/10 | 260/10 | .. |  | 620 | 500 |
|  | 0.600 | 600/10 | 100/100/12 | . | . | - | 610 | 490 |
| $\begin{gathered} 3^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=17800) \end{gathered}$ | $0 \cdot 400$ | $\begin{aligned} & 380 / 10 \\ & 484 / 8 \\ & 580 / 10 \\ & 700 / 6 \end{aligned}$ | $\begin{array}{r} 100 / 100 / 12 \\ 100 / 100 / 12 \\ 70 / 70 / 10 \\ 100 / 100 / 12 \end{array}$ | $\begin{array}{\|c} 350 / 10 \\ 250 / 8 \\ 200 / 10 \\ \ldots \end{array}$ | - | . | 730 | 490 |
|  | $0 \cdot 500$ |  |  |  | . | . | 650 | 435 |
|  | $0 \cdot 600$ |  |  |  | .. |  | 600 | 400 |
|  | 0.700 |  |  |  |  | - | 560 | 375 |
| $\begin{gathered} 3^{\mathrm{m}} \cdot 500 \\ (\mathrm{M}=19240) \end{gathered}$ | $0 \cdot 400$ | $\begin{aligned} & 360 / 10 \\ & 480 / 8 \\ & 580 / 6 \\ & 700 / 7 \\ & 800 / 8 \end{aligned}$ | $\begin{array}{r} 80 / 80 / 10 \\ 100 / 100 / 12 \\ 80 / 80 / 10 \\ 100 / 100 / 12 \\ 80 / 80 / 11 \end{array}$ | $\begin{aligned} & 300 / 10 \\ & 250 / 10 \end{aligned}$ |  | . | 740 | 425 |
|  | $0 \cdot 500$ |  |  |  | $300 / 11 \mathrm{~s}^{\text {r }} 2^{\mathrm{m}} \cdot 800$ | .. | 680 | 390 |
|  | 0.600 |  |  | 250/10 | . ${ }^{\text {. }}$ | . | 580 | 370 |
|  | 0.700 |  |  |  | .. | - | 580 | 370 |
|  | 0.800 |  |  | . | . |  | 570 | 325 |
| $\begin{gathered} 4^{\mathrm{m} \cdot 000} \\ (M=20540) \end{gathered}$ | $0 \cdot 400$ | 360/10 | 100/100/12 | 300/10 | $300 / 10 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}} \cdot 700$ | . ${ }^{\text {. }}$ | 820 | 410 |
|  | $0 \cdot 500$ | 480/8 | 100/100/12 | 300/10 | .. |  | 710 | 355 |
|  | 0.600 | 580/10 | 80/80/10 | 250/10 | -. | . ${ }^{\text {. }}$ | 660 | 330 |
|  | 0.700 | 700/10 | 100/100/12 | .. | .. | . ${ }^{\text {. }}$ | 655 | 330 |
|  | $0 \cdot 800$ | 800/10 | 80/80/11 | .. | . | .. | $590$ | $295$ |
|  | $0 \cdot 850$ | 850/8 | 80/80/11 | . | .. | $\cdots$ |  |  |
| $\begin{gathered} 4^{\mathrm{m} \cdot 500} \\ (\mathrm{M}=21700) \end{gathered}$ | $0 \cdot 500$ | 460/10 | 80/80/10 | $\begin{aligned} & 250 / 10 \\ & 300 / 10 \end{aligned}$ |  | . | 715 | 320 |
|  | $0 \cdot 600$ | 580/8 | 80/80/10 |  | $250 / 10 \mathrm{~s}^{\mathrm{r}} 2^{\mathrm{m}} \cdot 700$ | . | 650 | 290 |
|  | $0 \cdot 700$ | 700/10 | 100/100/12 | . |  | .. | 655 | 290 |
|  | 0.900 | 900/8 | 80/80/11 | -• |  |  | 610 | 270 |
| $\begin{gathered} 5^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=22750) \end{gathered}$ | $0 \cdot 600$ | 580/8 | 80/80/10 | $\begin{gathered} 340 / 10 \\ 220 / 10 \\ . . \\ . . \end{gathered}$ | .... | . <br> . <br> . | 675 | 270 |
|  | $0 \cdot 700$ | 680/8 | 80/80/10 |  |  |  | 645 | 260 |
|  | $0 \cdot 800$ | 800/7 | 100/100/12 |  |  |  | 620 | 250 |
|  | $0 \cdot 900$ | 900/8 | 80/80/12 |  |  |  | 625 | 250 |

Table VI. (24).-Cross-girders for Two-girder Bridges.

| Distanca between the Cross-girders, and Moments of Rupture. | Height of the Crossgirder. | Section. |  |  |  |  | Weight of the Crossgirders the Lineal Mètre of Flooring. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Vertical Rib. | Angle-irons. | Flanges. |  |  |  |
|  |  |  |  | First Plate. | Second Plate. | Third Plate. |  |
| $2^{\mathrm{m} \cdot} 000$$(\mathrm{M}=40250)$ | ${ }^{\mathrm{m}} \cdot 6.600$ | 540/10 | 80/80/10 | 310/10 | $310 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m} \cdot 300}$ | $310 / 10 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 000$ | kilos. $870$ |
|  | $0 \cdot 700$ | 640/10 | 80/80/10 | 250/10 | $250 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m}} \cdot 000$ | $250 / 10 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} 0000$ | 840 |
|  | $0 \cdot 800$ | 740/10 | 70/70/10 | 200/10 | $200 / 10 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 800$ | $200 / 10 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m} \cdot 800}$ | 820 |
|  | $0 \cdot 900$ | 880,10 | 80/80/10 | $320 / 10$ | . | . | 785 |
| $\begin{gathered} 3^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=47950) \end{gathered}$ | $0 \cdot 600$ | 540/10 | 100/100/15 | 350/10 | $350 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m}} \cdot 200$ | $350 / 10 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m} \cdot 200}$ | 740 |
|  | $0 \cdot 800$ | 740/10 | 80/80/10 | 250/10 | $250 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m} \cdot 000}$ | $250 / 10 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m} \cdot 800}$ | 600 |
|  | $1 \cdot 000$ | 974/10 | 80/80/10 | $300 / 13 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m} \cdot 000}$ | . | . | 530 |
| $\begin{gathered} 4^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=55300) \end{gathered}$ | $0 \cdot 800$ | 740/10 | 80/80/10 | $\begin{gathered} 300 / 10 \\ 220 / 9 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 800 \end{gathered}$ | $300 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m}} \cdot 200$ | $300 / 10 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 000$ | 490 |
|  | $1 \cdot 000$ | 1000/8 | 100/100/12 |  | $220 / 9 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m} \cdot 500}$ |  | 410 |
|  | 1.000 | 1000/8 | 80/80/10 | 250/10 | $250 / 10 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m}} \cdot 800$ | . | 400 |
| $\begin{gathered} 5^{\mathrm{m} \cdot 000} \\ (\mathrm{M}=61250) \end{gathered}$ | $1 \cdot 000$ | 1000/8 | 100/100/12 | $250 / 10 \mathrm{~s}^{\mathrm{r}} 6^{\mathrm{m} \cdot 000}$ | $250 / 10 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m} \cdot 600}$ | . | 340 |
|  |  |  |  |  |  |  |  |

Table VII. (31).-Cross-girders for Road Bridges.

| Bearing of the Crossgirder. | Height of the Crossgirder | $\begin{gathered} \text { Distance } \\ \text { between the } \\ \text { principal } \\ \text { Girders. } \end{gathered}$ | $\begin{gathered} \text { Moment } \\ \text { of } \\ \text { Rupture. } \end{gathered}$ | Section. |  |  |  | Weight ofthe Cross-girdersthe LinealMêtre ofFlooring. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Vertical Rib. | Angle-irons. | Flanges. |  |  |
|  |  |  |  |  |  | First Plate. | Second Plate. |  |
| $\begin{aligned} & \text { mètres. } \\ & 3 \cdot 000 \end{aligned}$ | m. ${ }^{\text {m. }}$, 250 | $3 \mathrm{~m} \cdot 400$ | 3600 | $\underset{250 / 8}{\text { mill. }_{2}}$ | $\begin{aligned} & \text { mill. } \\ & 80 / 80 / 10 \end{aligned}$ | .. | . | $\begin{aligned} & \text { kilos. } \\ & 120 \end{aligned}$ |
| , | 0.300 | ,' | ,' | 300/6 | 60/60/10 | . | . $\cdot$ | 100 |
| $4 \cdot 000$ | 0.300 | $4 \cdot 400$ | 5910 | 300/7 | 110/70/11 | . | . | 190 |
| , | $0 \cdot 350$ | ,' | ,' | 350/6 | 110/70/10 | .. | . | 180 |
| 5•000 | 0•300 | 5•400 | 10625 | 274/10 | 80/80/10 | $\left\{\begin{array}{c}300 / 13 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m}} \cdot 300 \\ \text { in length. }\end{array}\right\}$ | .. | 350 |
| , | $0 \cdot 400$ | ,' | ,' | 380/8 | 80/80/10 | $200 / 10 \mathrm{~s}^{\mathrm{r}} 3^{\mathrm{m}} \cdot 600$ | .. | 275 |
| 6.000 | 0•350 | $6 \cdot 400$ | 13500 | 310/8 | 80/80/10 | $\left\{\begin{array}{c}250 / 10 \mathrm{~s}^{r} \\ \text { the entire length. }\end{array}\right\}$ | $50 / 10 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m}} \cdot 000$ | 460 |
| ,' | 0.400 | ,' | ,' | 374/8 | s0/80/10 | $250 / 13 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m} \cdot 000}$ | .. | 395 |
| 7-000 | 0.400 | 7-400 | 16625 | 356/8 | 80/80/10 | $\left\{\begin{array}{c}250 / 11 \mathrm{~s}^{\mathrm{r}} \\ \text { the entire length. }\end{array}\right\}$ | $0 / 11 \mathrm{~s}^{\mathrm{r}} 4^{\mathrm{m}} \cdot 800$ | 565 |
| ', | $0 \cdot 500$ | ", | ' '' | 476/8 | 80/80/10 | $240 / 12 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 400$ | .. | 455 |
| 8.000 | $0 \cdot 500$ | 8.400 | 20000 | 476/8 | 100/100/12 | $220 / 12 \mathrm{~s}^{\mathrm{r}} 5^{\mathrm{m}} \cdot 500$ | . | 600 |

Table Vill.-Formule of Strains and Weights.

|  | General Case. | Case in which $\alpha=\beta$. |
| :---: | :---: | :---: |
| Strain upon the $n$th strut .. .. .. .. .. .. .. | $\frac{\delta}{2 \sin \cdot \alpha}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2} p^{\prime}\right]$ | $\frac{\delta}{2 \sin . \alpha}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right]$ |
| , $n$th brace.. .. .. .. .. .. .. | $\frac{\delta}{2 \sin . \beta}\left[(2 n-1) p+\frac{(m+n)(m+n-1)}{2 m} p^{\prime}\right]$ |  |
| , $n$th section of upper flange .. .. .. | $(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ | $(m-n+1)(m+n-1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ |
| , $n$th section of lower flange .. .. .. | $\left(p+p^{\prime}\right) \delta\left[\left(m^{2}-n^{2}\right) \frac{\delta}{2 h}+\frac{2 n-1}{2 \operatorname{tg} \cdot \alpha}\right]$ | $\left(m-n+\frac{1}{2}\right)\left(m+n-\frac{1}{2}\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$ |
| Maximum strain in the middle of the upper flange .. | $\frac{m^{2}\left(p+p^{\prime}\right) \delta^{2}}{2 / h} \text { or }\left(p+p^{\prime}\right) \frac{l^{2}}{8 h}$ | $\left(p+p^{\prime}\right) \frac{l^{2}}{8 h}$ |
| , , , lower flange .. | $\frac{\left(p+p^{\prime}\right) \delta}{2}\left[\left(m^{2}-1\right) \frac{\delta}{h}+\frac{1}{\operatorname{tg} \cdot a}\right]$ | $\left(2 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{4 h}$ |
| Total weight of the $2 m$ struts .. .. .. .. .. .. | $\frac{\delta h t \mathrm{~V}}{\sin .^{2} \alpha}\left[m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right]$ | $\frac{\delta h t \mathrm{~V}}{\sin .^{2} \alpha}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ |
| , braces $\quad$.. .. ${ }^{\text {. }}$ | $\frac{\delta h t}{\sin .^{2} \beta}\left(m^{2} p+\frac{7 m^{2}-1}{6} p^{\prime}\right)$ | $\frac{\delta h t}{\sin .^{2} \alpha}\left(m^{2} p+\frac{7 m^{2}-1}{6}-p^{\prime}\right)$ |
| Total weight of the upper flange .. .. .. .. .. | $m\left(p+p^{\prime}\right) \delta^{2} t \mathrm{U}\left[(4 m-1)(m+1) \frac{\delta}{6 h}-\frac{m}{\operatorname{tg} \cdot \alpha}\right]$ | $m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{6 h}$ |
| ," lower flange .. .. .. .. .. | $m\left(p+p^{\prime}\right) \delta^{2} t \mathrm{U}\left[(4 m+1)(m-1) \frac{\delta}{6 h}+\frac{m}{t g \cdot \alpha}\right]$ |  |
| ,, two flanges together .. .. .. | $m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{3 h}$ | $m\left(4 m^{2}-1\right)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{3 h}$ |
| 2nd Arrangement (Fig. 1658). $l=(2 m+1) \delta$. <br> Strain upon the $n$th strut .. .. .. .. .. .. .. | $\frac{\delta}{\sin . \alpha}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right]$ | $\frac{\delta}{\sin . \alpha}\left[n p+\frac{(m+n)(m+n+1)}{2(2 m+1)} p^{\prime}\right]$ |
| ,, $n$th brace .. .. .. .. .. .. | $\frac{\delta}{\sin \cdot \beta}\left[n p+\frac{(m+n)}{2(2 m+1)} p^{\prime}\right]$ |  |

Strain upon the $n$th section of upper flange ..

Maximum stiain in the middle of the flanges

braces
Total weight of the upper flange

$(r+n)(m-n+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$
$\left(m^{2}-n^{2}+m+2 n-1\right)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$
$m(m+1)\left(p+p^{\prime}\right) \frac{\delta^{2}}{2 h}$
$\frac{m(m+1) \delta h t \mathrm{~V}}{\sin ^{2} a}\left(p+\frac{7}{6} p^{\prime}\right)$
$\frac{m(m+1) \delta h t}{\sin .^{2} a}\left(p+\frac{7}{6} p^{\prime}\right)$
$m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{3 h}$
$\frac{2}{3} m(m+1)(2 m+1)\left(p+p^{\prime}\right) \frac{\delta^{3} t \mathrm{U}}{h}$

Table IX.a (91).-Girders Lightly Loaded on the Lower Side ( $\left.\frac{h}{1}=0 \cdot 11, \mathrm{U}=1 \cdot 40, \mathrm{~V}=1 \cdot 35\right)$.

|  | $\mathrm{N}=5$. | $\mathrm{N}=6$. | $\mathrm{N}=\%$. | $\mathrm{N}=8$. | $\mathrm{N}=9$. | $\mathrm{N}=10$. | $\mathrm{N}=11$. | $\mathrm{N}=12$. | $\mathrm{N}=13$. | $N=14$. | $N=15$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (0.40 | 0.003440 | 0.003521 | $0 \cdot 003575$ | $0 \cdot 003659$ | $0 \cdot 003723$ | $0 \cdot 003809$ | $0 \cdot 003879$ | $0 \cdot 003966$ | 0.004049 | 0•004064 | $0 \cdot 004045$ |
| $0 \cdot 50$ | $0 \cdot 003453$ | $0 \cdot 003533$ | $0 \cdot 003589$ | $0 \cdot 003673$ | 0.003739 | $0 \cdot 003825$ | 0.003897 | $0 \cdot 003985$ | 0•004069 | 0.004085 | 0.004066 |
| $q=\left\{\begin{array}{l}0.60\end{array}\right.$ | 0.003465 | $0 \cdot 003545$ | $0 \cdot 003602$ | $0 \cdot 003687$ | 0.003755 | 0.003841 | $0 \cdot 003915$ | $0 \cdot 004003$ | $0 \cdot 004090$ | $0 \cdot 004105$ | 0.004086 |
| $0 \cdot 70$ | 0.003477 | $0 \cdot 003557$ | 0.003616 | $0 \cdot 003701$ | $0 \cdot 003770$ | 0.003857 | $0 \cdot 003933$ | $0 \cdot 004022$ | $0 \cdot 004110$ | $0 \cdot 004125$ | 0.004106 |
| (0.80 | $0 \cdot 003489$ | $0 \cdot 003568$ | 0.003630 | $0 \cdot 003715$ | 0003786 | $0 \cdot 003874$ | 0.003951 | 0.004041 | $0 \cdot 004130$ | $0 \cdot 004145$ | $0 \cdot 004127$ |

Table IX. $\operatorname{s}\left(\right.$ Value of $\left.\frac{\mathrm{P}}{\left(p+p^{\prime}\right) l^{2}}\right)$.-Girders Heavily Loaded on the Lower $\operatorname{Side}\left(\frac{h}{1}=\frac{1}{8}, \mathrm{U}=130, \mathrm{~V}=125\right)$.

For Tables X. a, X. b, see page 827.


|  | $\mathrm{N}=5$. | $\mathrm{N}=6$. | $\mathrm{N}=7$. | $\mathrm{N}=8$. | $\mathrm{N}=9$. | $\mathrm{N}=10$ 。 | $\mathrm{N}=11$. | $\mathrm{N}=12$. | $\mathrm{N}=13$. | $\mathrm{N}=14$. | $\mathrm{N}=15$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.40 | $0 \cdot 003116$ | 0.003148 | 0.003154 | $0 \cdot 003171$ | $0 \cdot 003178$ | 0.003192 | 0.003200 | 0.003212 | 0.003221 | 0.003233 | $0 \cdot 003243$ |
| $0 \cdot 50$ | $0 \cdot 003165$ | $0 \cdot 003186$ | 0.003201 | $0 \cdot 003213$ | $0 \cdot 003226$ | - $0 \cdot 003238$ | $0 \cdot 003251$ | $0 \cdot 003263$ | $0 \cdot 003277$ | $0 \cdot 003289$ | 0.003303 |
| $0 \cdot 60$ | $0 \cdot 003215$ | $0 \cdot 003224$ | 0.003248 | 0.003255 | $0 \cdot 003275$ | $0 \cdot 003284$ | $0 \cdot 003303$ | 0.003314 | $0 \cdot 003332$ | $0 \cdot 003345$ | $0 \cdot 003364$ |
| $0 \cdot 70$ | 0.003265 | $0 \cdot 003262$ | 0.003295 | 0.003297 | 0.003323 | $0 \cdot 003330$ | $0 \cdot 003354$ | $0 \cdot 003364$ | 0.003388 | $0 \cdot 003400$ | 0•003424 |
| 0.80 | $0 \cdot 003314$ | $0 \cdot 003300$ | 0.003343 | $0 \cdot 003338$ | $0 \cdot 003372$ | 0.003376 | $0 \cdot 003406$ | $0 \cdot 003415$ | $0 \cdot 003443$ | 0.003456 | 0.003484 |

Table XIJ.-Weight of the Lineal Mètre of Railway Bridges under Two Lines of Rails.
Ètre of Railway Bridges under Two Lines of Rails.
(See 181-191.)

## BRIDGE。



Case of Limited Height (Girders Loaded on the Lower Side, or the Tubulai System).
Table XII. c.-Four-girder Bridges.


## Case of Unlimited Height (Girders under Rails).

Table XII. d.-Three-girder Bridges. Table XII. e.-Two-girder Bridges.

|  |  | $\begin{gathered} \text { Height } \\ \text { of the } \\ \text { Girders. } \end{gathered}$ | Weight the Mètre of Total Length |  |  |  | Total the Lineal Metre of Bearing | Height of the Girders. | Weight the Mètre of Total Length |  |  | Total the Lineal Mètre of Bearing. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Of the Two Outside Girders. | Of the Centre Girder | Oî the other Pieces. | Total with the addition of $\frac{1}{10}$. |  |  | Of the Two Girders | Of the other Pieces. | Total with the addition of $\frac{1}{10}$. |  |
| Solid <br> girders. | $\left\{\begin{array}{r}l= \\ 20 \\ 30 \\ 40 \\ 40 \\ 40 \\ \text { arrange- } \\ \text { ment. } \\ \text { First } \\ \text { Second } \\ \text { arrange- } \\ \text { ment. }\end{array}\right\}$ | mètres. | kilos. | kilos. | kilos. | kilos. | kilos. | mètres. | kilos. | kilos. | kilos. | kilos. |
|  |  | - 80 | 860 | 350 | 500 | 1880 | 1970 | $2 \cdot 15$ | 980 | 550 | 1680 | 1780 |
|  |  | - 40 | 1430 | 570 | 530 | 2780 | 2910 | $2 \cdot 70$ | 1600 | 580 | 2400 | 2500 |
|  |  | $3 \cdot 00$ | 2000 | 800 | 560 | 3700 | 3860 | $3 \cdot 40$ | 2260 | 630 | 3180 | 3320 |
|  |  | $3 \cdot 00$ | 1860 | 950 | 850 | 4030 | 4210 | $3 \cdot 40$ | 2200 | 780 | 3280 | 3430 |
|  |  | $3 \cdot 60$ | 2500 | 1250 | 900 | 5120 | 5350 | 4.00 | 2900 | 800 | 4070 | 4250 |
| Multiple latticegirders without vertical rods. |  | $2 \cdot 20$ | 740 | 280 | 520 | 1700 | 1770 | $2 \cdot 40$ | 870 | 560 | 1570 | 1640 |
|  |  | $3 \cdot 30$ | 1100 | 400 | 580 | 2290 | 2390 | $3 \cdot 60$ | 1270 | 630 | 2090 | 2180 |
|  |  | $4 \cdot 40$ | 1440 | 530 | 650 | 2880 | 3010 | $4 \cdot 80$ | 1680 | 700 | 2620 | 2740 |
|  |  | $4 \cdot 40$ | 1300 | 630 | 850 | 3060 | 3200 | 4.70 | 1600 | 830 | 2670 | 2800 |
|  |  | $5 \cdot 50$ | 1660 | 800 | 900 | 3700 | 3860 | $6 \cdot 00$ | 2000 | 880 | 3170 | 3310 |
|  |  | $6 \cdot 70$ | 2020 | 960 | 960 | 4330 | 4530 | $7 \cdot 00$ | 2470 | 920 | 3730 | 3900 |
|  |  |  |  |  | .. | .. | .. | $8 \cdot 50$ | 3060 | 980 | 4440 | 4640 |
|  |  |  |  |  |  |  |  | 9-50 | 3660 | 1020 | 5150 | 5380 |
| Triangular latticegirders. | $\left\{\begin{array}{c}l=20 \\ 30\end{array} \begin{array}{c}\text { First } \\ 40 \\ \text { arrange- } \\ 40 \\ 40 \\ 50\end{array}\right\}$ | $2 \cdot 20$ | 650 | 230 | 520 | 1540 | 1610 | $2 \cdot 40$ | 780 | 560 | 1470 | 1540 |
|  |  | $3 \cdot 30$ | 980 | 350 | 580 | 2100 | 2200 | $3 \cdot 60$ | 1160 | 630 | 1970 | 2060 |
|  |  | $4 \cdot 40$ | 1300 | 460 | 650 | 2650 | 2770 | $4 \cdot 80$ | 1540 | 700 | 2460 | 2580 |
|  |  | $4 \cdot 40$ | 1180 | 560 | 850 | 2850 | 2980 | $4 \cdot 70$ | 1450 | 830 | 2510 | 2620 |
|  |  | $5 \cdot 50$ | 1500 | 710 | 900 | 3420 | 3570 | $6 \cdot 00$ | 1870 | 880 | 3030 | 3160 |
|  |  | $6 \cdot 70$ | 1820 | S60 | 960 | 4000 | 4180 | $7 \cdot 00$ | 2280 | 920 | 3520 | 3680 |
|  |  |  |  |  |  |  |  | $8 \cdot 50$ | 2900 | 980 | 4270 | $44 \subset 0$ |
|  |  |  |  |  |  |  |  | $9 \cdot 50$ | 3500 | 1020 | 4970 | 5200 |
| CrossCattice-latergirderswithoutverticalrods. $\left\{\begin{array}{r}l=20 \\ 30 \\ 40\end{array} \quad \begin{array}{c}\text { First } \\ \text { arrange- } \\ \text { ment. }\end{array}\right\}$ |  | $2 \cdot 20$ | 640 | 230 | 520 | 1530 | 1600 | $2 \cdot 40$ | 750 | 560 | 1440 | 1500 |
|  |  | $3 \cdot 30$ | 940 | 340 | 580 | 2050 | 2140 | $3 \cdot 60$ | 1120 | 630 | 1930 | 2010 |
|  |  | $4 \cdot 40$ | 1240 | 440 | 650 | 2560 | 2630 | $4 \cdot 80$ | 1480 | 700 | 2400 | 2510 |
|  |  | $4 \cdot 40$ | 1120 | 540 | 850 | 2760 | 2880 | 4-70 | 1400 | 830 | 2450 | 2560 |
|  |  | 5. 50 | 1430 | 680 | 900 | 3310 | 3460 | $6 \cdot 00$ | 1800 | 880 | 2950 | 3080 |
|  |  | $6 \cdot 70$ | 1740 | 8:0 | 960 | 3870 | 4050 | 7•00 | 2200 | 920 | 3430 | 3590 |
|  |  |  |  |  |  |  | .. | 850 | 2700 | 980 | 4050 | 4230 |
|  |  |  |  |  | - |  | - | $9 \cdot 50$ | 3200 | 1020 | 4640 | 4359 |

A. -Roads of $6^{\mathrm{m}}$.


BUDDLE.
See Aqueduct. Arch. Drawbridge. Embankments. Materials of Construction, strength of. Oblique Arch. Piers. Pontoon. Retaining Walls. Viaduct. Weighbridge. Works on Bridges:-Perronet, 'Guvres,' folio and 4to, 1788. Gauthey, 'Traité des Ponts,' 3 vols., 4to, 1816. Ware, 'On Vaults and Bridges,' 8vo, 1822. Hann and Hoskin's 'Theory, Practice, and Architecture of Bridges,' 5 vols., royal 8vo, 1843-50. E. Clark, ' On the Britannia and Conway Bridges,' 2 vols., 8vo, plates, folio, 1850. Bow, 'On Bracing,' 8vo, 1851. Boudsot, 'Des Ponts Suspendus,' 4to, Paris, 1853. Humber, 'On Iron Bridges and Girders,' imperial 4to, 1857. Roy, 'Des Ponts et Viaducts en Maçonnerie,' 8vo, 1857. J. H. Latham, 'On the Construction of Iron Bridges,' 8vo, 1858. L. von Klein, 'Sammlung eiserner Brücken constructionen,' Stuttgart, 1863. Humber, ' On Iron Bridge Construction,' 2 vols., imperial 4to, 1864. J. Gaudard, ' De Divers Systèmes de Ponts en Fer', 8vo, Paris, E. Lacroix, 1865. Haupt's 'Theory of Bridge Construction,' 8vo, New York, 1865. Stoney, 'On Strains in Girders,' 2 vols., royal 8vo, 1866-69. W. Fränkel, ' Berechnung eiserner Bogenbrücken', Hanover, 1867. Shield's 'Theory of Strains in Girders,' royal 8vo, 1867. Baker, 'On Long-Span Railway Bridges,' 12mo, 1867. Unwin, 'On Wrought-Iron Bridges and Roofs,' $8 v o, 1869$. J. A. Roebling, 'Long and Short Span Railway Bridges,' folio, Van Nostrand, New York, 1869. Humber, 'Strains in Girders,' 12mo, 1869. Bree's 'Railway Practice,' 4 vols., 4to. Byrne's 'Essential Elements of Practical Mechanics.' Seguin Ainé, 'Des Ponts en Fil de Fer.' Also various Papers in 'Annales des Ponts et Chaussées,' and the 'Transactions Inst. C. E.,' the 'Inst. Mechanical Engineers,' 'Society of Engineers.'

BRIDGE-HEAD. Fr., Tête de pont; Ger., Brückenkopf; Ital., Testa di ponte, Testata; Span., Cabeza del puento.

See Fortification.
BRITANNIA-METAL. Fr., Métal britannique, Métal anglais • Ger., Britannia Metall; Ital., Lega Britannia.

See Alloys, p. 49.
BRONZE. Fr., Bronze; Ger., Bronze ; Ital., Bronzo; Span., Bronce.
See Aluminum. Bronze. Copper. Tin. Zinc.
BUDDLE. Fr., Baquet à rincer, lavoir; Ger., Waschtroy, Waschwerk. Ital., Lavatoio della miniera; Span., Mesa de larar.

The apparatus shown in Figs. 1802, 1803, was invented by Hundt, a Prussian engineer. Its introduction in the gold mines of Victoria has been attended with success. The arrangement of this buddle possesses the advantage of affording a large working area at the head, and at the same time effects a better separation of the waste than can be produced by round buddles of the ordinary construction. And when the lighter portions of the tailings have become separated from the heavier near the periphery of the circle, the area over which they are distributed gradually diminishes, which, by increasing the rapidity of the flow, enables them to be more readily carried off. In Australia this apparatus is employed for concentrating tailings from which a large proportion of gold has been previously extracted by the usual appliances. A is the spout which conducts the mixture of water and sand to the buddle: B, the outlet for carrying off the earthy impurities or final tailings; C, shaft communicating motion to the buddle arms $d$; $e$, distributing launders; and $g^{\prime}$, pipes attached to them. The pipe $f$ supplies clean water to the annular cistern $g$, from whence it passes by the pipes $g^{\prime}$, with rose apertures at the ends, and serves to dilute the mixture of water and tailings discharged by the launders
 $e$, on the annular incline at the periphery. The whole of this arrangement revolves on the shaft D . The final tailings fall
into the circular pit $i$, previous to being carried off by the channel B. To the wooden bars $k$ are attached pieces of canvas, which sweep over the surface of the stuff deposited in the buddle, and keep it even and free from ruts. The tailings, entering the receiver $h$, are distributed at the periphery of the buddle through the four launders $e$, which at their extremities are turned at right angles to the direction of their motion when in action; and at the same time clean water is distributed by the apertures pierced in the terminations of the pipes $g^{\prime}$. The speed given to this arrangement of arms, launders, and pipes, revolving on the shaft $D$, varies in proportion to the state of divisions of the sands to be treated. When these are rather coarse, the machine may make from six to eight revolutions a minute ; but when very fine stuff has to be operated upon, the speed is considerably increased. The influx of tailings and water must be regulated in accordance with the speed of the arms and the density of the material operated upon; and although no very definite instructions can be given with regard to this subject, a chart of the apparatus will enable any intelligent workman to make the necessary adjustments. The bed may be from 12 to 18 ft . in diameter, and it may have an inclination of from 6 to 9 in . from the edge to the centre. At the Port Phillip works the tailings cleaned by machines of this description are subsequently roasted, and passed through Chilian mills; but where, as in California, the enriched pyrites have to be transported to considerable distances, it would require to be more than once passed through the machine, or, after being once buddled, the heads might be further enriched, either in the rocker, or by a hand-buddle or shaking-table. A buddle of this kind can be filled in about four hours, and it will be found an excellent apparatus for enriching ores with but little waste. If it be intended to dress sulphides directly from the riffes, so as to render them almost entirely free from siliceous matter, the first heads will require to be re-washed once at least, and the second heads twice; but when this is done, it is necessary to be provided with other buddles besides those which first receive the tailings direct from the riffles, and which will be constantly in use for that purpose.

When the tailings to be washed are not conducted directly from the blanket boards, but are taken either from tyes or the heads of other buddles, they are charged with a shovel into a hopper connected with a circular sieve, working in water, which discharges into the spout A.

See Gold and Gold Mining.
BUFFER. Fr., Tampon, Appareil de choc; Ger., Buffer; Ital., Respingente.
A buffer is a cushion, or apparatus with strong springs, employed to deaden the concussion between a moving body and one, in motion or at rest, on which it strikes, as at the end of a railway carriage. A buffer is sometimes called a buffing apparatus, when it is composed of two or more parts or of two or more springy substances, as of steel, india-rubber, gutta-percha.

The central buffing and drawing apparatus, designed by E. D. Chattaway. is shown in Figs.
to 1806 . 1804 to 1806.
1804.

Side clevation.


Fig. 1804 is a side clevation, and Fig. - 805 a plan of the new coupling, working in connection with an ordinary draw-hook; and Fig. 1806 is an end view of the buffer-head. The buffer and draw-hook are constructed in one piece, and the buffer-head A, instead of being circular, is made of the peculiar form shown in Fig. 1806. The lower portion only is curvilinear, and the upper part is made narrow, and while presenting a buffing surface in front, it is shaped on its inner side similar to a draw-hook, for the purpose of receiving the coupling-link B, which works upon it as upon an ordinary draw-hook, and thus forms a complete hook and link connection. The draw-rod C is screwed near the end just within the buffer-head. Upon this screwed part is fitted the
adjusting nut and collar D with projection arms E, Fig. 1805, carrying the large coupling-link B. The coupling can thus be drawn hard up or slackened off by means of the pendulous lever $\mathbf{F}$.

The ordinary coupling and buffing arrangements as applied to railway wagons are open to several objections. As there is no means of tightening up the couplings, there is always a considerable amount of play between the adjacent vehicles of a train, frequently as much as 18 in . with wagons. The effect of this is, in the first place, to lengthen the train, thereby causing a greater expenditure of motive power, owing to increased effect of the wind and pressure of the flanges of the wheels against the rails, when the train is running through sharp curves; and, in the second place, to render the couplings liable to break or to become detached from the draw-hooks, in consequence of the train being started and stopped in detail by a series of sudden and violent jerks: a jerking action also causes a severe strain upon the draw-springs and often breaks them; besides, it acts injuriously upon the permanent way, and increases the risk of conveying goods of a fragile nature. As regards corner buffers, they are objectionable from the circumstance of the two buffers being seldom struck at the same instant or with the same degree of force, even on a perfectly straight line; while on curves the buffing action is mainly confined to one side of the train, severely straining the framing of the vehicles, and having a tendency to force the tenons out of the mortices. Besides tending to damage the rolling-stock, this action also tends to injure the permanent way, and to increase the cost of maintenance of both. It is also to be remarked that the ordinary buffers and couplings are always opposed to instead of being in harmony with each other, the couplings being slackened as the buffers are brought into play, and vice versâ ; and the reaction of the buffers has sometimes fractured the couplings.

A further objection to the ordinary couplings is the difficulty and delay frequently experienced in attaching and detaching wagons, since, unless the couplings be slack, it requires a violent shunting together of the train to allow the coupling-link to be slipped over the draw-hook; and if the draw-hooks be close to each other before the coupling has been lifted up, the train has to be drawn apart to admit of the link being raised, and then shunted together again in order to be attached. Independent of extra expenditure of engine-power and wear and tear of rolling-stock, this shunting is also a source of accident.

The most complete and effective buffing apparatus now in use is that of L. Sterne, shown in Figs. 1807 to 1811.

The buffers or springs, Figs. 1807, 1808, are built up of soft india-rubber rings and circular steel plates; the terminal plates of each of the springs being complete discs, some are formed with a hole through the centre. During the process of vulcanization the buffer-rings become inseparably united with the plates, and each spring thus becomes a perfectly air-tight chamber. When the pressure is brought to bear on the springs, the air within them becomes compressed, and offers a resistance proportionate to the amount of such compression and to the area of the dises upon which the

pressure of the air is exerted. The india-rubber rings not only perform the office of securely retaining the air within them, but they completely resist sudden concussions.

In order to avoid friction upon the rubber, by which it would be quickly destroyed, the circular plates are extended beyond the rings to a distance proportioned to the depth of these rubber rings. The india-rubber, therefore, cannot be injured, or come in contact with the inner surface of the cylindrical cover.

These buffers can be made of any desirable form, and airtight chambers, constructed in the manner just described, can be built up to any length required in practice. It will be seen that no mechanical fittings are required to make a perfectly air-tight chamber. The union of the rubber to the metal being of a solid and not merely adhesive character, the juncture is impervious.

These buffers are capable of resisting great compressive force, while at the same time they possess great sensitiveness as springs. Unlike a steel spring, which, if made to resist great force, is rigid until a considerable weight is brought to bear upon it, this bufferspring is sensitive to a very slight pressure, and yet absorbs a power of 15 tons, being 100 per cent. more than the resisting power of any steel springs used for this purpose. In most cases, where the action of buffers is required, the apparatus of Sterne, without
 alteration of principle, may be applied.

Fig. 1807 shows a cast-iron case, with a solid wrought-iron plunger, four rubber rings, and five steel plates.

Fig. 1808 is of a cast-iron case, containing a cast-iron cylinder-nlunger, pneumatic spring power, five rubber rings, and six steel plates.

Fig. 1809 represents a pneumatic rubber draw-spring. It has four rubber rings and three oval steel plates.

Sterne's buffer is in use on many railways. The arrangement, Fig. 1810, is that adapted to the carriages of the Metropolitan Railway.


Each buffer-rod carries a kind of shoe embracing the ends of a compensating travelling beam, which extends from one spindle to the other. This transmits the pressure fairly to the springs, at the same time enabling the buffer-rods to accommodate themselves freely to the position of the carriages passing round curves. The buffing spring occupies a length of $13 \frac{1}{2} \mathrm{in}$. when uncompressed, and is capable of being compressed 9 in . The light spiral springs at the end of the bufferspindles are merely placed there to prevent chattering. The draw-spring is formed by a pair of pneumatic springs composed of four rings of rubber each, inseparably united to three oval steel plates through which the draw-bar, Figs. 1810, 1811, passes. See Locomotives.

BUILDING. Fr., Construction, bâtiment ; Ger., Bauen ; Ital., Costruzione.
See Bond. Bridge. Focndations. Joints. Masonty. Railway Engineering. Roofs. Scaffolding.

BULLET-MAKING MACHINE. Fr., Machine à faire des balles; Ger., Kugel-Giessmachine.
In the manufacture of ordinary bullets for small arms the lead is first melted in an ordinary cast-iron vessel, fitted with a spout and movable plug at the bottom. By means of a removable spout, the liquid lead is conveyed to the cylinder A in Fig. 1813, where it remains until it is a solid, but not any longer, as the nearer it is to the fluid state, the easier it is shaped, so as to form a rod. The lead, being pushed up by hydraulic pressure, as in the lead-pipe machine of Cornell, flows out at the orifice in the die $B$, which may be made with different shaped orifices.

To convert the rod into bullets, it has first to be cut into portions, then each portion has to be compressed in ventilated dies, care being taken to prevent metallic contact, which is done by lubrication.


Such conditions may be attained by a great varicty of mechanical arrangements : Figs. 1811,1814, illustrate John Anderson's Bullet-making Machine, which was extensively employed at Woolwich

By referring to Fig. 1812, it may be seen that the rod of lead from the lead-pipe or lead-rod machine, after being coiled upon a portable reel, is wound upon the reels $\mathrm{C}, \mathrm{C}$, which are fixed upon the framing of the bullet machine, Fig. 1812.

The lead rod is then conducted by a pair of small rollers D, through the cutting-off lever $\mathbf{E}$, into the clips F, Fig. 1814, in which it is gripped while being sheared off; when the lever is raised up to perform this operation by the cam (G Fig. 1812) the length of lead rod thus cut off is determined by the adjustable stop.

To prevent the lead rod from slipping out of the die, it must be thoroughly clean and free from grease; it is therefore passed through a roll of tow or cotton waste I saturated with turpentine.

After the block of lead of the required length is cut off, the clips E, F, Figs. 1812, 1814, are opened, and the block is allowed to fall by gravitation to the bottom of the clips, and from thence is pushed into the forming-die J, Fig. 1812, by the punch K (the point of which is lubricated at each stroke); the finished bullet is then knocked out by the small punch L, into a shoot, by which it is conveyed to suitable boxes.

In order to ensure a clean and perfectly-formed bullet, it is necessary to ventilate the die; in this case the air escapes through the small punch L , which is bored out for that purpose.

When the portion of lead within the die has been compressed, the superfluous metal is squeezed out at the junction, thus forming a sort of frill, to remove which clips, Figs. 1812, 1814, are employed. These clips are pieces of steel containing a hole the size of the bullet; they rise in front of the die so soon as the punch has retired; at the next instant the bullet is pushed through the hole, thus leaving the frill behind.

When indentures have to be given to bullets, this is accomplished by passing them between a revolving disc and a corresponding fixed segment, the acting surfaces being of the required shape, and so placed with regard to each other that the entrance of the indenting channel shall be wider than the opening at the

1813.


point of exit. The revolving surface imparts a circular motion to the bullets, in a manner similar to that of milling coin.
J. D. Custar's Bullet Machine, Fig. 1815, has an arrangement of devices for cutting off the blank from a rod or thick wire, and depositing it in a radial groove $u$ in the flat face of a turn-
table which carries it in front of a cylindrical die. A piston or male die fitting the said cylinder forces the blank therein and against a female die. When the male die is retracted, and the female die thrust forward, it forces the blank out of the cylinder and upon the carrier S, which during the operation of the dies remains at rest. The bullet is then carried around the lathe, where it is again seized between the sliding mandrel and centre of the lathe, and trimmed off to the proper shape by a tool which is fed to the work by means of a lever operated by a cam attached to the driving machinery $\mathbf{C}$.

BUNG-CUTTING MACHINE. Fr., Machine à couper le tampon de bois; Ger., SpundzapfenSchneidmaschine ; Ital., Macchina da cocchiumi.

See Cask-making Machinery.
BUOY. Fr., Bouée, Amarque, Balise; Ger., Buje, Boje, Ankerboje; Ital., Gavitello; Span., Boya.

See Lighthouses; Beacons; and Buoys.
BURNISHER. Fr., Brunissoir; Ger., Polirstahl; Ital., Brunitoio.
See Hand-Tools.
BUSH. Fr., Coussinet ; Ger., Metallfutter ; Ital., Bronzina; Span., Coginete.
A bush is a perforated piece of metal, as hard brass, let into certain parts of machinery, to receive the wear of pivots, journals, and the like, as in the pivot-holes of a clock, the hub of a cart-wheel, and so on. Any similar lining of a hole with metal, as the vent of a gun, is termed the bush. See Details of Engines; Ordnance; and Small Arms.

BUTTERFLY-VALVE. Fr., Robinet papillon; Ger., Schmetterlings-Ventil; Ital., Valvola a farfalla.

See Valves.
BUTTRESS. Fr., Contre-fort, éperon, arc boutant; Ger., Strebepfeiler; Ital., Contraforte; Span., Ó machon.

A buttress is a projecting support to the exterior of a wall, most commonly applied to churches in the Gothic style, but also to other buildings, and sometimes to mere walls. A prop, a shore, or support, is also called a buttress. See Coast Defences. Construction. Fortification.

BUTT - WELD. Fr., Soudure par rapprochement; Ger., Stumpfe Schweissfuge; Ital., Bollitura affrontata.

See Forging.
Cable. Fr., Câble; Ger., Tau; Ital., Canapo, Cavo; Span., Cable.
A cable is a large, strong rope or chain, employed to retain a vessel at anchor, and for other purposes. It is commonly made of hemp or iron, but sometimes of iron or copper wire, as in the case of the cable of a suspension bridge, or of a submarine connection in the electrical telegraph. Practical information, however limited, upon the strength of rope and chain, is of great value and importance to the seaman, since there is scarcely an operation connected with his duties that can be performed without the use of a chain or a rope.

William Macdonald, Superintendent of the Liverpool Chain-testing Works, in 1860, before a committee of the House of Commons, gave the following tabulated statements, which show the total number of fathoms of chain proved by him from 1855 to 1859.

Stid Chain.


Close Link.

| Date. |  |  | Total Number <br> of <br> Fathoms proved. | Number of Fathoms defective. | Number of Fathoms not defective. | Number of Fathoms condemned. | Percentage <br> of defective <br> Chains. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1855 |  |  | 3799 | 1918 | 1881 | 278 | $50 \frac{1}{2}$ |
| 1856 |  |  | 8752 | 4636 | 4116 | 37 | $52 \frac{3}{4}$ |
| 1857 .. |  |  | 10190 | 7398 | 2792 | 955 | 72 |
| 1858 .. |  |  | 8731 | 5469 | 3262 | 44 | $62 \frac{1}{2}$ |
| 1859 (to August 2nd) |  |  | 5412 | 2066 | 3346 | 227 | 38 |
| Total | .. |  | 36884 | 21487 | 15397 | 1541 | $58 \frac{1}{2}$ |

Deductions and rules respecting the strength of materials, based on the experiments of Tredgold, Barlow, Fairbairn, and Hodgkinson, are not reliable, for these and other experimentalists
employed incomplete testing apparatus; and in general they operated upon fragments of materials too small to warrant their conclusions with regard to large and complicated structures. This remark does not apply to David Kirkaldy, who, from his skill and the accuracy and great range of his testing machinery, furnishes results in every respect reliable. We add a tabulated statement respecting the relative strength of welded joints, taken from Kirkaldy's work, 'Results of an Experimental Inquiry into the Tensile Strength and other Properties of various kinds of Wrought Iron and Steel.'

In the experiments made by Kirkaldy to ascertain the strength of welded joints, the pieces were cut through the middle, and then scarfed and welded in the ordinary manner by the same smith who prepared the bolts for his other experiments, and a few were prepared by a chainmaker, for comparison. The results varied greatly,-fourteen, operated on by the smith, show a loss, compared with the original whole bar, from $4 \cdot 1$ to $43 \cdot 8$ per cent., the mean loss being $20 \cdot 8$ per cent.; four by the chain-maker, from $2 \cdot 6$ to $37 \cdot 4$; mean, $15 \cdot 1$ per cent. Of the former, four broke solid, away from the weld : eight, partly through solid portion and partly at the weld; two separated at the weld. Of the latter, two broke solid; one broke partly solid and partly at the weld; and one gave way at the weld. It may be noticed that different sizes and also different qualities were used, but the results were alike uncertain, and are further proof of the correctness of the following observations by James Nasmyth, which appeared in 'The Engineer,' 8th March, 1861, a considerable time after the experiments of Kirkaldy were made :-
"Of all the processes connected with the working of malleable iron there is none that has a more intimate relation to the security of life and property than that of welding, or the process by which we are enabled to unite together, in one mass, the several portions of malleable iron, of which the generality of works in that material are formed. Every single link in a chain cable, every wheel-tire in a railway train, directly owes its trustworthiness to the manner in which the process of welding has been performed, in so far as that any imperfection in any one single member of the set of cable links or railway wheel-tires may involve a most fearful loss of life, of which, of late years especially, we have had such distressing and melancholy experience.
"In order to render clear the following remarks as to the cause of, and means that should be employed to prevent, defective welding, it is necessary to explain the nature of the process of welding, which consists in inducing upon malleable iron, by means of a very high heat, a certain degree of adhesiveness, so that any two pieces of malleable iron, when heated to the requisite degree, will, if brought into close contact, adhere or stick together with a greater or less tenacity, according to the amount of force applied to urge them into close contact.
"It is, therefore, to the means of thoroughly expelling this vitrified oxide from between the surfaces of the iron where the welded junction is to take place that we must direct our attention; for so long as any portion of this adhesive viscid substance is permitted to exist and interpose itself between the surfaces we desire to unite, no sound junction can take place, and once it has made a lodgment no after-heating or hammering, be it ever so severe, will cause its thorough expulsion. It is, therefore, to the thorough expulsion of the vitreous oxide in the first stage of the welding that we must direct the most careful attention; and it would, in no small degree, tend to prevent those fearful accidents of which defectively-welded ironwork is the principal cause.
"Since the chief cause of defective welding arises from portions of the vitreous oxide of the iron being shut up between the surfaces at the part presumed to have been welded, and that, besides the impossibility of ascertaining, in the majority of cases, after the process of welding has been gone through, whether or not this vitreous oxide has been thoroughly expelled, and the surfaces at the welding brought into perfect metallic union, and that no after-heating or hammering can dislodge the vitreous oxide when once it has effected a lodgment, our best security is to form the surfaces of the iron at the part where the welding is desired to take place, so that when applied to each other, at the welding heat, their first contact with each other shall be in the centre of each."

Kirkaldy tested two steel bars welded at the makers' works, the result still more unsatisfactory than with iron-one showing a loss of $45 \cdot 0$, the other $59 \cdot 6$ per cent.; whilst other two parted at the weld during the operation of forming the heads, previous to testing. At the commencement of these experiments several attempts were made to form heads on the steel specimens, in the same manner as on the iron, by welding on rings; but as they either failed at the weld, or the steel was found to be burned, that method was abandoned.

With the view of ascertaining the effect of heating iron to the welding point, and then allowing it to cool slowly without being hammered, an experiment was made by Kirkaldy with a bar of Glasgow B. Best, the results of which are stated in his table N, 1095. Although its breaking strain was nearly the same as that borne by another piece, 1094, off the same bar, in the ordinary condition, yet it will be found that the ductility of the iron had been injured by the high temperature and want of hammering; for it was found that instead of 42.2 per cent. it only contracted $27 \cdot 8$, or 34 per cent. less than the latter.

Figs. 1816 to 1829 exhibit mooring and other chains, with the articles required to be appenced to such chains, and the proportions of iron chain cables, as supplied to the English Royal Navy.

Fig. 1816 shows the pin and lead pellet, through forelock, and part of the ring or shackle on the anchor-shank. Figs. 1816 to 1821 present two views of each part of a cable where connections are made.

The dimensions given on the links, \&c., signify so many diameters of the iron of the common links of the cable, thus forming the scale for all sizes. In the mooring chains and gear, the unit is the thickness of the iron of the link.

Referring to Figs. 1816 to 1821, and large shackle for connecting any length of cable of $12 \frac{1}{2}$ fathoms: B, end links, without stay-pins; C, enlarged links, with stay-pins; D, common links, hickness put $=1 ; E$, swivel in the middle of every other length of $12 \frac{1}{2}$ fathoms; $F$, joining hackle for connecting either end of any length with any other length of the same size.

Welded

| $\begin{aligned} & \text { Index } \\ & \text { No. } \end{aligned}$ | Names of Makers or Works, and how Treated. | Original |  | Weight on Steelyard. | Breaking Strain. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Diam. | Area. |  | Total on Specimens. | Welded. |
|  |  | in. | sq. in. | Ibs. | lbs. | ${ }^{\text {lbs. }}$ |
| 1491 | Govan B. Best. Welded by smith | $\cdot 76$ | -4536 | 590 | 25590 | 56415 ) |
| 1492 | " <br> , | -76 | - 4536 | 569 | 2500222797 | 5511950963 |
| 1493 |  | -75 | - 4417 | 407 | $20466\}^{22797}$ | $46335{ }^{\text {a }}$ |
| 1494 |  | -76 | - 4536 | 421 | 20858 ) | 45983 |
| 1495 | $\left\{\begin{array}{cccc} \text { Govan Ex. B. Best." Welded by } \\ \text { chain-maker } & \text {.. } & \text {.. } & \text {.. } \end{array}\right\}$ | - 76 | - 4536 | 605 | 26010 ( | 57341 |
| 1496 | " <br> " <br> " <br> " | - 76 | - 4536 | 598 | 25814 21411 | 56909 48984 |
| 1497 | $" \quad " \quad " \quad "$ | -76 | -4536 | 420 | 20830 | 45921 |
| 1498 | $" \quad " \quad \text { " } \quad \text { " }$ | -68 | -3632 | 140 | 12990 | 35765 |

1816. 


$x y=$ pin and lead pellet through the bolt and forelocs.

1819.

Figs. 1822, 1823, George Elliott's splicing shackle.
Figs. 1824, 1825, Hardy's mooring swivel; 1826, splicing tails; 1827, 1828, swivel and shackle for mooring chain; and 1829 , links for pendant, or bridle chains.

Before 1842 no positive information on the strength of hempen cables was established. However, some experiments on the smaller kinds had boen made, and the strength of a good $10 \frac{1}{2}-\mathrm{in}$.

Joints.

cable or $10-\mathrm{in}$. hawser-laid rope was considered to be about 20 tons; these formed a scale by which the strength of other sizes was calculated, but there was no satisfactory evidence in proof of this scale with regard to any size. It was therefore ordered that a series of experiments should be made
 in Woolwich Dockyard to ascertain by positive proof the strength of all cables, both hempen and chain, of various sizes, from the smaller kind up to the largest then in use. The experiments were accurately recorded by Nicholas Tinmouth; but the testing machine used by Tinmouth was far inferior to the one employed at present by David Kirkaldy. The sizes of hemp selected for experiment, Table I., were 25 in . circumference, $22 \frac{1}{2}$ in., 20 in ., and so on. The sizes of the chain cables were, as shown in Table II., from $2 \frac{1}{8}$ to $\frac{5}{8}$ of an inch.

It is necessary to state that the $2 \frac{1}{8}$-in. chain. was not tested, on account of the probability that the testing machine would be seriously injured in the event of this chain breaking under a heavy strain. There were six experiments upon each of the other nine sizes of chain cable, amounting in the whole to fifty-four; and eight experiments upon each of the nine hemp cables, making together seventy-


These experiments are easily distinguished in the Table by their extending through all the columns from side to side. The strength of all the intermediate sizes, both of hemp and chain, was calculated by the following rule:-Divide the difference of the two strains by the difference of the squares of the diameters, or circumference, for a constant multiplier. This multiplier into the difference of the squares of any two sizes will give the number of tons to be added to the strain upon the smaller. As the actual strength of any instrument can only, as a whole, be considered equal to its weakest part, the minimum column of strength in the Tables must be considered the safest. The right-hand column has been calculated from the weakest of all the experiments, and may be serviceable where risk is apprehended and great caution necessary. With reference to the strength of the hawser-laid rope, and rigging or crane chain, in Tables III. and IV., the experiments were made at various times for isolated purposes; and although not in the same progressive order as in the case of the cables, yet the strength was ascertained by the same machine with the same degree of accuracy and embracing a sufficiently numerous class of sizes to render the calculation of all intermediate kinds equally correct.


Table I.-For Ascertaining the Strength of Chain Cables.

| Size. | No. of Yarns. | Weight, Fathoms. | Breaking Weight, in Tons. |  |  |  |  |  |  |  | Mean. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Maximum. | Intermediate Strains. |  |  |  |  |  | Minimum. |  |  |
| $\frac{\mathrm{in}}{26}$ | 3528 | $\begin{gathered} \text { lbs. } \\ 14112 \end{gathered}$ | $122 \cdot 2$ | . | . | .. | .. | . |  | '105.9 | $111 \cdot 6$ | $101 \cdot 5$ |
| $25 \frac{1}{2}$ | 3393 | 13572 | $117 \cdot 5$ | .. |  | . |  | .. | .. | $101 \cdot 9$ | $107 \cdot 3$ | $97 \cdot 6$ |
| $\dagger 25$ | 3267 | 13068 | 113. | 107. | $106 \cdot 5$ | 102. | $101 \cdot 5$ | $99^{\text {• }}$ | 99. | 98. | $103 \cdot 2$ | $93 \cdot 8$ |
| $24 \frac{1}{2}$ | 3122 | 12488 | $114 \cdot 4$ |  |  |  | .. | .. | .. | $94 \cdot 4$ | 102.5 | $90 \cdot 1$ |
| 24 | 3006 | 12024 | $115 \cdot 7$ | . |  |  | .. | . | . | 91. | $101 \cdot 9$ | $86 \cdot 5$ |
| 231 $\frac{1}{2}$ | 2880 | 11520 | 117. | .. | .. | .. | .. | .. | .. | $87 \cdot 6$ | $101 \cdot 3$ | $82 \cdot 9$ |
| 23 | 2763 | 11052 | 118•3 | . |  |  | . |  | .. | $84 \cdot 2$ | $100 \cdot 7$ | $79 \cdot 4$ |
| $\dagger 22 \frac{1}{2}$ | 2646 | 10584 | $119 \cdot 5$ | $109 \cdot 5$ | $101 \cdot 7$ | $93 \cdot 5$ | $99^{\text {. }}$ | 96.5 | 94. | 81. | $100 \cdot 1$ | 76. |
| 22 | 2529 | 10116 | $111 \cdot 4$ | .. | .. | .. | .. | .. | .. | $77 \cdot 9$ | 95. | $72 \cdot 6$ |
| $21 \frac{1}{2}$ | 2412 | 9648 | 103.5 | .. | .. | .. | . | .. | .. | $74 \cdot 9$ | $90 \cdot 1$ | $69 \cdot 4$ |
| 21 | 2304 | 9216 | $95 \cdot 8$ | . | .. | . | . | .. | - | 72. | $85 \cdot 3$ | $66 \cdot 2$ |
| $20 \frac{1}{2}$ | 2196 | 8784 | $88 \cdot 3$ | .. | . | .. | .. | .. | .. | $69 \cdot 2$ | $80 \cdot 6$ | $63 \cdot 1$ |
| $\dagger 20$ | 2088 | 8352 | 81. | $78 \cdot 5$ | $78 \cdot 2$ | 78. | $77 \cdot$ | $75 \cdot 5$ | $74 \cdot 2$ | $66 \cdot 5$ | $76 \cdot 1$ | 60. |
| 192 | 1980 | 7920 | $76 \cdot 7$ | .. | .. | .. | .. | .. | .. | $62 \cdot 1$ | $71 \cdot 3$ | $57 \cdot 1$ |
| 19 | 1881 | 7524 | $72 \cdot 6$ | .. | .. | .. | . | .. | . | $57 \cdot 9$ | $66 \cdot 6$ | $54 \cdot 2$ |
| 181 ${ }^{\frac{1}{2}}$ | 1782 | 7128 | $68 \cdot 6$ | .. | . | . | . | . |  | $53 \cdot 8$ | $62 \cdot 1$ | $51 \cdot 4$ |
| 18 | 1692 | 6768 | $64 \cdot 7$ | .. |  | .. | .. | . | . | $49 \cdot 8$ | $57 \cdot 7$ | $48 \cdot 6$ |
| $\dagger 17 \frac{1}{2}$ | 1597 | 6388 | 61. | $59 \cdot 7$ | $54 \cdot 7$ | $54 \cdot 5$ | 52 | 50. | $49 \cdot 2$ | 46** | $53 \cdot 4$ | $46 \cdot *$. |
| 17 | 1512 | 6048 | $57 \cdot 3$ | .. | .. | .. | .. | .. | .. | $44 \cdot 9$ | 51. | $43 \cdot 4$ |
| 161 ${ }^{\frac{1}{2}}$ | 1422 | 5688 | $53 \cdot 9$ | .. | . | .. | . | . | . | $43 \cdot 8$ | $48 \cdot 7$ | $40 \cdot 8$ |
| 16 | 1332 | 5328 | $50 \cdot 5$ | .. | . | .. | .. | .. | .. | $42 \cdot 8$ | $46 \cdot 5$ | $38 \cdot 4$ |
| 1512 | 1251 | 5004 | $47 \cdot 3$ |  |  | .. | .. | .. |  | $41 \cdot 9$ | $44 \cdot 3$ | 36. |
| $\dagger 15$ | 1179 | 4716 | $44 \cdot 2$ | $43 \cdot$ | $42 \cdot 7$ | $42 \cdot 5$ | 42. | $41 \cdot 7$ | $41 \cdot 5$ | 41. | $42 \cdot 3$ | $33 \cdot 7$ |
| 142 | 1098 | 4392 | $41 \cdot 6$ | .. | .. | .. | .. | .. | .. | $38 \cdot 4$ | $39 \cdot 9$ | $31 \cdot 5$ |
| 14 | 1026 | 4104 | $39 \cdot 1$ | .. | .. | .. | .. | .. | .. | 36. | $37 \cdot 6$ | $29 \cdot 4$ |
| $13 \frac{1}{2}$ | 954 | 3816 | $36 \cdot 7$ | .. | .. | .. | . | .. | .. | $33 \cdot 6$ | $35 \cdot 4$ | $27 \cdot 3$ |
| 13 | 882 | 3528 | $34 \cdot 4$ | .. | .. | .. | .. | .. | .. | $31 \cdot 3$ | $33 \cdot 3$ | $25 \cdot 3$ |
| $\dagger 12 \frac{1}{2}$ | 810 | 3240 | $32 \cdot 2$ | $32 \cdot 2$ | 32. | 32. | $31 \cdot 2$ | $31 \cdot 2$ | 31. | $29 \cdot 2$ | $31 \cdot 3$ | $23 \cdot 4$ |
| 12 | 756 | 3024 | $29 \cdot 8$ | .. | .. | .. | .. | .. | .. | $26 \cdot 6$ | $28 \cdot 6$ | $21 \cdot 6$ |
| 111 $\frac{1}{2}$ | 693 | 2772 | $27 \cdot 6$ | .. | . | . | . | . | . | $24 \cdot 2$ | $26 \cdot 1$ | $19 \cdot 8$ |
| 11 | 630 | 2520 | $25 \cdot 5$ | . | .. | . | .. | . | . | $21 \cdot 8$ | $23 \cdot 7$ | $18 \cdot 1$ |
| $10 \frac{1}{2}$ | 576 | 2304 | $23 \cdot 4$ | .. | .. | .. | .. | .. | .. | $19 \cdot 6$ | $21 \cdot 4$ | $16 \cdot 5$ |
| $\dagger 10$ | 522 | 2088 | $21 \cdot 5$ | 21. | $19 \cdot 7$ | $17 \cdot 7$ | $17 \cdot 7$ | . | .. | $17 \cdot 5$ | $19 \cdot 2$ | 15. |
| $9 \frac{1}{2}$ | 468 | 1872 | 19. | .. | .. | .. | .. | . | .. | $15 \cdot 7$ | $17 \cdot 1$ | $13 \cdot 5$ |
| 9 | 432 | 1728 | $16 \cdot 7$ | .. | .. | .. | .. | .. | .. | 14. | $15 \cdot 2$ | $12 \cdot 1$ |
| 812 | 396 | 1584 | $14 \cdot 6$ | .. | .. | .. | .. | .. | .. | $12 \cdot 4$ | $13 \cdot 4$ | $10 \cdot 8$ |
| 8 | 315 | 1260 | $12 \cdot 6$ |  |  | .. |  | $\ldots$ |  | $10 \cdot 9$ | $11 \cdot 7$ | $9 \cdot 6$ |
| +712 | 288 | 1152 | $10 \cdot 7$ | $10 \cdot 5$ | $10 \cdot 5$ | $10 \cdot 3$ | $10 \cdot 3$ | $10 \cdot 3$ | 10 | $9 \cdot 5$ | $10 \cdot 2$ | $8 \cdot 4$ |
| 7 | 252 | 1008 | $9 \cdot 3$ | .. | .. | .. | .. | .. | .. | $8 \cdot 2$ | $8 \cdot 8$ | $7 \cdot 3$ |
| $6 \frac{1}{2}$ | 216 | 864 | $8 \cdot 1$ | . | .. | . | .. | . | .. | 7. | $7 \cdot 5$ | $6 \cdot 3$ |
| 6 | 189 | 756 | 7. | .. | . | . | .. | .. | .. | $5 \cdot 8$ | $6 \cdot 3$ | $5 \cdot 4$ |
| $5 \frac{1}{2}$ | 162 | 648 | $5 \cdot 9$ | . | $\cdots$ | . | .. | $\cdots$ |  | $4 \cdot 8$ | $5 \cdot 3$ | $4 \cdot 5$ |
| $\dagger 5$ | 135 | 540 | 5. | $4 \cdot 9$ | $4 \cdot 6$ | $4 \cdot 2$ | 4. | 4. | 4. | $3 \cdot 9$ | $4 \cdot 3$ | $3 \cdot 7$ |
| $4 \frac{1}{2}$ | 108 | 432 | 4. | .. | .. | .. |  |  |  | $3 \cdot 1$ | $3 \cdot 4$ | $3 \cdot$ |
| 4 | 90 | 360 | $3 \cdot 2$ |  |  | .. | .. | .. |  | $2 \cdot 5$ | $2 \cdot 7$ | $2 \cdot 4$ |
| $3 \frac{1}{2}$ | 69 | 276 | $2 \cdot 4$ |  |  |  | .. | .. | . | $1 \cdot 9$ | $2 \cdot 1$ | $1 \cdot 8$ |
| 3 | 54 | 216 | $1 \cdot 8$ | . | . | . | . |  | .. | $1 \cdot 4$ | $1 \cdot 5$ | $1 \cdot 3$ |

The lines marked ( $\dagger$ ) contain the results given by experiments, and the intermediate lines those found by calculation.

Table II.-For Ascertaining the Strength of Chain Cable.

| Size. | Testing Strain, in Tons. | Weight,100 Fathoms. | Breaking Strain, in Tons. |  |  |  |  |  | Mean. | Calculated from * weakest. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Maximum. | Intermediate Strains. |  |  |  | Minimum. |  |  |
| ${ }_{2}{ }_{2}{ }^{\frac{1}{4}}$ | 911 | $\begin{aligned} & \text { lbs. } \\ & 27216 \end{aligned}$ | $130 \cdot 3$ |  |  |  |  | $121 \cdot 8$ | $125 \cdot 9$ |  |
| $2 \frac{1}{8}$ | $81 \frac{1}{4}$ | 24276 | $116 \cdot 2$ |  | . | . | . | $108 \cdot 6$ | $112 \cdot 3$ | $95 \cdot 8$ |
| $\dagger 2^{88}$ | 72 | 21504 | 103. | $102 \cdot 5$ | 101* | $97 \cdot 5$ | 97* | $96 \cdot 25$ | 99.5 | 95 |
| $\dagger 17$ | $63 \frac{1}{4}$ | 18900 | 99. | $97 \cdot 75$ | $93 \cdot 5$ | 90. | 89. | 88. | $92 \cdot 8$ |  |
| $\dagger 1 \frac{3}{4}$ | $55 \frac{1}{8}$ | 16464 | $85 \cdot 25$ | $81 \cdot 5$ | $80 \cdot 5$ | 67. | $65 \cdot 5$ | 65. | $74 \cdot 1$ | 65. |
| 15 | $47 \frac{1}{2}$ | 14196 | 75. |  |  |  |  | $59 \cdot 5$ | $66 \cdot 5$ | 56 |
| $\dagger 1 \frac{1}{2}$ | $40 \frac{1}{2}$ | 12096 | $65 \cdot 5$ | $65 \cdot 5$ | $59 \cdot 25$ | 57.75 | $55^{\circ}$ | $54 \cdot 5$ | $59 \cdot 5$ |  |
| $1{ }^{18}$ | 34 | 10164 | $53 \cdot 6$ | .. | .. | .. | .. | $44 \cdot 4$ | $48 \cdot 5$ | $40 \cdot 1$ |
| $1{ }^{\text {崖 }}$ | $28 \frac{1}{2}$ | 8400 | $42 \cdot 8$ |  |  |  |  | $35 \cdot 3$ | $38 \cdot 5$ | $33 \cdot 1$ |
| $\dagger 1 \frac{1}{8}$ | 223 | 6804 | 33. | 31.75 | 29. | $29^{\circ}$ | $27 \cdot 5$ | 27. | $29 \cdot 5$ | $26^{\text {• }}$ |
| +1 | 18 | 5376 | $27 \cdot 25$ | 26. | $24 \cdot 75$ | $23 \cdot$ | 22.75 | 22. | $24 \cdot 3$ | 21.2 |
| $\dagger \frac{7}{8}$ | $13 \frac{3}{4}$ | 4116 | $22 \cdot 5$ | $21 \cdot 5$ | $21 \cdot 1$ | $20 \cdot 7$ | $20 \cdot 5$ | $20 \cdot 3$ | $21 \cdot 1$ | $16 \cdot 2$ |
| $\dagger \frac{3}{4}$ | $10 \frac{1}{8}$ | 3024 | 15. | $14 \cdot 25$ | 14. | $12 \cdot 75$ | $12 \cdot 62$ | $12 \cdot 5$ | $13 \cdot 5$ | $11 \cdot 9$ |
|  | $8 \frac{1}{2}$ | 2541 | $12 \cdot 3$ |  |  |  |  | $10 \cdot 8$ | $11 \cdot 4$ | 10 |
| $\dagger \frac{5}{8}$ | 7 | 2100 | $9 \cdot 87$ | $9 \cdot 75$ | $9 \cdot 5$ | $9 \cdot 5$ | $9 \cdot 5$ | $9 \cdot 37$ | $9 \cdot 5$ | $8 \cdot 2$ |
| $\frac{9}{\frac{9}{16}}$ | $5{ }_{4}^{1} \frac{1}{2}$ | 1701 1344 | $6 \cdot 3$ | .. | $\ldots$ | ... | $\ldots$ | $\ddot{5} \cdot 9$ | $\ddot{6}$. | $\ddot{5} \cdot 3$ |

The lines marked ( $\dagger$ ) contain the results of experiments, and the intermediate lines those derived from calculation.

Table III.-For Ascertaining the Strength of Hawser-laid Rope.

| Size. | No. of Yarns. | Weight, 100 Fathoms. | Strain, in Tons. |  |  |  | Mean. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Maximum. | Intermediate Strains. |  | Minimum. |  |
| $\begin{gathered} \text { in. } \\ \dagger 12 \end{gathered}$ | 1173 | $\begin{gathered} \text { lbs. } \\ 2940 \end{gathered}$ | $45 \cdot 5$ | $40 \cdot 5$ | 39. | 35. | 40 |
| $11 \frac{1}{2}$ | 1077 |  | $41 \cdot 7$ | .. | - | $32 \cdot$ | $36 \cdot 7$ |
| 11 | 987 | .. | $38 \cdot 2$ | .. | .. | $29 \cdot 3$ | $33 \cdot 6$ |
| 101 | 900 | $\ddot{\square}$ | $34 \cdot 9$ | . | .. | $26 \cdot 7$ | $30 \cdot 7$ |
| 10 | 816 | 2136 | $31 \cdot 7$ | .. | .. | $24 \cdot 2$ | $27 \cdot 9$ |
| $9 \frac{1}{2}$ | 738 | - | $28 \cdot 6$ | .. | .. | $21 \cdot 8$ | $25 \cdot 2$ |
| 9 | 660 | 1712 | $25 \cdot 7$ | .. | .. | $19 \cdot 6$ | $22 \cdot 6$ |
| $8 \frac{1}{2}$ | 591 |  | $23 \cdot$ | $\ldots$ | . | $17 \cdot 5$ | $20 \cdot 2$ |
| 8 | 522 | 1379 | $20 \cdot 4$ | .. | .. | $15 \cdot 5$ | 18. |
| $7 \frac{1}{2}$ | 459 | .. | 18. | .. | .. | $13 \cdot 6$ | $15 \cdot 8$ |
| 7 | 399 | .. | $15 \cdot 8$ | .. | .. | $11 \cdot 8$ | $13 \cdot 8$ |
| $6 \frac{1}{2}$ | 345 | $\because$ | $13 \cdot 7$ |  | $\ldots$ | $10 \cdot 2$ | 12. |
| $\dagger 6$ | 294 | 834 | $11 \frac{3}{4}$ | $10 \frac{3}{4}$ | 10. | $8 \cdot 7$ | $10 \cdot 3$ |
| $5 \frac{1}{2}$ | 249 | 712 | $9 \cdot 8$ | .. | . | $7 \cdot 3$ | $8 \cdot 7$ |
| 5 | 204 |  | $8 \cdot 2$ | .. | 7. | 6. | $7 \cdot 2$ |
| $4 \frac{1}{2}$ | 168 | 413 | $6 \cdot 7$ | .. | 5. | 5. | $5 \cdot 9$ |
| 4 | 132 | .. | $5 \cdot 3$ | .. | .. | $4 \cdot$ | $4 \cdot 7$ |
| $3 \frac{1}{2}$ | 102 | $\because$ | $4 \cdot 1$ | .. |  | $3 \cdot 2$ | $3 \cdot 7$ |
| 3 | 75 | 203 | $3 \cdot 1$ | .. | $2 \cdot 5$ | $2 \cdot 4$ | $2 \cdot 8$ |
| $2 \frac{1}{4}$ | 54 | .. | $2 \cdot 2$ | .. |  | $1 \cdot 8$ | $2 \cdot 1$ |
| 2 | 33 | .. | $1 \cdot 5$ |  | $1 \cdot 7$ | $1 \cdot 3$ | $1 \cdot 4$ |
| $\dagger 1 \frac{3}{4}$ | 27 | .. | $1 \cdot 28$ | 1-28 | $1 \cdot 23$ | $1 \cdot 13$ | $1 \cdot 23$ |
| $\dagger 1 \frac{1}{2}$ | 21 | $\bullet$ | -90 | -89 | - 88 | -86 | -88 |
| $\dagger 1 \frac{1}{4}$ | 15 | . | -60 | -56 | -55 | -53 | - 56 |
| $\dagger 1$ | 12 | .. | -58 | $\cdot 51$ | -49 | -46 | - 51 |
| $\dagger \frac{3}{4}$ | 9 | .. | - 51 | $\cdot 46$ | $\cdot 46$ | -2 | -46 |
| $\dagger \frac{1}{2}$ | 6 | . | - 28 | - 28 | $\cdot 28$ | -28 | 28 |

The lines marked ( $\dagger$ ) contain the results of experiments, and the intermediate lines those derived from calculation.

Table IV.-For Ascertaining the Strength of Round-link Crane Chain.

| Size. | Weight, 100 Fathoms. | Strain, in Tons. |  |  |  | Mean. | Testing Strength. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Maximum. | Intermediate Strains. |  | Minimum. |  |  |
| $\mathrm{in}^{\text {in }}$ | $\begin{aligned} \text { lbs. } \\ 15569 \end{aligned}$ | 75. | 74-7 | $74 \cdot 5$ | 68. | 73 | $31 \cdot 6$ |
| $1 \frac{1}{2}$ | .. | 64. | .. | .. | $58 \cdot 2$ | $62 \cdot 3$ | $27 \cdot$ |
| $1 \frac{7}{16}$ | .. | 59. | .. | .. | $53 \cdot 8$ | $57 \cdot 4$ | $24 \cdot 7$ |
| $1 \frac{18}{}{ }^{18}$ | .. | $54 \cdot 2$ | .. | .. | $49 \cdot 6$ | $52 \cdot 8$ | $22 \cdot 6$ |
| $1 \frac{5}{16}$ | .. | $49 \cdot 7$ | .. | .. | $45 \cdot 5$ | $48 \cdot 4$ | $20 \cdot 6$ |
| $1{ }^{1 / 4}$ | .. | $45 \cdot 3$ | .. | .. | $41 \cdot 7$ | $44 \cdot 1$ | $18 \cdot 8$ |
| $1 \frac{3}{16}$ |  | $41 \cdot 2$ | .. | .. | 38. | $40 \cdot 1$ | 17. |
| $1{ }^{1}{ }^{\circ}$ | 7481 | $37 \cdot 3$ | .. | .. | $34 \cdot 5$ | $36 \cdot 3$ | $15 \cdot 3$ |
| $1 \frac{1}{16}$ |  | $33 \cdot 6$ | .. | .. | $31 \cdot 2$ | $32 \cdot 7$ | $13 \cdot 6$ |
| $1^{16}$ | 6490 | $30 \cdot 1$ | .. | .. | $28 \cdot 1$ | $29 \cdot 3$ | 12. |
| $\frac{1}{1} \frac{5}{6}$ | 5600 | $26 \cdot 8$ | $\cdots$ | . | $25 \cdot 2$ | $26 \cdot 1$ | $10 \cdot 5$ |
| \% | 4500 | $23 \cdot 7$ | $\because$ | .. | $22 \cdot 5$ | $23 \cdot 1$ | $9 \cdot 1$ |
| $\dagger^{\frac{1}{13}}$ | 4000 | $20 \cdot 9$ | $20 \cdot 3$ | .. | $20^{\circ}$ | $20 \cdot 4$ | $7 \cdot 9$ |
| ${ }^{\frac{3}{4}}$ | 3449 | $17 \cdot 8$ | .. | .. | $16 \cdot 6$ | $17 \cdot 3$ | $6 \cdot 8$ |
| $\frac{13}{13}$ | 2900 | $14 \cdot 9$ | .. | .. | $13 \cdot 5$ | $14 \cdot 6$ | $5 \cdot 6$ |
| $\frac{5}{8}$ | 2538 | $12 \cdot 3$ | .. | .. | $10 \cdot 8$ | 12. | $4 \cdot 6$ |
| $\frac{9}{16}$ | 2001 | 10. | .. | .. | $8 \cdot 7$ | $9 \cdot 7$ | $3 \cdot 8$ |
| $\frac{1}{2}$ | 1583 | $7 \cdot 9$ | .. | .. | $6 \cdot 9$ | $7 \cdot 7$ | 3. |
| $\frac{7}{16}$ | 1060 | 6. | .. | .. | $5 \cdot 2$ | $5 \cdot 9$ | $2 \cdot 3$ |
| - ${ }^{16}$ | 827 | $4 \cdot 4$ | .. | .. | $3 \cdot 8$ | $4 \cdot 3$ | $1 \cdot 6$ |
| $\frac{5}{16}$ | 581 | 3. | .. | .. | $2 \cdot 7$ | 3. | $1 \cdot 1$ |
| ${ }_{\frac{1}{4}}^{16}$ | 392 | $1 \cdot 9$ | .. | .. | 1.7 | $1 \cdot 9$ | $\cdot 75$ |
| ${ }^{3} 8$ | .. | $1 \cdot 1$ | .. | . | $\cdot 97$ | 1. | -42 |

The lines marked ( $\dagger$ ) contain the results of experiments, and the intermediate lines those derived from calculation.
Experiments and Calculations, made by Nicholas Tinmouth, to Determine the Curvature of Chain Cables.-Given the length of a chain cable, the vertical distance between its extremities, the tension on the upper end, and its weight 100 fathoms; to find the angle between the upper part of the cable and a vertical line, and that between the lower part of the cable and a horizontal line. Suppose the length of chain cable 100 fathoms; vertical distance between its extremities 11 fathoms; strain on the upper end $40 \frac{1}{2}$ tons; size of cable $1 \frac{1}{2}$ in., whose weight the 100 fathoms is 108 cwt. or $5 \cdot 4$ tons. Since 100 fathoms weigh $5 \cdot 4$ tons, 750 fathoms will weigh $40 \frac{1}{2}$ tons, or 750 fathoms attached to the upper end and hanging freely, will have the same effect as the strain of $40 \frac{1}{2}$ tons. Take any vertical line A Q, Fig. 1830, equal to 100 fathoms; from A as a centre, and with a radius equal to 750 fathoms describe the arc $a b$. From Q as a centre, and with a radius equal to 750 minus 11 or 739 fathoms, 11 fathoms being the vertical distance between the

extremities, describe an are $c d$ intersecting the arc $a b$ in some point P. Join PA, PQ. On PA describe a semicircle. The point $Q$ will in this case be found to lie within the semicircle. Produce A Q to cut the semicircle in R. Join PR. From P as a centre with the radii PQ, PR, describe the arcs $\mathrm{Q} x, \mathrm{Q} y$, intersecting A P in $x$ and $y$. Then PA Q is the angle which the upper part of the cable will make with a vertical line; as P A $x=79^{\circ} 52^{\prime}$, Fig. 1831, and QPR the angle which the lower part of the cable makes with a horizontal line, as L Q $x$, Fig. 1831, $=2^{\circ} 28^{\prime}$. A $x$, Figs. 1830, 1831, is the vertical distance between the extremities of the cable. As the length A Q, Fig. 1830, of the cable is increased, the point Q approaches the point R, and the angle Q P R which the lower part makes with a horizontal line, diminishes. When the point $Q$ coincides with R or $\mathrm{A} Q$ is equal to $A R$, the angle $\mathrm{Q} P \mathrm{R}$ vanishes, and a horizontal line forms a tangent to the cable at its lowest part. Thus, in Fig. 1831, A Q R is the cable represented by A R, Fig. 1830, $=131 \cdot 89$ fathoms, and is a semi-catenary, the horizontal line R $y$, Fig. 1831, being a, tangent to

Tabia V.-Resulith of Caloulationa on mine Catenary Curve of a Cifain Carlem

it at its lowest point R; the vertien distance between its extremitios being I) IR or $\boldsymbol{\Lambda} y=11 \cdot(88$ finthoms, which corrosponds with $\Lambda y$, Fig. 1830, measured on its proper semle. $\Lambda$ Q, Figs, 1830, 1831, is usegment of the semi-catemery $\Lambda \mathrm{I}$. Hence a diagrom being constructed as above, if the point Q, Jig. 18:30, shombld be fomblo to within the semicircle, the cable forms a segment of a
 18:31, lie in the carcumference of the semicircle, the colbe forms a complete semi-entenary, to the lowest point of which a borizontal line forins a tangent. If the point ( b be found to lio without the semicirelons q, Fig. 1830, the lomgth A IR will form a semi-matemary; and tho remaining portion $\mathrm{R}_{4}$, which is boyond tho somicircle will rest on the gromad, as R 1 , Fig. 1831.
1831.


If it bo required to aseertain what length of chain eable will form a eomplete semi-eatemary, ut the proposed depth $\Delta x$, or 11 fathons; let the are $x$ Q, lig. 18:30, lo continned motil it intersects the semicirele in some point $S$ : join $\Lambda S, P S$, thon $\Lambda S$ is the length repuired; which is the grembest lowgth of cable thont can be omployed with a depth of 11 finthoms, so that no part of it shall rest on the gromal, as $\Lambda n \mathrm{~s}$, Fig. 1s:31. The angle P $\wedge$ Q, Fig. 1830, or P $\Lambda x$, Fig. 1831, which the upper part of the cmble makes with a vertical line, bmy be fomm from the following expression: cos. $\mathrm{P} \Lambda Q=\begin{gathered}(\mathrm{P} \Lambda)^{2}+(\Lambda Q)^{2}-(\mathrm{P} Q)^{2} \\ 2 \times \mathrm{P} \Lambda \times \Lambda(Q\end{gathered}$. Substitnting for $\mathrm{P} \Lambda, 750$ fathoms; for PQ,739) fathoms: and for $\Lambda Q, 100$ finthoms:

$$
\operatorname{cos.} \mathrm{P} \wedge Q=\frac{(750)^{3}+(100)^{2}-(739)^{2}}{2+750 \times 100}=0.17586 \text { to rad. } 1 .
$$


The mighe () PR, Fig. 1830, or La ( $r$, Fig. 18:3, which the lower part of tho cable makes with a horizontal line, may bo fomed from tho following expression:

$$
\text { sin. } \mathrm{Q} \mathrm{P} \mathrm{R}=\frac{(\mathrm{P} \Lambda)^{2}-(\mathrm{P} Q)^{2}-(\Lambda Q)^{2}}{2 \times \Lambda Q \times \mathrm{PQ}}
$$

00 Fathoms in Length and $1 \frac{1}{2}$-in. in Diameter, under given Strains and Depths of Water.


Substituting, as before, the proper values of $\mathrm{PA}, \mathrm{PQ}$, and A Q ,

$$
\sin . \mathrm{QPR}=\frac{(750)^{2}-(739)^{2}-(100)^{2}}{2 \times 100 \times 739}=0.043159 \text { to rad. } 1
$$

And from a table of sines we have angle $\begin{gathered}\mathrm{Q} P \mathrm{R} \\ \text { or } \mathrm{Q} x\end{gathered}=2^{\circ} 28^{\prime} 25^{\prime \prime} \cdot 09$
The greatest length of cable that can be employed with a depth of 11 fathoms so that no part of it may rest on the ground is AS, which is equal to $\sqrt{(\mathrm{AP})^{2}}-(\mathrm{PS})^{2}$. Substituting for A P and PS their values $\begin{array}{c}\text { A S, } \\ \text { or } \\ n\end{array}$ S. Fig. 1830181$\}=\sqrt{(750)^{2}-(739)^{2}}=127 \cdot 9$ fathoms.

The angle P A S, which this length of cable makes with a vertical line may be found from the expression $\sin . \mathrm{PAS}=\frac{\mathrm{PS}}{\mathrm{PA}}$. Now $\mathrm{PS}=\mathrm{PQ}=739$ fathoms, and $\mathrm{PA}=750$ fathoms. Hence, by substitution, sin. PAS $=\frac{739}{750}=0.9853$ to radius 1 ; and from a table of sines. we have $\left.\begin{array}{l}\text { angle PA S, Fig. } 1830 \\ \text { angle P A } x \text {, Fig. } 1831\end{array}\right\}=80^{\circ} 10^{\prime \prime} 29 \cdot 88$.
The position which a chain cable would assume under any given circumstances may also be obtained experimentally, by the suspension of a small chain of any convenient length.

Let it be required to find by experiment on a chain $\frac{9}{16}$ of an in. in diameter and 25 ft . long, the position which a $1 \frac{1}{2}$-in. chain cable would take, whose length is 100 fathoms, vertical distance between the extremities 11 fathoms, and strain on the upper end $40 \frac{1}{2}$ tons:

The weight of 100 fathoms of $1 \frac{1}{2}$-in. chain cable is 108 cwt .
$\frac{9}{16}$-in. $\quad, \quad 15$ cwt. 0 qr. 21 lbs.
The proportionate strain to be applied ${ }^{1 \overline{1}}$ to the $\frac{9}{16}-\mathrm{in}$. chain may be found from the following expression:

$$
\begin{aligned}
& =\begin{array}{l}
\text { Weight of chain the } 100 \text { fathoms } \\
\text { Weight of cable the } 100 \text { fathoms } \\
\text { cwt. qr. longth of chain } \\
\text { ft. longth of cable }
\end{array} \text { strain } \\
& =(\text { by substitution }) \frac{15021}{108} \times \frac{25}{600} \times 40 \frac{1}{2}=4227 \cdot 552 .
\end{aligned}
$$

The length of the experimental chain being $\frac{1}{24}$ the length of the cable, the vertical distance between the extremities of the chain should be taken $\frac{1}{24}$ of 11 fathoms, or 2 ft .9 in .

The curve formed by the cable suspended under these circumstances was similar to that derived
from calculation for a $1 \frac{1}{2}$-in. chain cable of 100 fathoms on a $\frac{1}{24}$ scale. The deflection at the middle of the length from a straight line joining the extremities of the chain measured 5 in . This deflection, multiplied by 24 , gives the deflection on the full scale 10 ft ., agreeing with the result of calculation.

Let one end of a chain cable be made fast, and a given strain act on the other end; both extremities being in the same horizontal line, to find by construction the depth of the lowest part of the chain ; let the size of the cable be $1 \frac{1}{2}$ in., length 100 fathoms, strain $40 \frac{1}{2}$ tons. Take a vertical line A R, Fig. 1830, equal to one-half the length of the cable, or 50 fathoms; from R draw the line $\mathbf{R P}$ at right angles to $A \mathrm{R}$; from A as a centre, with a radius equal to 750 fathoms, or the number of fathoms of a $1 \frac{1}{2}$-in. chain cable due to $40 \frac{1}{2}$ tons strain, describe an arc $a b$ cutting $R P$ in $\mathbf{P}$; join $\mathbf{P A}$; from P as a centre, with a radius $\mathrm{P} R$ describe the arc $\mathrm{R} y$ cutting AP in $y$. A $y$ is the required depth of the lowest part of the chain.
The same may also be found from the expression

$$
\text { Depth of lowest point of cable }=\mathrm{T}-\sqrt{\mathrm{T}^{2}-s^{2}} \text {; }
$$

where $\mathbf{T}$ is equal to $\mathbf{P A}$, the tension at $\mathbf{A}$ in fathoms of chain, or 750 fathoms and $s$ is equal to A R the half-length of cable, or 50 fathoms. Hence, by substitution,

$$
\begin{aligned}
& \text { Depth of lowest point of cable }=750-\sqrt{(750)^{2}-(50)^{2}}, \\
&=750-748 \cdot 331=1 \cdot 668 \text { fath. } \\
& \frac{6}{10 \cdot 008} \mathrm{ft} .
\end{aligned}
$$

The deflection of a cable from a horizontal line may also be found experimentally, by the suspension of a small chain of any assumed length.

Let the size of the experimental chain be $\frac{9}{16}$, length 25 ft ., weight the 100 fathoms, 15 cwt .0 qr . 21 lbs.

The proportionate weight to be attached

$$
\begin{aligned}
& =\frac{\text { Weight of chain the } 100 \text { fathoms }}{\text { Weight of cable the } 100 \text { fathoms }} \times \begin{array}{l}
\text { length of chain } \\
\text { length of cable }
\end{array} \times \text { strain } \\
& \text { cwt. qr. lbs. fons. cwt. qrs. lbs. }
\end{aligned}
$$

The length of the chain in this experiment being $\frac{1}{2,4}$ the length of the chain cable, the curro observed is the curve on a $\frac{1}{24}$ scale, which the cable will form.

The deflection was found to be $4 \frac{15}{16}$, which being multiplied by 24 gives the deflection, or depth of the lowest point of the cable of 100 fathoms with $40 \frac{1}{2}$ tons strain, equal to 9 ft . $10 \frac{1}{2} \mathrm{in}$. Let the same be tried with $12 \frac{1}{2} \mathrm{ft}$. of $\frac{3}{8}-\mathrm{in}$. chain, weighing $7 \cdot 96$ cwt. to the 100 fathoms.

Proportionate weight to be attached

$$
\begin{aligned}
& =\frac{\text { Weight of chain the } 100 \text { fathoms }}{\text { Weight of cable the } 100 \text { fathoms }} \times \frac{\text { length of chain }}{\text { length of cable }} \times \text { strain } \\
& =\text { (by substitution) } \frac{7 \cdot 96}{108} \times \frac{12 \frac{12}{2}}{600} \times 40 \frac{1}{\text { f.t. }}=1 \quad 0 \quad 27 \\
& \text { tons. cwt. qr. lbs. }
\end{aligned}
$$

The length of chain in this experiment being $\frac{1}{48}$ of the length of cable, the curve observed is that on a $\frac{1}{48}$ scale, which the cable will form. The deflection was found to be $2 \frac{5}{16}$ of an in., which multiplied by 48 is 9 ft .3 in ., the depth of the lowest point of the cable.

Let the size of the cable be as before $1 \frac{1}{2}$ in., and the strain $40 \frac{1}{2}$ tons, but the length $12 \frac{1}{2}$ fathoms. Take a vertical line A R, Fig. 1831, equal to one-half the length of the cable, or $6 \frac{1}{4}$ fathoms; from $\mathbf{R}$ draw the line $\mathbf{R} \mathbf{P}$ at right angles to $A R$; from $A$ with the radius 750 fathoms, being the number of fathoms of a $1 \frac{1}{2}$-in. chain cable due to a strain of $40 \frac{1}{2}$ tons, describe an arc $a b$ cutting $R \mathbf{P}$ in $\mathbf{P}$; join $\mathbf{P A}$; from $\mathbf{P}$ with the radius $\mathbf{P} \mathbf{R}$ describe the arc $\mathbf{R} y$ cutting $\mathbf{A} \mathbf{P}$ in $y$. A $y$ is the required depth of the lowest point of the chain.

The same may also be found from the expression

$$
\text { Depth of lowest point of cable }=T-\sqrt{T^{2}-s^{2}} \text {; }
$$

where $\mathbf{T}$ is equal to $\mathbf{P} \mathbf{A}$, the tension of $\mathbf{A}$ in fathoms of chain, or 750 fathoms; and $s=\mathbf{A R}$, the half-length of the cable, or $6 \cdot 25$ fathoms. Hence, by substitutior

| Depth of lowest point of cable | $=750-\sqrt{(750)^{2}-(6 \cdot 25)^{2}}$ |
| ---: | :--- |
|  | $=750-749 \cdot 974=0 \cdot 026$ fath |
| $\frac{6}{0 \cdot 156} \mathrm{ft}$. |  |
| $\frac{12}{1 \cdot 872} \mathrm{in}$. |  |
|  | $=1 \frac{7}{8}$ in, nearly. |

The deflection may also be found by experiment. Suppose a length of 25 ft . of $\frac{9}{16}$ chain, weighing 15 cwt .0 qr . 21 lbs . to the 100 fathoms, be taken for the experiment.

Then the proportionate weight to be attached is

$$
\begin{aligned}
& =\frac{\text { Weight of chain the } 100 \text { fathoms }}{\text { Weight of cable the } 100 \text { fathoms }} \times \frac{\text { length of chain }}{\text { length of cable }} \times \text { strain } \\
& \text { cwt. qr. lbs. ft. tons. ton. cwt. qrs. lbs. } \\
& =(\text { by substitution }) \frac{15 \quad 0 \quad 21}{108} \times \frac{27}{75} \times 40 \frac{1}{2}=1 \quad 17 \quad 3 \quad 24 \frac{1}{2} .
\end{aligned}
$$

The length of the chain in this experiment being $\frac{1}{3}$ the length of the chain cable, the curve observed is that which the cable will form on a scale of $\frac{1}{3}$.

The deflection was found to be $\frac{13}{1} \frac{1}{6}$ of an in., which, multiplied by 3 , is equal to $2 \frac{7}{16} \mathrm{in}$. On the addition of $\frac{1}{2}$ cwt. to the above weight, the deflection was $\frac{5}{8}$ of an in., which, multiplied by $3=1 \frac{7}{8}$, agreeing with the calculation. As the nip on the roller where the weight was attached was considerable, it is probable that the chain was not sufficiently free.

The experiment was also tried with a $\frac{3}{8}-\mathrm{in}$. chain, weighing $7 \cdot 96 \mathrm{cwt}$. the 100 fathoms, and $12 \frac{1}{2}$ ft. long.

The proportionate weight to be attached

$$
\begin{aligned}
& =\frac{\text { Weight of chain the } 100 \text { fathoms }}{\text { Weight of cable the } 100 \text { fathoms }} \times \frac{\text { length of chain }}{\text { length of cable }} \times \text { strain } \\
& =(\text { by substitution }) \frac{7 \cdot 96}{108} \times \frac{12 \frac{1}{2}}{75} \times 40 \frac{1}{2}=0.49766=9 \\
& \text { tons. cwt. qrs. lbs. } \\
& =\begin{array}{lll}
9 . & 22.32
\end{array}
\end{aligned}
$$

The length of the chain in this experiment being $\frac{1}{6}$ of the length of the cable, the curve observed is that, on a scale of $\frac{1}{6}$, which the cable will form.

The deflection was found to be $\frac{5}{16}$, which, multiplied by 6 , is equal to $\frac{30}{16}=1 \frac{7}{8} \mathrm{in}$., which agrees with the calculation.

The weight attached was $1 \frac{1}{2} \mathrm{lb}$. more than should have been employed.
Where the result of the experiment does not exactly agree with that of calculation, it is owing an imperfection in the experiment. See Anchor. Founding, Casting, and Forging. Materials of Construction, Strength of. Rope-making Machinery.

CAGE, SAFETY. Fr., Cuffat des mines de houille; Ger., Kübcl, Fahrstuhl; Ital., Gabbia di sicurezza.

The safety-cage of J. P. Harper with conducting ropes of iron wire is represented and the mechanical arrangements illustrated in Figs. 1832 to 1835. Fig. 1832 is of this cage in working order; Fig. 1833, sectional elevation of compressors when not in action; Fig. 1834, sectional elevation of the compressors when first brought into action; and Fig. 1835, elevation of compressors when in extreme action, showing the conducting uron-wire rope in a compressed state.


Harper's safety-cage differs both in principle and detail from any that have preceded it, and certainly stands alone in this respect, that it is specially designed, constructed, and adapted for the ordinary wire-rope conductors.

Safety-cages have hitherto been brought out more particularly for adaptation to the wooden guides, but, as these are becoming superseded by the iron-wire ones on account of their durability and cheapness-if safety-cages are to be universally adopted, they must be made applicable to wire-rope conductors.

Safety-cages can be used to greater advantage in connection with the iron-wire than with the wooden guides, for, with the wooden ones, sharp-pointed or toothed instruments are usually brought to clutch or dig into the conductor; and if it so happens that the length of guide-rods on which the apparatus is brought to play be rotten, unsound, infirmly fixed, or in any way defective, the safety apparatus is of no avail; indeed sometimes, be the rods ever so sound and in good repair, we hear of accidents occurring, owing to the guide-rods being split and ripped down by the apparatus brought to bear on them.

Accidents of this description are less liable to occur with the iron-wire guides. It has been found that it would not do to employ an instrument with a sharp point, a toothed or uneven surface, to bear on the conductors, for, as the circumference of a wire rope is uneven, they would cut and injure the more prominent parts.

This safety-cage acts effectually without damaging or depreciating in the least the wire conductors, of whatever description they may be. The apparatus is extremely simple, substantial, inexpensive in its construction, is not easily deranged, and can be applied to cages and hoists, of whatever description or size, whether conducted at the ends, corners, or sides, and carrying any weight. The principle by which the apparatus, on the severance of the winding-rope from the cage, is brought to bear on the conductors is by compression; and its operation is (of its own accord) to bring the cage to an immediate stand by means of catches or compressors, which fasten on, encircle, and compress not only the sides but the whole circumference of the conductor. When the winding-rope is readjusted by the tension of the rope, the compressors at the same instant release their grasp of the conductors, and the whole is again in working order.

The general features of the cage will be readily seen on examining Fig. 1832. The catches, or more properly the compressors, are of peculiar shape: Figs. 1833 to 1835 show their positions when out of play and held from the guides by the tension of the rope, the position they assume when first brought into play, and the iron-wire guide in extreme compression. This will, of course, vary in proportion to the weight on the cage at the time they are brought into requisition.

Fig. 1832 is a design for double cages in a 13 -ft. shaft, fitted with wire-rope conductors, each cage having two, three, or four guides as may be required, and single or double tiered to carry either two or four wagons.

The cage, Fig. 1832, though conducted by four guides, the apparatus is brought to bear on two only, that is, across the corners. The compressors, A, Fig. 1835, made of malleable iron and case-hardened, are of peculiar construction, made in a fork-like shape, and of any proportionate elevational depth: see Fig. 1834. They work on each side of the conductors on axles B attached to the cage, and when not in play are held by the tension of the winding-rope at an acute angle ; and, when brought into play, slide into each other, and encircle the conductor, assuming a nearly horizontal position, and forming, both in plan and elevation, a circular hole the full depth of the compressors, but less in diameter than the conductor, so that as the angle is made to increase in proportion to the weight on the cage, so will the compression also increase on the conductors.

The compressors A are constructed with lever ends, and when the cage hangs freely on the winding-rope they are held away from the conductors by side rods D , attached on each side of the cage to main vertical and shouldered rods E working through guide-boxes F , in which are inserted spiral springs (merely to give impetus to the compressors) bearing on the inside shoulders of the main vertical rods E , which are connected on each side of the cage by a cross-bar $G$, working over the top or cover of the cage $\mathbf{H}$, and attached to the winding-rope. To prevent the compressors A wrenching apart, they are connected by a front tie-plate C. K represents ordinary cast-iron guide-boxes through which the conductors slide.

It will be seen that when the cage hangs freely on the rope the shoulders on the main vertical rods E are brought up by means of the cross-bar G to the under-sides of the guide-boxes F , and the compressors, now being held away from the conductors, are allowed to pass freely between the forks of the catches; but immediately on the rope becoming disconnected from the cross-bar $G$ the concealed spiral springs, hitherto held in subjection, are released, give impetus to and assist in bringing the compressors A into instantaneous play, which, then sliding into one another, so close round and compress the conductor into a circular hole, less in diameter than the conductor itself, through. which it is impossible for it to pass; hence the cage is brought to an immediate stand.

In the arrangements by which the compressors are connected to the winding-rope, it will be perceived that both sets must work simultaneously on the guides, and are such as cannot be easily deranged.

CAISSON. Fr., Caisson; Ger., Versenkkasten; Ital., Cassone a cilindro; Span., Cajon.
Floating bodies, constructed of sheet iron, designed to open or close the entrances of a docks or basins, are termed caissons.

The engravings, Figs. 1836, 1837, represent one of the caissons for closing dock or basin entrances at Haulbowline Navy Yard, Cork, constructed by A. Clarke. It is one of the class termed sliding caissons, and is drawn into a recess formed for its reception at right angles to the entrance of the dock or basin, when the entrance is opened to allow ships to pass in or out. Such caissons are also used as roadways for the traffic, usually very heavy, passing across the entrance of the dock or basin.

The object sought is to preserve the roadway or deck of the caisson level with the wharf on one side of the entrance and with the cover of the recess on the other side when the caisson is in its piace and being used as a roadway; likewise to keep the sides of the entrance level and free from obstruction when the caisson is drawn into its recess, and the operation of taking ships into
or out of the dock or basin is in progress. These objects have hitherto been accomplished by making the deck of the caisson capable of being lowered, when it is necessary to draw the caisson into and under the cover of the recess, or by lowering the caisson bodily by sinking it to a sufficient depth to admit of its passing below the cover.

In the present instance a third method has been adopted, that of raising and lowering a part of the cover of the recess sufficiently to allow the caisson to pass down a series of slopes at the bottom of the recess and entrance; these slopes being so adjusted that when the caisson is drawn completely into its recess it has become lowered to a very small extent, and at the same time so thrown out of the level as to fit under the cover, which for this purpose is formed with a slight inclination towards the entrance. The part of the cover which was raised to allow the caisson to pass is then lowered, and the wharf is clear for docking operations.

The flotation of the caisson is so adjusted that the portion of its weight borne by the masonry of the entrance is small, being only sufficient to keep the caisson steady when moving. The power, therefore, required to force it up the slopes on which it slides is but trifing, and means are provided by which it can be made to float completely if necessary.

The machinery for lifting the cover of the recess, and that for moving the caisson, will be worked by a small steam-engine. A pair of endless chains on either side of the upper part of the recess supply the moving power for the caisson.

Referring to the drawing, A is the caisson in its place across the entrance, B is the recess into which it is drawn, C is the cover of the recess, and D the portion of the cover which is raised, as 1836.

shown by the dotted line, to allow the caisson to pass into the recess; E E E are the slopes on which the caisson slides, and the dotted line $\mathbf{F}$ shows the position of the caisson when drawn into the recess, after which the cover $\mathbf{D}$ is lowered into its former position. G $\mathbf{G}$ are the endless chains which are attached to the swivelling cross-head H connected to the caisson. I, I, I, are rubbing-pieces; K is a trap for mud; and L the skids on which the caisson slides, which are so formed as to clear the sliding ways E of any mud which may have accumulated.

The caisson can be used as an ordinary floating caisson when disconnected from the cross-head H ; and by temporarily placing one of these caissons on either side of the basin entrance, and pumping the water ont from bet ween them, access to the caisson recess can be obtained for cleaning and repairs.

Many chest-like vessels, in both civil and military engineering, are termed caissons. In military engineering a chest that contains ammunition, and also the wagon or tumbril which conveys military stores, is called a caisson. A chest filled with explosive materials to be laid in the way of an enemy, as under some work at the possession of which he is
1837.
 aiming, and to be fired when the enemy is exposed to its effect; a wooden box or frame of strong timbers, used for laying the foundations of a bridge in situations where the coffer-dam cannot be employed; and other structures, receive the appellation caisson.

See Bridge. Docks. Fortification. Locks. Ordnance.
Calender. Fr., Calandre; Ger., Calender; Ital., Sopressa; Span., Calandria.
Calender, Five-roller, by Thomson Brothers and Co., of Dundee. -The arrangement of this calender is illustrated in Fig. 1838. It may be modified to finish either linen or cotton goods. This machine may be employed to effect :-Plain calendering, by passing the cloth through the rollers; Chesting, or rolling cloth on the top roller under heavy pressure, either without or after passing it through the other rollers; and Mangling, or rolling the cloth on the second roller from the top; to effect this object, the end of the piece being sewed to the body of the cloth, the calender is kept revolving under heavy pressure backwards and forwards. A drag or slip is made between the centre driving-roller and the lower cast-iron roller when the machine is employed for Glazing; the centre driving-roller may be heated either by steam or by hot bolts, the latter being necessary for high glaze. The top and bottom rollers A E, Fig. 1838, are of cast iron, 20 in . in diameter, with wrought-iron journals to sustain the pressure to which they may be subjected. The centre roller C, 11 in. diameter, is made hollow, and fitted at the ends with stuffing-boxes for the admission of steam. The two paper rollers, B D, 24 in . diameter with 54 in . width of paper, have wroughtiron journals and end-plates. The two upper rollers have small pivots set into the ends of the journals for the purpose of suspending them by rods from top-llocks while the cloth is being drawn off by the stripping motion, after the operation of chesting or mangling. The machine, Fig. 1838, is driven by a shaft connected by a wheel and pinion to the centre roller. This shaft carries two pulleys II, for open and for cross belts, each pulley being 34 in . diameter, and the belts 8 in . broad;

## CALENDER.

a double friction clutch is placed between the pulleys, for the purposes of starting, reversing, or stopping the machine-the handle $h$ being retained in a central position by a spring catch. The calender does not require to be attached to side walls; a foundation capable to carry its weight is sufficient. The crane motion for lifting the levers and top rollers is fixed to the frame, and the top rollers are held in position, when suspended, by a friction strap which also serves to lower the rollers. The blocks for the journals of the rollers are not attached to the framing, the brasses

being set forward by side screws when they wear in the blocks, thus keeping the rollers in the centre; the upper part of the block being made hollow, so that the journal may be lubricated. Pressure is applied by weights suspended by the chain at Z, which is coilcd on the screw pulley, and is equal to about 22 tons, which includes the weight of the rollers. By reversing the chain on the screw pulley, the weights, when a light pressure is required, may be made to counterbalance the weight of the levers and top rollers.

Calender with Five Rollers; designed and constructed by A. More.-Fig. 1839 is an end view; Fig. 1840, a side elevation. The same letters of reference denote the same parts in each view. $\mathbf{A}, \mathrm{A}, \mathrm{A}$, three cylinders or rollers made of paper, the construction of which will be noticed afterwards. B B, two cast-iron cylinders, made hollow to allow of the introduction of hot bolts within them; or of steam when it is required. CC, the two side frames into which are fitted the several brass bushes for the cylinders to turn upon. D D, top guides into which the cross-head G and elevating screws HH work. EE, top-pressure levers connected by a strong rod of iron with the under-pressure lever F. This system of levers is connected with the cross-head G by two strong links of iron. The elevating screws H H pass through the cross-head, and rest upon a strong cast-iron block, into which is fitted the brass bush of the top paper roller. By means of the screws, the cross-head and levers can be raised or depressed as required, and when the calender is working warm and requires to be stopped, the elevating screws are screwed up for the purpose of lifting the paper rollers off the hot cylinders, to prevent their being injured by the heat.

The construction of the paper rollers or cylinders is as follows:-Upon each end of a journal
of malleable iron, of sufficient strength to withstand the necessary pressure without yielding, is fastened a strong plate of cast iron, of the same diameter as the roller to be made; the plate is secured in its proper place by a ring of iron, cut in two, and let into a groove or check turned in the journal. When the roller is finished, the annular pieces are kept in their groove by a hot hoop put upon the outside of them and allowed to cool. A plate is fitted on the other end, of exactly the same size, and in the same manner. In building the rollers, one of the plates is taken off the journal, but the other is allowed to remain in its place. The paper sheets of which the rollers are

to be made have each a circular hole cut in the centre of it, of exactly the same diameter as the journal. The sheets are then put upon the journal, and pressed hard against the fixed plate. When the journal is filled with paper, it is put into a strong hydraulic press and pressed together, always adding more paper to make up the deficiency caused by compression, until the mass will press no harder. The half rings are then put in their place, to prevent the plate from being pressed back by the elasticity of the paper. The roller is now to be dried sufficiently in a stove, the heat of which causes the paper to contract so as to be quite loose. The roller is then again taken to the press, and the unfixed plate being removed, more paper is added, and the whole again compressed until the roller is hard enough for the purpose to which it is to be applied. It is next turned truly in a lathe till it acquires a very smooth surface. The woodeut, Fig. 1841, shows the manner in which the calender is geared to make it a glazing calender. In this cut, $a$ marks the top cylinder of the calender, upon which is keyed a spur-wheel $b$; and $c$ is the under cylinder, upon which is also keyed a spur-wheel $d$. The intermediate or carrier wheel $c e$, when drawn into gear, reduces the speed of the under cylinder $c$ one-fourth. Now, the cylinder $\alpha$ being the one that gives motion to all the rollers, and revolving always at the same speed, the cloth, in its passage through all the rollers below the cylinder $a$, is carried through at a speed one-fourth less than if it passed only below the cylinder $a$; consequently, when it comes into contact with $a$, it is rubbed and thereby glazed, in consequence of the cylinder $a$ moving one-fourth quicker than the cloth, as above stated. The woodcut, Fig. 1842, shows the manner in which the rollers are lifted clear of each other when the machine is stopped. In this, ee are two rods of iron attached to the block or seat of the top roller; $b f g$, three bridges of malleable iron,
 rods when once they are adjusted to their proper places by pinching-screws. The bridge $b$ is placed $\frac{1}{2}$ an in. clear of the bearing of the cylinder $a$, when all the rollers are resting upon each other ; the bridge $f$ is placed 1 in . below the bearing of the paper roller $h$; and the bridge $g$ is placed $1 \frac{1}{2}$ in. below the bearing of the cylinder $c$. When the pressure-screws of the calender are lifted, the blocks of the top roller being attached to them, the rods $e c$ are lifted also, aud along with them the different rollers, as the bridges successively come into contact with their respective bearings.

The manner of passing the cloth through the calender varies very much according to the amount of finish required upon it. The various methods are accomplished by different arrangements of the gearing, so that a calender calculated to do all the different kinds of finishing becomes a very complicated machine, on account of the quantity of gearing required. For common finishing
the method of passing the cloth through the calender is as follows:-The cloth is passed alternately over and under a series of rails placed in front of the machine, so as to remove any creases that may be in it, and is then introduced between the lower roller A and cylinder B; returns between the lower cylinder B and the centre roller A; passes again between the central A and the upper B, and again returns between the top pair A B, where it is wound off on a small roller (hid in the drawings by the framing of the machine), pressing against the surface of the top roller $A$. When this small roller is filled with cloth it is removed, and its place supplied by another, to be in succession filled as the motion of the machine progresses.

Water-Mangle, with Two Copper and Three Wooden Rollers; designed and constructed by A. More. -This machine, Figs. 1843, 1844, differs nothing in principle and little in general construction

from the five-rollered calender above described, except in this, that it is intended for wet goods. it is drawn on a scale slightly less, but the views given and the lettering of the parts correspond to those of the preceding figures. A, A, A, three wooden rollers, and BB the two copper rollers of the mangle. These last consist of a copper cover upon a cast-iron body, through which passes a wrought-iron journal, differing from those of the wooden rollers in being round, whereas these are square between the bearings. The smaller of the two copper rollers, namely, the third in order, is in this arrangement the driver, the mangle being driven like the calender, by a system of reversing gear not shown in the drawings.

The pressure in the mangle is brought on by a system of levers, which differs slightly from that described. In this indeed there are strictly two distinct pressures: that brought on the axis of the middle roller by the lever $\mathbf{E}$, which is connected by a link with the weighted lever $\mathbf{F}$; and that transmitted through the whole system of rollers by the single-weighted lever D. The weight of this last is regulated by means of a set-screw which turns in a nut in the jaws of the lever D, and bears upon the set-block which rests upon the journal of the top roller. This pressure is thus transmitted downwards from the top roller throughout the whole set, and at the middle roller B is added to the pressure obtained by the lever $\mathbf{E}$. By this arrangement the pressure between the three under rollers is greater by the pressure of $\mathbf{E}$ than it is between the upper pair; but for very high pressure the lever $\mathbf{D}$ may be locked by set-pins, and the set-screws turned down by the hand-wheel G , until the requisite degree of pressure is obtained.

The manner of passing the cloth through this machine is the same as that already described in the calender, with this single exception, that before the cloth enters between the lower roller $\mathbf{A}$ and the small cylinder B, jets of water from a pipe, perforated with small holes, extending the whole width of the machine, are allowed to play upon the cloth, so as to impart to it sufficient moisture for causing it to receive the requisite degree of smoothness preparatory to the starching process, and at the same time allow the cylinder $B$ to free it from any impurities that may be remaining in it, by forcing them back with the expressed water.

Calender.-Description.-A, two cast-iron frames; B C D, three cylinders, Figs. 1845, 1846; E F G, three cog-wheels; HI, two force-screws: K L, two fly-wheels with handles. The cylinder $B$, which is in cast iron and hollow, is heated by another iron cylinder heated red hot. The material of the cylinder C is pasteboard; its axle is of wrought iron. These three cylinders must be perfectly round and parallel.

The wheel $\mathbf{F}$ forms the communication between $\mathbf{E}$ and $\mathbf{G}$, which rest upon the cylinders $\mathbf{B}$ and D. The relation of $\mathbf{F}$ to the circumference of the cylinders is such that when the machine is set
to work these cylinders slide, causing friction, and thus give a gloss to the cloth. The friction is variable according to the nature of the cloth or tissue.


In order to set the machine in motion, the flywheels K and L being turned in order to press the screws $H$ and I against the pillows of the first cylinder $B$, the cloth is placed between the rollers
 in the direction indicated by the arrows.

CALIPERS. Fr., Compas, Compas d'épaisseur; Ger., Taster; Ital., Compasso da grossezze; Span., Compas curvo.

See Compasses.
CALKING. Fr., Calfater; Ger., Kalfatern ; Ital., Calfatare; Span., Calafateo.
The process of copying or transferring a drawing by covering the obverse side of a design with black lead, or red chalk, and tracing lines through on a waxed plate, or wall, by pressing lightly over each stroke of the design with a point, which leaves an impression of the colour on the plate, paper or wall, is termed calking, which is often spelled calquing.

CALKING IRON. Fr., Calfat ; Ger., Das Kalfateisen; Span., Hierro de Calafateador ó Estopero.

An instrument, like a chisel, used in calking ships, boilers, caissons, and so on. These little implements are of various forms, fashioned to suit different sorts of work: one form is shown in Fig. 1847; $a$ the joint of the
 plates, $b$ the tool driven by a hand-hammer. See Hand-Tools.

CALORIMETER. Fr., Calorimètre; Ger., Wärmemesser ; Ital., Calorimetro; Span., Calorímetro.

The apparatus invented by Lavoisier and Laplace for measuring the amount of heat contained in certain bodies is often designated as a calorimeter; this apparatus Laplace operates by the melting of ice around the body to be tested. See Pyrometer.

CAM. Fr., Came, ou camme; Ger., Zahn-Daumen; Ital., Palmola, Dente; Span., Escéntrico.
A projecting part of a wheel or other moving piece, so shaped as to give an alternating or variable motion of any desired velocity, extent or direction, to another piece pressing against it, by sliding or rolling contact. Cams are much used in machines that involve complicated and irregular movements, as in the sewing and pin-making machines.

The cam-wheel A, Fig. 1848, revolving on its axis C, raises and lets fall the beam D; the heart-cam $B$, revolving on its axis $E$, gives an irregular motion to the rod $S$; and the cam $G$, revolving on its axis $\mathbf{F}$, moves H , and may be made to operate an alarm bell. The cam and the toggle-joint should be considered two of the mechanical pourers; then we should have, 1 , the lever ; 2 , inclined plane; 3, wheel and axle; 4, screw; 5, pulley; 6, wedge; 7, cam; 8, toggle-joint. When valves are employed instead of slides, for altering the flow of steam to the steam-cylinder, it is usual to move them by cams. In the rough sketch, Fig. 1849, P is a pipe connected with the boiler, Q a port to the cylinder, and V a valve, which is here represented single, but it is usually of the double-beat kind, shown in Fig. 1850. The valve V, Fig. 1849, closes the passage from P to Q, the valve-rod R V, passing through a stuffing-box in the cover of the valve-box, terminates in a roller R, which bears upon a cam S, fixed on a shaft T, caused to rotate by the engine. This cam is a disc, partly circular, with part of it, S , projecting to a greater distance from the centre. As long as the roller $R$ bears upon the circular portion, the valve $V$ remains down upon its seat; but as the projecting part of the cam is brought by the revolution of the shaft under the roller, the valve-rod is pushed up, and the valve lifted to allow the passage of the steam. When it is desirable that the valve should be kept open during a greater or less portion of the revolution of $T$, the cam is sometimes made with steps of various extent, on any of which the roller may be made to bear at pleasure, as shown in plan and section, Fig. 1851. To prevent the valve being
damaged from repeated blows against its seat, a dash-pot is employed, which is a small cylinder, partly filled with fluid, and having a loosely-fitting piston to ease the blow of the falling weight. The stationary under-part of the dash-pot is usually filled with water, and the plunger rises and falls with the valve-stem. See Dash-Рot. In Fig. 1852 is represented the gearing for operating the steam and exhaust valves of the Ridgwood pumping engines, belonging to the Brooklyn Water Works, New York.


The following description will show the different functions which cams have to perform in working this gearing. The steam-piston is shown at half-stroke on its downward movement, the various parts of the gear being drawn in exact accordance with that position and direction, the upper steam and lower exhaust valves being open. The frame or yoke $a$, with the inclines $b$ and $b$, on its sides, receives its motion from the beam, and moves in the same direction as the steam-
piston, and is for the purpose of closing the steam and exhaust valves; two levers $c$, with tne rollers on the upper ends, transmit the motion from the inclines through the rod $d$, arms $e$ and $f$, rod $g$, and $\operatorname{arm} h$, to the exhaust-valve rock-shaft; the exhaust-valve being closed when the lever $c$ has made half its movement, or on reaching its vertical position. The steam-valves are closed by the same cam-yoke $a$, through the rod $i$, lever J, arm K, rods $l$, $l$, and rock-arm or cam $m$, communicating motion to the closing-arm A of the steam-valve. The closing-arm A is in the form of a segment; the face being made of two curves, the difference in their radii being equal to the lift of the steam-valve: this segment or cam is adjustable by hand, that is, the closing-face can be brought sooner or later under the toe of the lever B or upper steam-valve, performing the closing or "cut off" at the point required. The water-cylinder C is used exclusively for opening the steam and exhaust valves, and is furnished with a piston, admission and exhaust ports, valves, \&c., like any steam-cylinder. The piston-rod is attached to a cross-head, and connected to the levers $c$, by the rod D. The admission and exhaust valve are operated from the double-curved arm or cam E, on the rock-shaft $\mathbf{E}$. The slot in the arm $\mathbf{E}$ is composed of two curves joined together in the vertical centre by an inclined slot; the difference in the radii of the curves being equal to the movement of the valve, and the length of the incline determines the time of action. On the end of the lever $G$ is a roller revolving freely on a journal, and fitting the slot on the arm E, the vibrating motion of the arm raising or depressing the roller from one curve to the other, imparting a like motion to the other end of the lever G, and through the rod H and right-angled arm I to the valve-rod K. It will be observed that this valve motion is intermittent. There is another valve in the water-cylinder chest which we call the supply-valve; as it opens and closes the communication between the water-chest and the rising main, and is operated from the roll-lever $c$, through the connecting-rod $\mathbf{L}$ and lever M, attached to the valve-rod; this motion to the supply-valve is also intermittent, and made by the slot in the rod L and lever M. The ends of the lower levers $\mathbf{B}, \mathbf{B}^{\prime}$, of the steam-valves are connected to weighted plungers $\mathbf{N}, \mathbf{N}^{\prime}$, working in small open cylinders, or dash-pots $\mathbf{R}^{\prime}, \mathbf{R}^{\prime}$, one for each valve; the gravitation of the weighted plungers opens the steam-valves through the rods $O$ and lever $P$, attached to the valve-stems, and the time taken to open the valve wide being regulated by the velocity of the water forced out of the dash-pots, a small cock being fitted to each for this special adjustment. This completes the description of the various parts of the gearing and their separate duties; we shall now show the combined action of the whole in relation to the steam-piston.

The steam-piston having reached midway on its downward stroke, of course the frame or yoke $a$ is also at mid-position in the same direction, and the upper steam-valve open-its weighted plunger unlatched and at the bottom of the cylinder ; the lower exhaust-valve is also open, leaving free communication between the under-side of the steam-piston and the condenser. These are positions of the parts as illustrated in Fig. 1852. Now, as the cut off usually takes place at or about half-stroke in this engine, it will be seen, by reference to the figure, that the closing part on the periphery of the arm $A$ is just entering under the toe or cam $Q$ of the lower lever or upper steam-valve; and very little more movement of the arm A will close the valve-that is, the lower will be lifted by the action of the arm, transferring its motion through the rod $O$ and lever $P$ to the valve-stem, forcing it downward and closing the valve, at the same time the lower lever B on its lift is carried with its weighted plunger to the top of its dash-pot R. The latch-bolt $s$, shown in the figure as withdrawn, enters a socket in the plunger by the action of a spring on its back, and holds the plunger there until it is time to open the upper steam-valve again on its next downward stroke; the closing-arm A moves on without producing any further motion to the lower lever B, its toe simply resting on the curve of the arm as it passes under it. The steam-valve being now closed, the balance of the down-stroke is made by the exhaust steam; the frame or yoke $a$ is descending, and the water-cylinder piston C is at full stroke towards the steam-cylinder; the admission-valve K of the former has just accomplished its half-movement, that is, it is square over its cylinder ports. When the frame or yoke $a$ has deseended so that the lower end of the upper incline $b$ comes in contact with the roller $t$, on the lever $c$, the water-cylinder valve K has completed its movement, opening the back port to the exhaust passage, and removing the water on the back of the piston, and leaving the front port open ready for the admission of pressure on the front side of the piston. The motion of this admission-valve ceases now, or at least until the frame $a$ has arrived at the same position on the up-stroke, when a similar but a reverse motion takes place. This cessation of motion to the admission-valve is accomplished by the rock-arm E having moved its curved slot or cam over the roller, the movements to the valve attachments being produced while the inclined part of the slot is passing under the roller of the lever G. The frame or yoke $a$ is still descending under the operation of the expanding steam; the upper incline $b$ has forced the lever $c$ over its vertical position or half-movement, closing gradually the lower exhaust-valve through the various connections, rock-arms, and toes. The duty of the upper incline $b$ ends here for the down-stroke; at the same instant the admissionvalve covering the supply port between the water-cylinder chest and rising main is opened by the action of the rod L and lever M, and water under pressure admitted to enter the aiready open port of the water-cylinder, carrying the water-piston to the end of its stroke, and with the lever $c$, through the connecting-rod D , which movement opens the upper exhaust-valve, and permits the steam that has just expended its power on the down-stroke to escape to the condenser; by this same movement the short arm V comes in contact with the end of the slot in the latch-bolt of the lower steam-valve, withdrawing it, and allowing the lower steam-valve to be opened by the gravitation of the weighted plunger N , through similar rods, levers, and so on, as described for the upper steam-valve. The engine is now reversed and commencing its upward stroke; but the direction reversed, the lower incline $b$, on the frame $a$, performing a similar duty on the upper stroke as the upper one did on the down-stroke, and so on continually. In such movements cams perform important operations. See Bank-note Printing Machine. Battery. Brick-making Machines. Mechanical Moveyents. Pin-mafing Machines. Press. Sewing Machines.

## CAMERA LUCIDA.

The camera lucida is a little bit of peculiarly-shaped glass, placed on a prop, and it enables any person, although that person be ignorant of the rules of perspective and but little accustomed to make drawings, to take with a pencil or pen on paper the lines and shades of any machine or other object with ease and accuracy.

This useful little instrument, conveniently mounted for use by Elliott Brothers, 449, Strand, is shown in Figs. 1853, 1854; the small four-sided glass prism is set at K .

Fig. 1856 is an enlarged section of this prism at right angles to the edges. $A$ is a right angle, and $B$ an angle of $135^{\circ}$. The angle at the eye and the one opposite are equal; each of those angles must therefore be equal to $67 \frac{1}{2}^{\circ}$ : $135=90+\frac{90}{2}$, and $67 \frac{1}{2}=45+\frac{45}{2}$.

The bit of glass, or glass prism, which is not much larger than a thumbnail of ordinary size, is held in a brass frame K, Fig. 1855, which is attached to an upright rod, having at its lower end a screw clamp, to fix it to the edge of a dra wing-board or table, Figs. 1853 , 1854. The prism K is fixed at the height of about 10 or 12 in . from the table, and has its upper face, A, Fig. 1856, that is K, 1855 , nearly horizontal. Rays coming from an object, R S, Fig. 1856, and falling nearly perpendicular on the first surface, enter the glass prism, and undergo total reflection from the contiguous surface, then they fall at the same angle on the next surface, and are totally reflected again; lastly, they emerge nearly perpendicular to the horizontal surface, as represented in Fig. 1856. The eye receives the emergent rays, and perceives the image $r s$, with all the shades,

lines, and colours of $R \mathrm{~S}$, on a sheet of paper upon a drawing-board or table to which the instrument is made fast. If the lines of the image are traced with a pencil, a very correct design
is obtained; but there is some difficulty in seeing both the image and the point of the pencil, for the rays from the object give an image which is farther from the eye than the pencil. This difficulty is readily surmounted by a little practice, and by placing between the eye and the prism a lens, which gives to the rays from the pencil and those from the object the same divergence. In this case, however, it is necessary to place the eye very near the edge of the prism, so that the aperture of the pupil is divided into two parts, one of which sees the image, and the other the pencil. The arrangement of the horizontal face, Fig. 1855, effects this object to the greatest nicety. Amici's camera lucida, represented in Fig. 1857, is preferable to that of Wollaston, Fig. 1856, inasmuch as it allows the eye to change its condition to a considerable extent, without ceasing to see the image and the pencil at the same time. It consists of a triangular glass prism, B Dr, right angled at B, having one of its perpendicular faces B D turned towards the object PQ that has to be drawn at $p q$; the other side of the prism $\mathbf{B} r$ is placed at right angles to an inclined plate of glass, $n \mathrm{~m}$. The rays P Q A, proceeding from the object, and, entering the prism, are totally reflected from the base at $\mathbf{C}$, and emerge in the direction $s t$. They are then partially reflected from the glass plate $m n$ at $t$, and form a vertical image of the object PQ , which is seen by the eye in the direction $t p q$. The eye, at the same time, sees through the glass $m n$, the point of a pencil applied to the paper, and the outline of the object may be traced: when straight lines have to be traced, a ruler can be applied.

CANAL. Fr., Canal, Ger., Kanal; Ital., Canale; Span., Acéquia, Canal.

1. Canals differ in this regard from rivers, that they have a regular bed, having throughout the same inclination and the same profile; and they carry down the same volume of water throughout their length. In case one of these conditions is not fulfilled, where, for instance, after a certain slope, another is assumed, there will result two canals, the one succeeding the other.

If from the point $o$, Fig. 1858, at the bottom of the canal, a horizontal line o $p$ is drawn, its corresponding vertical $q p$ will be the slope of the canal for the length oq. It is called the absolute slope, if $o$ and $q$ are the extremities of the bed of the canal ; and the relative slope, or the slope per foot, if $o q$ is 1 ft . long. Calling the slope $p$, if L is taken for any length of a canal, $D$ being the difference of level between the extremities of this portion, we have $p=\frac{\mathrm{D}}{\mathrm{L}}$; or, if $e$ represents the angle of inclination $p=\sin$. $e$.

The section of a canal, or any water-course, is the area of the section made by a plane perpendicular to the axis of the current; in a rectangular canal, if $l=$ breadth and $h=$ depth, $s$, or area of section, is $s=l h$; if it is trapezoidal, $l=$ breadth at bottom, and $n$ the slope of the sides, or the ratio of the base to the height, then $s=(l+n h)$ $h$ or $s=(l+\cos . f . h) h$, where $f$ is the inclination of the sides to the horizon.


That part of the contour of the fluid section in contact with the bed or bottom, as well as sides or berms, is called the wetted perimeter of the section. Designating it by $e$ for rectangular canals, we have $c=l+2 h$; for trapezoidal, $c=l+2 h \sqrt{n^{2}+1}=l+\frac{2 h}{\sin . f}$

Dubuat gives the name of mean radius of the section, for the ratio of the area to that of the wetted perimeter, or $\frac{s}{c}$.

Let us now examine the nature of the motion of water in canals, that is to say, the nature and expression of the forces which produce it; thus establish the formulæ of this motion, with their various applications; and, finally, ascertain the quantity of water which canals can receive at their heads or inlets.
2. Nature of Motion in Canals.-Gravity is the sole force that acts upon a mass of water left to itself, in a bed of any form; it produces all the motion which takes place.

Whenever its action upon each fluid particle (whether it be that which it exerts directly downwards, or that indirectly produced by the lateral pressure of the adjoining particles) is destroyed, so that the fluid mass is brought to a state of rest, its surface will be horizontal. Reciprocally, when the surface of a fluid is horizontal, exception being made for any impulse before impressed upon it, all action of gravity will be destroyed, and no motion can take place. But as soon as this surface is inclined, motion takes place, and continues, even if the bottom of the bed is horizontal, and even if it should have a counter-slope for some distance. Whence the principle, admitted in hydraulics, that the motion of particles in a water-course is due wholly to the slope at the surface; this slope it is which is the immediate cause of motion, and enables gravity to act.
3. Let us examine the mode of action of this force, and what is its measure in the different cases that may occur, which are represented in Fig. 1858.

Suppose, then, a canal, in which the surface of water is parallel to the bottom of the bed, and consider the very small section A. The fluid particles which are on the bottom $a^{\prime} b^{\prime}$, will descend by the direct action of gravity, as down an inclined plane. Those which are above, up to the surface $a b$, forming, as it were, threads laid upon the first, will descend in the same manner. The effective portion of gravity, that which is not destroyed by the resistance of the bed, and which causes the motion, will be represented by the height $a c$. and this height will be $g \sin . \imath ; i$ being the inclination of the surface $a b$ to the horizontal $b c$. The indirect action of gravity, or the lateral pressure experienced by each particle, being the same in all directions, by reason of the parallelism of $a b$ and $a^{\prime} b^{\prime}$, will not occasion any motion.

Let us admit, now, a current with a surface more inclined than its bed, and represent a small
section of it by B. Take, then, into consideration, any particle, $m$, traversing the section in the direction $m n$. This particle, or rather the linear system of particles $m n$, will experience: 1st. The direct action of gravity, which we represent by the height $m$ of the inclined plane $m n$, or by its equal $c d, m d$ being taken equal to $n b$. 2nd. The indirect action due to the inequalities of pressure upon the two extremes of the system $m$ and $n$; at the upper extremity $m$, conformably to the rules of hydrostatics, the pressure is represented by the height of the fluid column $m a$; at the lower extremity it is represented by $n b$; the resultant of these two pressures, that which produces motion, will equal then $m a-n b=a d$ : as for the pressures which each particle of the system experiences at its sides, perpendicular to $m n$, they will be equal to each other, and reciprocally destroy each other, and have no effect. Thus the system $m n$ will be urged downwards by the two forces a d and $c d$, or by their sum $a c$, which is $g \sin . i, i$ being the inclination of the surface.

When the bed is horizontal, as in the section C, the direct action of gravity upon the particles in contact with the bottom will, it is true, be entirely destroyed by the resistance of the bottom; but the indirect action, or the inequalities of pressure, will amount to $a a^{\prime}-b b^{\prime}=a c=g \sin$. $i$. For all other particles $m$, the moving force will be as above, $m f+(m a-n b)=c d+d a=a c=g \sin$. $i$.

Finally, if the bottom has a counter-siope, as in D, the particles upon it will be urged back or up stream, by its relative gravity, $k a^{\prime}=c d$; but, on the other hand, they will be urged downward by the difference of the pressing columns $a a^{\prime}$ and $b b^{\prime}$, or by $a d$. Hence it follows that they will be impelled in this last direction by $a d-c d=a c=g \sin$. $i$.
4. It follows, from these different facts, that, in a water-course of any form, each particle, in traversing a section having an inclination of surface equal to $i$, receives from gravity an impulse represented by $g \sin . i$; that is to say, that if the impulse continues during one second, it will produce a velocity equal to $g \sin . i$; this, then, is the expression of the accelerating force, and is dependent solely upon the indication of the surface.

This slope, so to speak, may vary at every step, or it may be constant for a long space, in which case a longitudinal section of the surface of the current forms a right line. This is frequently the case in canals, properly so called, of a constant slope and profile; the surface lines and the bottom lines can neither converge nor diverge, and must be parallel ; the surface will then have the same inclination as the bottom, and the sin. $i$ will be $=\sin . c$, or $=p(2)$, and the accelerating force will be $=g p$.
5. From what has been said, water running in a canal is constantly subject to the action of an accelerating force; so that, if it encounter no other opposing force, it will descend with an accelerated motion, and its velocity would never be uniform. Nevertheless, it often attains this uniformity in a very short space of time, after which the acceleration is inappreciable. Experience proves this to be a fact; it is to be seen in most canals, even those of great slope. Thus Bossut, causing water to run in a wooden canal 656 ft . long, with a slope of 1 in 10 , and having divided the canal into spaces of 108 ft . each, has found that each division, excepting the first, has been traversed in the same time. There must then be, after a certain period of time, a retarding force, which destroys at each instant the effect of the accelerating force, and which is equal to it. Thus, water will move along with a velocity acquired in the first moments of its rumning; a phenomenon similar to that produced in nearly all motion; in that of machines, for example.

But in canals there can be no retarding force but that which comes from the resistance of the bed. This resistance cannot be called in question; from experiments made with a tube 2.06 ft . long, there was a discharge of $5 \cdot 22$ cubic ft. in $100^{\prime \prime}$; and when its length was doubled to $4 \cdot 12 \mathrm{ft}$., dimensions in other respects the same, it took $117^{\prime \prime}$ to discharge the same volume. Thus the velocity in the tube was diminished in the ratio of 117 to 100 ; and it can only be that the canal, by reason of its increased length, offered a greater resistance to the velocity ; it therefore resisted motion.
6. Let us examine the nature of this resistance.

When water passes over the surface of a body, there being no repulsion or negative affinity between the two substances, it wets this surface; that is to say, a thin lamina of fluid is applied to it, penetrating its pores, and it is retained there, both by this engagement of its particles, and by the mutual attraction of the particles for each other.

It is over such a revetment or watery covering, fixed against the sides of the canal, that the water which it conducts must pass. The thin sheet of this mass, immediately in contact with this covering, by sliding along and rubbing against it, mingles its particles with those of the covering -it adheres, and its velocity is retarded. In consequence of the mutual adhesion of the particles, this stoppage, gradually diminishing, is communicated from one to another of the adjacent layers, till it is felt by the most distant fillets. The mass, in consequence, receives a mean velocity less than would take place without the action of the sides and the viscosity of the fluid.

The cause of this diminution of velocity has often been attributed to the friction of the water against the sides of its bed. Such a friction, if it occurs at all, is of a nature entirely different from that of solid bodies against each other; it depends neither upon the pressure nor the nature of the rubbing-surfaces. Dubuat is convinced, by direct experiments, that the resistunce of water is independent of its pressure. He has never yet found any variation in the friction of water upon glass, lead, pewter, iron, woods, and different kinds of earth.

This last fact might be accounted for by observing that in all cases the friction can only take place upon the aqueous layer which covers the sides of the bed. But a friction independent of pressure? It would seem quite natural to admit that the resistance could proceed from no other source but the adhesion of the particles of water in motion, both among themselves and with those of the fluid-covering of the sides of the bed.

This adhesion has been measured by weights. Dubuat found that, to detach tin plates from tranquil water with which they had been brought in contact, there was needed, beside their own weight, an effort of 0.96 lb . avoirdupois to 1.03 lb . the square foot of surface.

Venturi, by means of a remarkable experiment, affords a direct evidence of the effect of adhesion,
which enables the particles of water in motion to catch up and carry in their train those which are contiguous to them in a fluid mass at rest. To a reservoir A, Fig. 1859, kept constantly full, was fastened a box filled with water, in which was placed a trough CD, open at its ends, and its kottom resting on the edge D. A small tube was placed in the reservoir, with its end at O. As soon as this was opened, the jet which issued, passing through the water which had found its way into the trough, drew with it the part adjacent; this was replaced by that immediately next it, which in its turn was replaced by the water in the box; so that, in a short time, the water fell from the level of $q \mathbf{D}$ to $g h$.
7. Since the resistance is from the action of the sides of the bed, the greater the extent of these sides, that is to say, the greater the wetted perimeter for any unit of length, the greater the amount of resistance.

But this resistance of the perimeter will be shared among all the particles of the section, since their motion is con-
 nected by a mutual adhesion; thus, the greater the number of particles, or the greater the section, the less will the velocity of each, and consequently their mean velocity, be changed. The effect of resistance will be in the inverse ratio of the section.

On the other hand, the resistance will increase with the velocity. The greater this is, the greater will be the number of particles drawn at the same time from their adhesion to the sides; and, further, it must draw them more promptly, and consequently expend more force; so that the resistance will be in the double ratio of the velocity. The viscosity of the fluid occasions still another resistance, which becomes more sensible, compared to the first, as the velocity is smaller. Dubuat has observed this important fact, and Coulomb, through a series of experiments, found that it is simply proportional to the velocity. Thus the expression of ratio between the resistance and velocity involves two terms; in one, the velocity is as the second power; in the other, as the first; this last, which is but a small fraction of the velocity, will disappear in great velocities; it is always inferior to the other, when the velocity exceeds $0 \cdot 23 \mathrm{ft}$., but below this it preponderates. In short, the resistance experienced by water from its motion in a canal, is proportional to the wetted perimeter, to the square of the velocity, plus a fraction of velocity, and is in the inverse ratio of its section. Experience proves that this is very near the truth.

With the symbols already adopted, in calling $b v$ the fraction of the velocity in question, and $a^{\prime}$ a constant multiplier, the expression of resistance will be $a^{\prime} \frac{c}{s}\left(v^{2}+b v\right)$.
8. After what has just been said upon the resistance of the bed and its effects, the different fillets of a fluid in motion in a canal will have a velocity the greater as they are removed from the sides of the bed; thus they will have different velocities. Nevertheless, in estimating the discharge of a canal, we may admit that the whole mass of water in motion is endowed with a mean velocity; which will be such as, being multiplied by the section of the canal, will give the volume of water passed in one second. So that if Q represents this volume, $s$ being the section and $v$ the mean velocity, we have $\mathrm{Q}=s v$.
9. From what has been stated above, it follows that the greatest velocity of a current will be at its surface-in its middle, if the transverse profile is regular-if it is not, then in portions very nearly corresponding with the greatest depths; it is there that is generally found the thread of water, or fillet of the greatest velocity.

This velocity of the surface, being that most easily determined by experiment, the knowledge of its ratio with the mean velocity is a subject of great interest in practice; it will enable us to determine this last velocity so as easily to calculate the discharge. The investigation of this ratio has been the object of many hydraulic observers, as we shall see in the article on Rivers; we confine ourselves here to what concerns canals.

Dubuat has made precise experiments upon this subject. They are in number thirty-eight. They were made with two wooden canals 141 ft . in length; the one of a rectangular form $1 \cdot 6 \mathrm{ft}$. wide-the section of the other a trapezium whose small base was $\frac{1}{2} \mathrm{ft}$., with its sides inclined $36^{\circ} 20^{\prime}$ to the horizon (making $n=1 \cdot 36$ ) the depth of water varied from $0 \cdot 17 \mathrm{ft}$. to 0.895 ft ., and the velocity from 0.524 ft . to 4.26 ft . Dubuat concludes, from these experiments, that the ratio of velocity at the surface, to that of the bottom, is greater according as the velocity is less, and that this ratio is entirely independent of the depth; that to the same velocity of surface corresponds the same velocity of bottom. He has observed also that the mean velocity is a mean proportional between that of the surface and that of the bottom. Calling $u$ the velocity of the bottom, V that of the surface, and $v$ the mean velocity, he gives the results of his observations by the formula

$$
u=(\sqrt{\mathrm{V}}-\cdot 298868)^{2} \text { and } v=\frac{\mathrm{V}+u}{2}=(\sqrt{\mathrm{V}}-\cdot 149434)^{2}+\cdot 022332
$$

Prony, after discussing the experiments of Dubuat, has thought this the more convenient formula, $v=\mathrm{V} \frac{\mathrm{V}+7 \cdot 78188}{\mathrm{~V}+10 \cdot 34508}$.

Here is a small Table of some values of $v$ corresponding to values of V , as given by this formula. Prony, taking a mean term, has thought that, in practice, we may take $v=0.8 \mathrm{~V}$; that is to say, in order to have the mean velocity of a current of water, we may diminish that of the surface one-fifth.
10. Formula of Motion.-We have two kinds of motion to consider. Most frequently, the surface of a current in a long and regular canal

| V. | V. | $v$ |
| :---: | :---: | :---: |
|  | mètres. | feet. |
| 0.25 | .8202 | 0.77 V |
| 0.50 | 1.6404 | 0.79 V |
| 1. | 3.2809 | 0.81 V |
| 1.50 | 4.9213 | 0.83 V |
| 2. | 6.5618 | 0.85 V |

assumes a constant slope, which is the same as that of the bottom of the bed, and this surface becomes parallel to this bed. Then all transverse sections are equal; the mean velocity is the same in each, and the motion is uniform.

But it often happens that the surface varies from point to point, and is not the same with that of the bottom; so that, at different points of the canal, the sections, and consequently their velocities, are no longer equal. Still, the quantity of water admitted in the canal remaining the same, upon each isolated point, the section of the fluid mass will be constantly the same, and the velocity then will always have an equal value: all, then, is constant, and the motion, without being uniform, will be permanent.
11. Uniform Motion.-We have already remarked that, when the water in a canal becomes uniform, the retarding force equals the accelerating force ; and that the expression for this last, in such kind of motion, is $g p$; so that we have $g p=a^{\prime} \frac{c}{s}\left(v^{2}+b v\right)$ : or. making $\frac{a^{\prime}}{g}=a$, we have

$$
p=a \frac{c}{s}\left(v^{2}+b v\right)
$$

If, at a portion of the canal where the motion is uniform, we take two points upon the surface of the fluid, whose distance apart we represent by $\mathrm{L}^{\prime}$, and difference of level or absolute slope by D , we have $p=\frac{\mathrm{D}}{\mathrm{L}^{\prime}}$, and $\mathrm{D}=a \frac{c \mathrm{~L}^{\prime}}{s}\left(v^{2}+b v\right)$. If we take the canal throughout its entire length, which we called L , and H being the difference between the head and foot of the same; from this difference H , we must take a height dae to the velocity $v$ of uniform motion, and it will be presently shown (27) that $\mathrm{H}-\frac{v^{2}}{2 g}=a \frac{c \mathrm{~L}}{s}\left(v^{2}+b v\right)$.
12. It remains to determine the two constant coefficients $a$ and $b$.

Prony, in combining the results of thirty experiments made by Dubuat, has undertaken and executed this determination. Some years afterwards, Eytelwein following the steps of Prony, but extending his observations upon ninety-one canals or rivers, in which the velocity varied from 0.407 ft . to 7.94 ft ., and the fluid section from $\cdot 151 \mathrm{sq}$. ft. to $28 \cdot 030 \mathrm{sq}$. ft., found $a^{\prime}=\cdot 0035855$, or $a=\cdot 000111415$, and $b=\cdot 217785$, the English foot being the unit.

Thus, putting for $g$ its value $=32 \cdot 18 \mathrm{ft}$., the fundamental equation for the motion of water in canals will be $p=\cdot 000111415 \frac{c}{s} v^{2}+\cdot 0000242647 \frac{c v}{s}$; or, observing that $v=\frac{\mathrm{Q}}{s}$ (8), Q being the discharge, $p s^{3}=\cdot 000111415 c \mathbf{Q}^{2}+\cdot 0000242647 c \mathbf{Q} s$. Of the four quantities, $\mathbf{Q}, p, s$, and $c$, or, remembering that $s=(l+n h), h$ and $c=l+2 h\left(\sqrt{\left.n^{2}+1\right)}\right.$ (1), of the four quantities, $\mathrm{Q}, p, h$, and $l$, three being given, this equation enables us to ascertain the fourth. As for $n$, the slope to be given to the banks, it will be indicated by the nature of the soil in which the canal is dug.
13. It is seldom that the velocity is found among the list of problems to be resolved; still, for any case where its direct expression is required, the first of the two equations above gives

$$
v=-0 \cdot 1088946+\sqrt{8975 \cdot 414 \frac{p s}{c}+\cdot 01185803}
$$

or, more simply, and with sufficient accuracy, $v=\sqrt{8975 \cdot 414 \frac{p s}{c}}-0 \cdot 1088946$.
14. Consequently, we have from $\mathrm{Q}=s v$,

$$
\begin{gathered}
\mathbf{Q}=s\left(-0 \cdot 1088946+\sqrt{8975 \cdot 414 \frac{p s}{c}+\cdot 01185803}\right), \text { or } \\
\mathbf{Q}=s\left(\sqrt{8975 \cdot 414 \frac{p s}{c}}-\cdot 1088946\right)
\end{gathered}
$$

15. In great velocities, those of $3 \cdot 2809 \mathrm{ft}$., for instance, or any above this, where the resistance is simply proportional to their square, we have

$$
v=94 \cdot 738 \sqrt{\frac{p s}{c}}, \text { and } \mathrm{Q}=94 \cdot 738 s \sqrt{\frac{p s}{c}}
$$

Let there be, for example, a canal, whose section is a trapezium $13 \cdot 124 \mathrm{ft}$. wide at top, 3.2809 ft . at bottom, and 4.92 ft . deep; with a slope of $0 \cdot 001$. Required, the quantity of water which it will convey.

We have $p=0.001 ; l=3.2809 \mathrm{ft}$. ; $h=4.9214 \mathrm{ft}$. With regard to $u$, or ratio of base to height of banks, the height is that of the trapezium, and the base is one-half the differeuce between the two bases; so that $n=\frac{13 \cdot 124 \mathrm{ft} .-3 \cdot 2809 \mathrm{ft} \text {. }}{2 \times 4.9214}=1$. From this, $s=(l+n h) h=$ $(3 \cdot 2809+4 \cdot 9214) 4 \cdot 9214=40366$ sq. ft. ; and $c=l+2 h \sqrt{n^{2}+1}=17 \cdot 2 \mathrm{ft}$. Consequently, Q, the quantity sought, is

$$
\mathrm{Q}=40366\left(\sqrt{8975 \cdot 414 \frac{40 \cdot 36 \times \cdot 001}{17 \cdot 2}+\cdot 01185805}-\cdot 108895\right)=180 \cdot 87 \mathrm{cub} . \mathrm{ft}
$$

If we neglect the term $\cdot 01185805$ under the radical, we have for $Q=180 \cdot 843$, which enly differs from the above by 027 .

The formula above for great velocities would give

$$
\mathrm{Q}=94 \cdot 738 \times 40 \cdot 366 \sqrt{\frac{001 \times 40 \cdot 366}{17 \cdot 2}}=180 \cdot 65 \mathrm{cub} . \mathrm{ft} .
$$

16. The slope is directly given by the fundamental equation which we have already estaelished (12).

The Canal de l'Ourcq furnishes both an example of the mode of its determination, and some remarks worthy of attention.

There were 106.61 cub. ft. of water a second to be disposed of; the projected navigation required there a depth of 4.9214 ft ; and in order that the water should always be at hand for the service of the fountains in Paris, it was necessary that it should have at least a velocity of $1 \cdot 1483$ ft .; the soil was such as to admit of a slope of $1 \frac{1}{2}$ base to 1 of height.

We have, then, $\mathrm{Q}=106 \cdot 61$ cub. ft .; $v=1.1483 \mathrm{ft} . ; h=4 \cdot 9214 \mathrm{ft}$.; and $n=1 \cdot 50$. Moreover, from the given terms of the problem, $s$ is known, for $s=\frac{\mathrm{Q}}{v}=\frac{106 \cdot 61 \mathrm{cub} . \mathrm{ft}}{1 \cdot 1483 \mathrm{ft} .}=92 \cdot 843 \mathrm{sq} . \mathrm{ft}$.; $l$ will also be known, since from the expression $s=(l+n h) h$, we deduce

$$
l=\frac{s-n h^{2}}{h^{-}}=\frac{92 \cdot 843-1 \cdot 50 \times 4 \cdot 9214^{2}}{4.9214}=11.483 \mathrm{ft} .
$$

consequently, we have $c=l+2 h \sqrt{n^{2}+1}=29 \cdot 227 \mathrm{ft}$.; whence the general equation,

$$
p=\cdot 0001114155 \frac{v^{2} c}{s}+\cdot 000024265 \frac{v c}{s}
$$

substituting the numerical quantities, gives $p=0.00005502$ : such is the slope indicated by the formulæ.

Girard, the engineer who planned the canal, arrived at very nearly the same result. But he has observed, with reason, that aquatic plants, growing always upon the bottom and berms of the canal, augment very much the wetted perimeter. and consequently the resistance; he remembered that Dubuat, having measured the velocity of water in the canal (du Jard) before and after the cutting of the reeds with which it was stocked, has found a result much less before the clearing. Consequently, he has nearly doubled the slope given by calculation, and has carried it up to $0 \cdot 0001056$; the length of the canal being 314966 ft ., this gives $33 \cdot 260 \mathrm{ft}$. of absolute inclination.
17. If the dimensions $l$ and $h$ were the one unknown, and the other one of the given quantities of the problem to be solved, we take the values of $c$ and $s$ as functions of these two dimensions, and substitute them in the fundamental equation (12); $l$ would then be deduced by the resolution of an equation of the third degree, and $h$ by that of an equation of the fifth degree.

Let us determine, for example, the width to be given at the bottom of a canal, appointed to conduct $123 \cdot 60$ cub. ft. of water, with a depth of $4 \cdot 9213 \mathrm{ft}$., the slope being $0 \cdot 0001$, and the soil of such a character as to require for slope the base to be twice the height. Thus, $\mathrm{Q}=123 \cdot 60 \mathrm{cub}$. ft. ; $p=0 \cdot 0001 ; h=4 \cdot 2649 \mathrm{ft}$. ; and $n=2$.

We substitute these two last quantities in the expressions of $s$ and $c(1)$, which in their turn are substituted in the general equation. This will involve, then, only the unknown term $l$; and, making all reductions, and arranging according to the powers of $l$, we have

$$
l^{3}+23 \cdot 913 l^{2}-46 \cdot 578 l-3832=0
$$

Substituting for $l$, we find, on trial, $l=11 \cdot 138 \mathrm{ft}$.
18. Most generally $l$ and $h$ are not given terms of the problem; we have only $\mathbf{Q}$ and $p$, or the volume of water which the canal ought to conduct, and the slope which it should have, leaving the engineer to determine the width and depth. To obtain these two unknown quantities, there is but one equation; the problem, therefore, is indeterminate. The engineer then supplies the gap, in giving such a figure as he deems best adapted to the profile of the projected canal; this figure, indicating the relation between the two dimensions, furnishes the equation which was hitherto needed.

In the choice of this figure, regard must be had to the object most important to be fulfilled, a.d that that is adopted which fulfils it with least expense of construction and of maintenance. When it is desired to convey the greatest possible quantity of water to the point where the canal empties, according to the formula of discharge (14 and 15), the volume of water brought down is so much the greater, as the section of the fluid mass is greater, and as the wetted perimeter is smaller; consequently, we must take a figure which, with the same perimeter, presents the greatest surface.
19. Geometry informs us that the circle has this property. The semicircle, and therefore a semicircular canal, has the same property, the ratio between the semicircle and semicircumference being the same as that between the circle and entire circumference.

Then follow the regular demi-polygons, and with the less advantage, as the number of their sides is less; and so among the most practicable forms we have the regular demi-hexagon, the demipentagon, and finally the half-square.

But these figures are not admissible for canals in earth excavations; their berms, not having sufficient slope, would cave in.

In order that they should be sustained without revetment, they should have a slope of from 1.50 to 2 of base to height, as there is more or less consistency in the soil; in the regular semi-hexagon, where the slope is larger than the other named polygons, it is only $0 \cdot 58$. A slope of 1 is only adopted in excavations of small importance or for temporary use ; but for canals, the slope of 2 to 1 is usually adopted, and sometimes $2 \frac{1}{2}$; such was the slope adopted at the canal of Languedoc.
20. As the usual profiles of canals are trapezoidal, the question of figure of greatest discharge is reduced to taking, among all the trapeziums with sides of a determinate slope, that which yields the greatest sectiou for the same wetted perimeter.

Since the section $s$, or $(l+n h) h$, should be a maximum, its differential will be zero, and we have $h d l+l d h+2 n h d h=0$.

Since the perimeter remains constant, the expression $c=l+2 h \sqrt{n^{2}+1}(1)$ being differentiated, gives us $0=d l+2 d h \sqrt{n^{2}+1}$. The value of $d l$, derived from this equation and substituted in the preceding, gives $l=2 h\left(\sqrt{n^{2}+1}-n\right)$.

With this value of $l$, we have $s=h^{2}\left(2 \sqrt{n^{2}+1}-n\right)=n^{\prime} h^{2}$, by making $2 \sqrt{n^{2}+1}-n=n^{\prime}$; and $c=2 h\left(2 \sqrt{n^{2}+1}-n\right)=2 n^{\prime} h$.

Putting these equivalents of $s$ and $c$ in the fundamental equation of motion (12), it becomes

$$
\frac{p n^{\prime 2} h^{5}}{2}=0 \cdot 0001114155 Q^{2}+0 \cdot 0000242651 Q n^{\prime} h^{2}
$$

This and the preceding equation give for $l$ and $h$ the maximum sought.
Let us take, for example, $\mathrm{Q}=70 \cdot 6632$ cub. ft., $p=\cdot 0012, n=1 \cdot 75$. The second of the above equations is reduced to $h^{5}-1 \cdot 2522 h^{2}-178 \cdot 04=0$.

Making, by a first approximation,

$$
\begin{array}{llllllll}
h=2 \cdot 82 \mathrm{ft} ., \text { we have } & . . & . . & . . & . . & . . & -9 \cdot 6593=0 . \\
h=2 \cdot 85 \mathrm{ft} & . . & . . & . . & . & . . & . . & . . \\
h=2.850567 \mathrm{ft.} & . . & . . & . . & . . & . . & . . & +0.0002=0 .
\end{array}
$$

So that the true value of $h$ will be 2.850567 ft . This will give for $l$, which is $2 h\left(\sqrt{n^{2}+1}-n\right)$, $=1.5107 \mathrm{ft}$.

These dimensions are those of the stream. But the depth of the excavation slould be greater. It would be well to increase it to
$\begin{array}{llll}. . & . . & . . & 3.937 \mathrm{ft} . \\ . . & . & . . & 1.5105 \mathrm{ft} . \\ . . & . . & . . & 15.29 \mathrm{ft} .\end{array}$
The breadth at bottom remains the same $\quad . . \quad$.. $\quad . . \quad . . \quad$....
There will then be, a running foot of cut, an excaration of .. .. .. $33 \cdot 1$ culb. ft .
In homogenous earth, so long as the depth of excaration does not exceed $6 \frac{1}{2} \mathrm{ft}$., and the upper width $16 \frac{1}{2} \mathrm{ft}$., the expense of digging will be proportional to the volume of excavation, and the figure of least section will therefore be the most economical.
21. As for those canals where there is no fear of caving in, such as those excavated in rock, or protected with masonry, which are more particularly termed Aqueducts, as well as those in wood and mill courses, they most always have a rectangular form. Still, as we have seen, the regular demi-hexagon of the same section will conduct more water; but simplicity, facility, and economy of construction have prevailed. We must remember that the dimensions of the rectangle should have a width nearly double the depth of the fluid mass it is destined to carry, and consequently it should be $\sqrt{\frac{2 Q}{v}}$.
22. Permanent Motion.-We have seen (10) that permanent motion differs essentially from uniform in this, that the mean velocity in each section, remaining constant, is not the same as in the adjacent sections; consequently, the sections of water are no longer equal to each other, their depth is not the same, the surface of the fluid is not parallel to that of the bed of the stream, and its inclination varies from one point to another. We have examples of such motion in canals too short for the velocity to acquire a uniformity, at the head and foot of long canals, and in those whose bottom is horizontal.
23. Let there be a current endowed with permanent motion, and let us regard that part of it comprised between A and M, Fig. 1860. Through these two points of the surface, and through N infinitely near to M, imagine transverse sections A O MP, and $\mathrm{N} p$, made perpendicular to the axis of the current. From the points $\mathbf{A}$ and $\mathbf{M I}$ we draw the horizontal lines $\mathbf{A} E$ and M $t$; EM will be the fall of the surface from $\mathbf{A}$ to $\mathbf{M}$, which we designate by $p^{\prime} ; t \mathrm{~N}$, or the elementary increment of the slope, will be $d p^{\prime}$ or M N sin. $i, i$ being always the angle $t \mathrm{M} \mathrm{N}$ of inclination of the surface to the horizon. Let us consider upon the section A O, taken up stream for the point of departure, the particle having the mean velocity of the section, whatever else may be its position, and let $m m^{\prime}$ be the path which it describes as far as MP. Call $z$ the length of this path, $t$ the time employed in traversing it, and $v$ the velocity of the particle on arriving at $m$. We have then $m^{\prime} n^{\prime}=d z ; d t$ will be the time in passing $d z$, and $d v$ the increment of velocity during this passage (which will be $-d v$ when motion is retarded).

The forces which act upon the particle $m$ while traversing $m m^{\prime} n^{\prime}$ are: 1st, on one side, gravity, which tends to accelerate its motion, and whose whole action, according to what we have said in (3), is $g \sin . i ; 2 \mathrm{nd}$, on the other side, the resistance of the bed, which tends to retard its motion, and whose expression is (7) $a^{\prime} \frac{c}{s}\left(v^{2}+b v\right)$.

These two forces acting opposite to each other, their resultant, or the effective accelerating force, will be equal to their difference. But in all variable motion, the accelerating force is also expressed by the increment of the velocity, divided by that of the time, or by $\frac{d v}{d t}$; we have then $\frac{d v}{d t}=g \sin , i-a^{\prime} \frac{c}{s}\left(v^{2}+b v\right)$.

Multiplying all the terms by $d z$ (remarking that $\frac{d z}{d t}=v$, the space, divided by the time, equalling the velocity; remarking, further, that $d z \sin . i=d p^{\prime}$, since for $d z$ or $m^{\prime} n^{\prime}$ we may take MN, which will not differ from it save in extreme cases, but by an infinitely small quantity of the second order, and that $\left.\mathrm{MN} \sin . i=t \mathrm{~N}=p^{\prime}\right)$, we have $v d v=g d p^{\prime}-a^{\prime} \frac{c}{s}\left(v^{2}+b v\right) d . z$.
Such is the equation established by Poncelet.
Integrating, determining the constant for the section A, when $p^{\prime}=0, z=0$, and $v=v_{0}$, we have

$$
\frac{v^{2}}{2}-\frac{v_{0}^{2}}{2}=g p^{\prime}-\int a^{\prime} \frac{c}{s}\left(v^{2}+b v\right) d z .
$$

But (8) $v=\frac{\mathrm{Q}}{s}$; and if we designate by $s_{n}$ the area of the section at the final point $M$, and by $s_{0}$ that at the initial point A, which let us divide by $g$, and remembering that $\frac{a^{\prime}}{g}=a=0.00002426 \tilde{5}$ (12), and that $b=0.000111415$, we have finally

$$
p^{\prime}=\frac{\mathrm{Q}^{2}}{2 g}\left(\frac{1}{s_{n}^{2}}-\frac{1}{s_{0}^{2}}\right)+\int\left(0.0001114155 \frac{c \mathrm{Q}^{2}}{s^{3}}+0.000024265 \frac{c \mathrm{Q}}{s^{2}}\right) d z:
$$

a formula which gives directly the slope of the surface from $A$ to $M$.
In the application, the quantity under the sign $\int$ may be integrated by approximation. For this purpose, divide the arc A M or $z$ into portions, A B, B C, C D, and so on, whose lengths are such that the divisions of the are may be taken, without sensible error, for right lines. Designate these lengths by $z_{1}, z_{2}, z_{3} \ldots z_{n}$, and the areas of the sections at A, B, C $\ldots \mathrm{M}$, by $s_{0}, s_{1}, s_{2}$, $s_{3} \ldots s_{n}$, and by $c_{0}, c_{1}, c_{2} \ldots c_{n}$, their respective wetted perimeters. We measure or take immediately these lengths, sections and perimeters upon the given stream, and all will be known in the integral, which will become

$$
0.0001114155\left(\frac{z_{1} c_{1}}{s_{1}^{2}}+\frac{z_{2} c_{2}}{s_{2}^{3}}+\ldots \frac{z_{n} c_{n}}{s^{n}}\right) \mathrm{Q}^{2}+0.000024265\left(\frac{z_{1} c_{1}}{s_{1}^{2}}+\frac{z_{2} c_{2}}{s_{2}^{2}}+\ldots \frac{z_{n} c_{n}}{s_{n}}\right) \mathrm{Q}
$$

Let us represent by $M$ the multiplicator of $Q^{2}$, and by $N$ that of $Q$; let us make also $\frac{1}{2 g}\left(\frac{1}{s_{n}^{2}}-\frac{1}{s_{0}^{2}}\right)=\mathrm{D}$, the equation will then be $p^{\prime}=(\mathrm{D}+\mathrm{M}) \mathrm{Q}^{2}+\mathrm{N} \mathrm{Q}$.
24. From this we deduce $\mathrm{Q}=-\frac{\mathrm{N}}{2(\mathrm{D}+\mathrm{M})}+\sqrt{\frac{p^{\prime}}{\mathrm{D}+\mathrm{M}}+\left(\frac{\mathrm{N}}{2(\mathrm{D}+\mathrm{M})}\right)^{2}}$.

In our article on Rivers we shall have occasion to apply this formula, with its details, to streams whose form and delivery were otherwise known, and we shall see that its deductions are not far from the truth.

In canals where the slope of the bed and the profiles are constant, the calculations are much simplified; the depth of water at any one station will be sufficient to know its section and wetted perimeter; moreover, the depths, with the inclination of the bed, will give that of the surface.

As an example, let us determine the volume of water which a rectangular mill-course, 8.202 ft . wide, with a horizontal bed, will conduct to a mill. At four points, distant $328 \cdot 1 \mathrm{ft}$. apart, we

| No. | $z \cdot$ | h | $c$ | s | $\frac{2 \cdot c}{s^{2}}$ | $\frac{z \cdot c}{s^{3}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\begin{gathered} \text { feet. } \\ 0 \end{gathered}$ | feet. | ${ }_{18 \cdot 306}^{\text {feet. }}$ | sq. ft. | 0 | 0 |
| 1 | 328.09 | $4 \cdot 901$ | $18.00 \pm$ | $40 \cdot 20$ | 3.655 | -0909 |
| 2 | 328.09 | $4 \cdot 845$ | $17 \cdot 892$ | $39 \cdot 74$ | $3 \cdot 717$ | -0935 |
| 3 | $328 \cdot 09$ | 4.573 | $17 \cdot 348$ | $37 \cdot 51$ | $4 \cdot 045$ | -1078 |
|  | P | -479 |  |  | 11.417 | $\cdot 2922$ |

take four depths, noted in column $h$ of Table. Since the canal is rectangular, and $l=8 \cdot 202 \mathrm{ft}$., then $s=8 \cdot 202 \mathrm{hft}$., and $c=8 \cdot 202+2 h \mathrm{ft}$. We calculate these values for the different stations, and then, through these, those of $\frac{z^{*} c}{s^{2}}$ and $\frac{z^{\bullet} c}{s^{3}}$. All are in the above Table.

The canal being horizontal, $p^{\prime}=5 \cdot 052-4 \cdot 573=479 \mathrm{ft}$. We have

$$
\begin{aligned}
& \mathrm{D}=\frac{1}{64 \cdot 364}\left(\frac{1}{37 \cdot 51}-\frac{1}{41 \cdot 44^{2}}\right)=. . \\
& . . \\
& . . \\
& \mathrm{I}=\text { sum of } \frac{z^{\circ} c}{s^{3}} \times 0.0001114155=. . \\
& . . \\
& \mathrm{z} \\
& \mathrm{~N}=\text { sum of } \frac{z^{\cdot c}}{s^{2}} \times 0.000024265=. . \\
& \frac{\mathrm{N}}{2(\mathrm{D}+\mathrm{M})}=4 \cdot 0092, \frac{p^{\prime}}{\mathrm{D}+\mathrm{II}}=13866,\left(\frac{. .}{\left(\frac{\mathrm{N}}{2(\mathrm{D}+} \overline{\mathrm{II})}\right.}\right)^{2}=16 \cdot 0742 \mathrm{ft} .
\end{aligned}
$$

So that $\mathrm{Q}=-4.0092+\sqrt{13866+16 \cdot 0472}=113 \cdot 81 \mathrm{cub} . \mathrm{ft}$.

With the formula for uniform motion in taking a mean height between the extreme heights, and for a slope per foot, $\cdot 479$ divided by $984 \cdot 27 \mathrm{ft}$., the sum of the $\boldsymbol{z} \cdot$, we have

$$
\mathrm{Q}=-4 \cdot 299+\sqrt{15072+18 \cdot 478}=118 \cdot 54 \mathrm{cub} . \mathrm{ft} .
$$

25. The equation (23) which gives the slope of the surface of the current knowing some of the sections, will further, by the taking of one depth only, enable us to trace in its progress the curve described by a fluid point of the surface of a water-course in a canal, whose slope, profile and discharge are otherwise known.

For the place, when the depth of water is given by the aid of the profile, it will be easy to establish its section and wetted perimeter; let us designate them by $s_{0}$ and $c_{0}$. Take a second station, at a distance $z$ from the first, so small, that in this distance there shall be but little variation in $s_{0}$ and $c_{0}$, and so that they may be regarded as constant in the expression of the resistance of the bell, and we have $p^{\prime}=\frac{\mathrm{Q}^{2}}{2 g}\left(\frac{1}{s_{1}{ }^{2}}-\frac{1}{s_{0}^{2}}\right)+a c_{0}\left(\frac{\mathrm{Q}^{2}}{s_{0}^{3}}-\frac{\mathrm{Q}}{s_{0}^{2}}\right) z^{\circ}$.

We may neglect the first part of the second member at the first trial, which amounts to supposing a uniform motion throughout the whole length $z^{\circ}$, and we shall have the first approximate value of $p^{\prime}$. This will enable us, knowing the slope of the bed, to assign very nearly the depth of the stream at the second station, and consequently gives us $s_{1}$. All will then be known in the above equation, and we have a second and more approximate value of $p^{\prime}$ than the first. If it is thought best, we are able from this to calculate a third, which shall be still more exact. In the same manner, we may determine the depth at the third and fourth stations, and so arrive at all the ordinates of the curve required to be constructed.
26. But this method involves nuch uncertainty, and many suppositions, and often leaves us much embarrassed. We can avoid, in part, these inconveniences, and go directly to the solution of the problem, by introducing the slope of the bed in the problem, according to the method of Belanger.

For this purpose let us take in hand the first differential equation of (23); and we remark that the angle $i$, or $t$ M N, or M N s, Fig. 1860, is composed of two other angles: first, M N $r$, which measures the inclination of the surface upon $\mathbf{N} r$, parallel to the bottom of the bed $\mathbf{P} p$; designate this by $j$ : second, the angle $r \mathrm{~N} s$, which this bottom makes with the horizon, and which we have already called $e$; so that $i=j+e$, and consequently, $\sin . i=\sin . j \cos . e+\sin . e \cos . j$. But sin. $e=p(1)$, cos. $e=\sqrt{1-p^{2}}$, and cos. $j=1$, considering the smallness of the angle $j$; thus $\sin . i=\sin . j \sqrt{1-p^{2}}+p$, and the equation becomes

$$
\begin{equation*}
\frac{d v}{d t}=g \sin . j \sqrt{1-p^{2}}+g p-a^{\prime} \frac{c}{s}\left(v^{2}+b v\right) \tag{A}
\end{equation*}
$$

The term $\frac{d v}{d t}$ may take a finite form, which will depend upon the figure of the bed. When the canal is of small extent, we usually consider the slope as uniform, with a mean width $l$. From this supposition results $s=l h$ and $c=l+2 h$; so that $v=\frac{\mathrm{Q}}{s}=\frac{\mathrm{Q}}{l h}$, and $d v=-\frac{\mathrm{Q} l d h}{l^{2} h^{2}}$; moreover (23), $v=\frac{d z}{d t}$ or $d t=\frac{d z}{v}=\frac{l h d z}{\mathrm{Q}}$; then $\frac{d v}{d t}=\frac{\mathrm{Q}^{2} l d h}{l^{3} l^{3} d z}=\frac{\mathrm{Q}^{2} l}{l^{3} h^{3}} \sin$. $j$, since $\frac{d h}{d z}=\frac{\mathbf{M} r}{\mathrm{MN}}=-$ $\tan . i$ or $-\sin . j$.

Substituting this value in the equation [A], neglecting $p^{2}$, which will always be small compared to 1 , substituting for $g, a^{\prime}$ and $b$ their numerical values (12), and evolving sin. $j$, we have

$$
\sin . j=\frac{\mathrm{P} l^{3} h^{3}-\left\{0 \cdot 0001114155(l+2 h) \mathrm{Q}^{2}+0 \cdot 0000242651(l+2 h) l h \mathrm{Q}\right\}}{031073 l \mathrm{Q}^{2}-l^{3} h^{3}} .
$$

We have taken for the curve of a fluid thread of the surface of the stream, a polygon, each of whose sides has a finite length MN=z , and whose inclination relative to the bed is $j$ : the difference $\mathbf{M} r$ between the depths of the two extremities of a side will be its slope compared to this bottom; designating it by $p^{\prime \prime}$, we have $\sin . j=-\frac{p^{\prime \prime}}{z^{\prime}}$, and consequently,

$$
p^{\prime \prime}=\frac{p l^{3} h^{3}-\left\{0 \cdot 000111415(l+2 h) \mathrm{Q}^{2}+\cdot 0000242651(l+2 h) l h \mathrm{Q}\right\} z^{.}}{l^{3} h^{3}-\cdot 031073 l \mathrm{Q}^{2}} .
$$

The series of values of $p^{\prime \prime}$ will enable us to trace the polygon, or required curve.
Instead of comparing the slopes to the bed, we might compare them with the horizon, and thus have their value $p^{\prime}$, in observing that $p^{\prime}=p^{\prime \prime}+p$.

Lnlets.-Canals, with the exception of those for navigation, at their points of departure receive their water from reservoirs or retaining basins placed at their heads, and which most frequently are portions of the river whose level has been raised for this purpose by dams.

The head of the canal, at the point for receiving water, is either entirely open, or furnished with gates. Let us examine these two cases.
27. Canals of Open Entrance.-Water, on its entrance into an open canal, forms a fall, its level being lowered for a certain distance; then it is elevated a little by slight undulations, beyond which the surface takes and maintains a form very nearly plane and parallel with the bed, its slope and profile being always considered as constant. The velocity is accelerated from the top to the foot of the fall; it then diminishes during the elevation of its surface, and, soon after, its motion continues in a manner sensibly uniform. Dubuat, who has made a particular study of the circumstances of motion at the entrance of canals, and throughout their course, has found such an order of things established, that when the motion has become regular and uniform, the velocity of the surface is very nearly that due to the entire height of the fall, and that the head due to the mean
velocity is equal to the difference between the height of the reservoir and that of the uniform section. So that if II represent the height of water in the reservoir above the sill of entry into the canal, $h$ the height of the uniform section, that is to say, the constant depth of the current after it has attained a uniform motion, and $v$ the velocity of this motion, we have $H-h=0 \cdot 015536 v^{2}$; or rather, $0.015536 \frac{v^{2}}{m^{2}}, m$ being the coefficient of contraction which the fluid mass experiences at its entrance into the canal, a contraction which occasions a greater fall.

Dubuat, from several experiments made with wooden canals (9), with heights of reservoir H from $\cdot 394 \mathrm{ft}$. to 2.887 ft ., has found that $m$ varies from 0.73 to 0.91 ; but he remarks that, in great canals, where the height due to the velocity is small compared to the depth, the contraction will be less, and he thinks there would be no sensible error in taking $m=0.97$. Eytelwein assumes 0.95 for large canals, and 0.86 for the narrow, such as is adopted for most mill-courses. He, as well as Dubuat, supposes, for these coefficients, that the bottom of the canal is at the same level with the bottom of the reservoir, and that it is but a prolongation of it. If this were not the case, there would be a contraction at the bottom, and the value of $m$ would be a very little smaller.
28. The fall which takes place at the entrance of a canal, by diminishing the depth $h$, lessens the discharge $Q$, of which this depth is an element. So that, in order that the canal should receive all the water which it can afterwards convey, we must prevent the fall.

Theoretically, to accomplish this end we must enlarge the upper part of the canal for a length somewhat beyond $015536 \frac{v^{2}}{p} \mathrm{ft}$., so that the mean widths of the new profile should increase as they approach the reservoir, with an inverse ratio to the velocity of the stream at each of these widths, beginning with 0 , its value in the reservoir, till, by the uniform acceleration of its descent, it reaches $v \mathrm{ft}$. at the foot of the enlarged part. According to this law, the width at the reservoir should be infinite, since the velocity is zero. Such a case would be impracticable, and any approach to it would involve much labour and expense.

Consequently, the engineer who, without involving himself in unnecessary expense, desires to obtain for the canal all the water that can reasonably be expected, will be content to widen the approach, and in doing this must be governed by local circumstances. For instance, if the head is to be laid in masonry, he will give to the approach the form of the contracted vein; that is to say, taking the width of the canal as a unit, we shall have for length of the enlarged part $0 \cdot 7$, and 1.4 for width at the mouth, as comprising the full sweep to be given to the angles. But it is not worth while to exaggerate the advantages from these widenings, as the discharge by them will hardly be increased by more than some hundredths.
29. Dubuat also concludes, from his observations, that the velocity and section are uniformly established at a certain distance from the reservoir, just as if uniformity commenced at the origin of the canal. In this case we may suppose the fall to be made suddenly on its entrance to the canal, and thence the fluid surface maintains a uniform slope. Its value is obtained ( 1 and 11) by dividing the difference of level of the two points by their distance apart; one may be taken at the origin of the canal, and according to our supposition its level will be less than that of the reservoir, by a quantity equal to the height of the fall $\mathrm{H}-h$. Consequently, if D is the difference of level between the reservoir and any point of the surface at the distance $L$ from the reservoir, but where the motion has acquired its uniformity, $p$ being always the effective slope, we have

$$
p=\frac{\mathrm{D}-(\mathrm{H}-h)}{\mathrm{L}}=\frac{\mathrm{D}-0.015536 v^{2}}{\mathrm{~L}}
$$

30. With these given quantities we can resolve the various questions pertaining to a canal from a reservoir, supposing always that the motion becomes uniform, which will not be the case uuless the canal has a certain length, or should it have no inclination, or approach $90^{\circ}$, and so on.

Let us resume the equation, $\mathrm{H}-h=0.015536 \frac{v^{2}}{m^{2}}$, and in place of $v$ substitute its value, given
in (13), and we have $\mathrm{H}-n=\frac{0 \cdot 015536}{m^{2}}\left(\sqrt{8975 \cdot 414 \frac{p s}{c}}-\cdot 108895\right)^{2}$.

$$
\text { Moreover, we have } \mathrm{Q}=s\left(\sqrt{8975 \cdot 414} \frac{p s}{c}-\cdot 108895\right)
$$

By means of these two equations, in giving to $s$ and $c$ their expression, as functions of the dimensions of the canal, and substituting the preceding value of $p$, when $p$ is not directly given we can determine either the discharge, or the slope, or one of the dimensions; the other quantities being known. We give an example.

Suppose we purchase the site where it is intended to locate the entrance to the canal, with the condition that it shall be rectangular in form, open to the height of the dam, with a width of 13.124 ft ., and whose sill is to be 6.562 ft . below the ordinary low-water line. We wish to conduct this water to a mill distant $869 \cdot 438 \mathrm{ft}$., so that the surface of the stream, on its arrival there, shall not be over 1.4436 ft . below the low-water mark of the reservoir above. What will be the quantity of water conducted to the mill?

The cutting being made in the dam, the rectangular canal $13 \cdot 124 \mathrm{ft}$. by 6.562 ft . deep is fitted in; the clause of the grant forbids any attempt to enlarge the approach; and every alteration within the appointed limits would diminish the discharge.

Since the canal is rectangular, and $13 \cdot 124 \mathrm{ft}$. wide, we have $s=13 \cdot 124 h \mathrm{ft}$., and $c=13 \cdot 124 \mathrm{ft} .+2 h$; moreover, $p=\frac{1 \cdot 4436-(\mathrm{H}-h)}{869 \cdot 4384}=\frac{h-5 \cdot 1184}{869 \cdot 4384} \mathrm{ft}$., H being $6 \cdot 562 \mathrm{ft}$. Although the canal is large,
so that the coefficient of contraction would probably be above 0.95 , yet, to be prudent, we will take a mean between those indicated by Eytelwein, and call it $m=0.905$. With these values the first of the two equations above will be
$\begin{aligned} 6 \cdot 562-h & =\frac{0 \cdot 015536}{\cdot 905^{2}}\left(\sqrt{\left.5975 \cdot 414 \frac{13 \cdot 124 h(h-5 \cdot 1184)}{869 \cdot 4384(13 \cdot 124+2} h\right)}-\cdot 108895\right)^{2} . \\ \text { Reducing } 6 \cdot 562-h & =\cdot 018969\left(\sqrt{135 \cdot 47 h \frac{(h-5 \cdot 1184)}{(13 \cdot 124+2 h)}}-\cdot 108895\right)^{2} \text { gives us the value of } h .\end{aligned}$
To obtain it, put successively for this unknown quantity in the second member, several numbers; first, $6.23 t$ gives $h=5.889$ ft.; which in its turn gives $6 \cdot 114$. In this manner we obtain successively $5 \cdot 968,6 \cdot 053,6 \cdot 001,6 \cdot 040,6 \cdot 014,6 \cdot 034,6 \cdot 020,6 \cdot 027,6 \cdot 0237 \mathrm{ft}$. Thus the true value of $h$ falls between these two last numbers; let us take the smallest, $h=6.0237 \mathrm{ft}$. Then

$$
p=\frac{6 \cdot 0237-5 \cdot 1184}{869 \cdot 4384}=0 \cdot 001041 \mathrm{ft}
$$

All the quantities required to ascertain the discharge being known, we introduce them into the second equation, and so obtain $Q=417 \cdot 795$ cub. ft . Such is the volume of water per second which the canal will lead to the mill.

When the velocity of the current is required to be 3.28 ft . or more, we substitute the expression for velocity given in (15), and the two equations to be used will be

$$
\mathrm{H}-h=\frac{139 \cdot 44}{m^{2}} \cdot \frac{p s}{c} \text { and } \mathrm{Q}=94 \cdot 738 s \sqrt{\frac{p s}{c}} \text {; }
$$

or, supposing a mean width $l$, and taking always $m=\cdot 905$,

$$
\mathrm{H}-h=170 \frac{p l h}{l+2 h} \text { and } \mathrm{Q}=94 \cdot 738 l h \sqrt{\frac{p l h}{l+2 h}} .
$$

The slope $p$ will be given either directly, or by the expression $p=\frac{\mathrm{D}-(\mathrm{H}-h)}{\mathrm{L}}-$.
In the above example, the values of $\mathrm{H}, l$ and $p$, put in the first of these equations, which is of the second degree, will give readily $h=6.027 \mathrm{ft}$.; also, $p=\cdot 001045$ and $\mathrm{Q}=418.86 \mathrm{cub}$. ft .; results nearly identical with the preceding.
31. Among the questions relating to the admission of water in canals, there is one of too much interest to millwrights for us to pass it by without a notice in this work.

The force of a current to move machinery depends not only upon the quantity of water which it conveys, but also upon the height from which it falls; so that this force will be measured by the product of the quantity with the height of the fall of water. The greater the slope given to the canal, the greater will be the amount of water brought, and this is one of the factors of the product; but, at the same time, the fall (the other factor) is diminished, and it will be found that the product having been at first augmented with the slope, will after that be diminished, and then continue to decrease. There is then a maximum of power, which it is essential to determine and put in use. Without employing analytical formulæ, this determination can be arrived at in a simple manner, as will be seen in the following example.

Let us resume that given in the last number, and let us suppose the height of fall there to be 14.764 ft . The water taken by the canal has arrived at the mill with a loss of level of 1.447 ft .; consequently, the effective fall will only be $13 \cdot 317 \mathrm{ft}$. In multiplying this by the quantity of water brought down, 418.86 cub. ft., we have for the product $5577 \cdot 9$ cub. ft.; the corresponding slope was $0 \cdot 001045$. Let us increase this slope successively to $0 \cdot 0015, \cdot 002, \cdot 0025$, and $\cdot 003$; the respective products of the quantity by the fall will be $1859 \cdot 42,1931 \cdot 12,1939 \cdot 94$, and $1907 \cdot 45$ cub. ft . The slope of $\cdot 003$ has already occasioned a diminution; in trying that of $\cdot 0026$, the product will be $1938 \cdot 18$ cub. ft.; whence we conclude that the maximum of effect lies between the slopes of $0 \cdot 0025$ and $\cdot 0026$. Finally, as the variations of the product are very small between 0.002 and 0.003 , we adopt, between these limits, those best suited to the locality and nature of the machinery used; there may be some for which a great fall will be preferred.
We will remark that the given solutions of all the problems in question can be regarded only as simple approximations; for in order that they should be exact, the bases on which they rest, that is to say, the conclusions which Dubuat has drawn from experiments, should be explicitly confirmed by observations made upon great canals; and it would, moreover, be necessary to be quite sure that the water, before it reaches the extremity of the canal, has attained a uniform motion, and we have but limited means of coming to a positive assurance.

If water which is in the reservoir of a river to which a canal has been adapted, should arrive there directly, with an acquired velocity, the height of fall which takes place at the entrance will be less than that indicated (27) by a quantity equal to the height due to this velocity.

Canals with Gates.-When a canal receives its water through openings of a system of gates, established at its head, which is generally the case with mill-courses, either the upper edge of the orifice will be completely and permanently covered by the water already passed into the canal, or it will not.
32. If the head above the centre of the orifice is great, so as to exceed two or three times the height of the orifice, its upper edge will not be covered by the water below, and the discharge will be the same as if there had been no canal. Experiments with orifices in thin sides and furnished with additional canals, leave no doubt upon this subject; they justify an assertion, long since made by Bossut, the exactness of which has been questioned.

Bossut fitted to an orifice $\cdot 0886 \mathrm{ft}$. high and $\cdot 4429 \mathrm{ft}$. wide, made at the bottom of a reservoir, a horizontal canal of the same width, and 111.55 ft . in length; he produced in it currents under
heads of 12.468 ft ., 7.802 ft ., and 3.937 ft ., and he received at the extremity of the canal the same quantity of water that issued from the orifice when the canal was taken away.

The cause of this equality is apparent. When the water is urged by a great head, and consequently issues with great velocity, the contraction it experiences on all sides renders the section smaller immediately beyond the interior plane of the orifice, so that, on issuing, it touches neither the sides nor the bottom of the canal; it acts as if it were projected in air, and the discharge continues the same that it would if this were really the case. Beyond the contracted section the vein dilates, it is true; it joins the sides of the canal; it meets with resistance, and runs less swift; but then it is too far from the orifice to react against what issues from it, so as to reduce its discharge. This will always be given by the formula $m l^{\prime} h^{\prime} \sqrt{2 g \mathrm{H}}, l^{\prime}$ and $h^{\prime}$ being the width and depth of the orifice; $m$ will have the same value as for orifices in thin partitions.

Generally, we take 0.70 for the coefficient of ordinary gates of flumes. See Poncelet and Lesbros' experiments.

Without adopting another coefficient for each particular case, the volume of water which enters a canal furnished with large gates, and under a great head, may be had approximately by the formula $0.70 l^{\prime} h^{\prime} \sqrt{2 g \mathrm{H}}$.
33. When the water, impelled beyond the gates by a great head, falls into the cana1, it meets a resistance which diminishes gradually its first velocity, and so increases the section of its current. If the width of the canal is constant and equal to the opening of the gate, it will be the depth which receives the gradual increase; so that the surface of the fluid below the orifice, or rather below the point of greatest contraction, up to that where the increase of depth ceases, will present a counter-slope. Frequently masses of water will be detached from the summit, and will, rolling back, return towards the orifice; usually they will be retained, being as it were repelled by the velocity of the stream; though sometimes they will return even to the gate, and re-cover the orifice, though but for a moment. Even in this case the discharge will be the same as if there were no canal, and it will be calculated by the formula of the preceding number.
34. These phenomena do not occur when the head is small. Water, on issuing from the gates, is in contact with the sides of the canal; it experiences a retarding force, which is communicated to the fluid at the instant of its passage through the orifice; the discharge, and therefore its coefficient, is lessened ; but we have no further guide for its determination. There may be some cases where, with a very small head, the gate is without sensible influence; thus Eytelwcin has found the same discharge, whether the gate was wholly raised, or slightly dipped on the down-stream side.

But in case it is immersed any considerable depth, and the fluid vein at its issue is entirely covered over with still water, the height due to the velocity of issue will be the difference between the elevation (above any given point) of the surface above the gate and of that below the gate. For the elevation below the gate we take the height or depth of water in the canal, when its motion has become regular ; as that immediately at the gate would be found too small. Consequently, if $h$ is tho height in the canal, $\mathbf{H}^{\prime}$ the height up stream above the sill of the inlet, the discharge of the orifice of the gate, and consequently that of the canal, will be expressed by $m l^{\prime} l^{\prime} \sqrt{2 g\left(\mathrm{H}^{\prime}-h\right)}$. But the discharge of the canal, the motion having become uniform, is also (14) $s\left(\sqrt{8975 \cdot 414 \frac{p s}{c}}-\cdot 108895\right)$. We have then $m l^{\prime} h^{\prime} \sqrt{2 g\left(\mathrm{H}^{\prime}-h\right)}=s\left(\sqrt{8975 \cdot 414 \frac{p s}{c}}-\cdot 108895\right)$; an equation which enables us to solve the various questions relative to canals furnished with gates at their heads.

Suppose, for instance, we would determine the quantity $h^{\prime}$; we must raise the gate, at the entrance of a long rectangular canal of $4 \cdot 265 \mathrm{ft}$. width and 001 slope, in order that the water may have a depth of 2.625 ft . ; the width of the gate is 3.609 ft ., and the height of the reservoir 3.937 ft . We take $m=0.70$ (32). We have then $l^{\prime}=3 \cdot 069 \mathrm{ft} . ; \mathrm{H}^{\prime}=3.937 ; h=2.625 ; l=4 \cdot 265$; $p=0 \cdot 001 ; s=4 \cdot 265 \times 2 \cdot 625=11 \cdot 195 \mathrm{sq}$. ft .; $c=4 \cdot 265+2 \times 2 \cdot 625=9.515 \mathrm{ft}$. These numerical quantities, substituted in the equation above, give us $23 \cdot 209 h^{\prime}=35 \cdot 180$; whence $h^{\prime}=$ 1.514 ft .

The Suez Canal.-The plans and sections of the Suez Canal given in Figs. 1861 to 1867, are, with some trifling additions, the same as those compiled by Sir W. T. Denison from 'Compagnie Universelle du Canal Maritime de Suez: Carte générale de l'Isthme, etc., 1866 ;' and the plans of Port Saïd and of the Port of Suez, from 'Percement de l'Isthme de Suez: Actes constitutifs de la Compagnie Universelle du Canal Maritime de Suez, avec cartes et plans;' documents publiés par F. de Lesseps, the engineer of this great work. Our additions are taken from 'Histoire de l'Isthme de Suez,' par Olivier Ritt, 1869. Sir W. T. Denison's paper "On the Suez Canal" was read in the Institution of Civil Engineers, 16th April, 1867, and afterwards published in pamphlet form, edited by James Forrest, the secretary of the Institution. The thick black line in the plan, Fig. 1861, indicates the course, and Fig. 1862 is a vertical longitudinal section, of the canal ; the horizontal lines in Figs. 1861, 1862, are to a scale of 09 in . to a mile, and the ordinates, Fig. 1862. $\cdot 01 \mathrm{in}$. to a foot. Fig. 1864 is a cross-section at Port Saïd, and Fig. 1863 a cross or lateral of the canal at Suez; the scale of these sections is 008 in, to a foot. Fig. 1865 is a plan of Port Said and the harbour on the Mediterranean; Fig. 1866 is a plan of the port of Suez, on the Red Sea: the plans of these places are given to a scale of 1000 yds . to an inch. The motions and actions of the water of the canal may be calculated, by the rules just established, from the levels, inclinations, and dimensions registered on Figs. 1867, 1863, and 1864.

Col. Denison, in his notes, as the work was being completed, observed :-The scheme of the Suez Canal may be said to comprise two distinct undertakings. The first, and principal, is the con-
1861.
1861.


## CANAL.

struction and maintenance of a broad and deep salt-water channel on one level between Port Saïd on the Mediterranean, and Suez on the Red Sea. The second, preliminary in point of time, and indeed essential to the construction, as well as to the beneficial use of the canal, is the maintenance of a supply of fresh water sufficient for the wants of the population congregated along the line of canal, and specially at its two extremities.


The arrangements for the second of these undertakings have been completed. A canal, commencing at a place called Zagazig, to which water is brought from the Nile by one of the many branches from the main stream, has been carried to Suez, passing within about a mile or two of Ismailia, the central point of the main canal, and the head-quarters of the establishment of the Company. It is navigable for the whole distance, the fall from Zagazig to Suez being overcome by locks. From the point where the canal turns southward to Suez, a branch is carried first to Ismaillia, where it provides a supply for the inhabitants, and for some hydraulic machinery, by which water is forced into a double line of 9 -in. pipes, through which fresh water is carried along the side of the canal, supplying the various establishments along the line of about 50 miles in length, and a population at Port Saïd already numbering upwards of 10,000 ; and secondly, for a distance
of about a mile to the east of Ismailia, within which distance it is made to drop by two detached locks to the level of the Mediterranean. At the Suez extremity the fresh-water canal terminates in a lock, by which vessels drop into the creek which brings goods and passengers from the anchorage to the town.

In the immediate vicinity of the anchorage a dry dock, capable of taking in the largest steamer, is constructed.

The salt-water canal commences to the south-east of outer port, the ground being dredged out to the necessary depth, and to a width sufficient to give ample space for the exit and entrance of vessels. The canal sweeps a way in a curved line towards the north, passing to the eastward of the fresh-water canal through the lowest portion of the land, which is, in point of fact, very little above the level of the Red Sea. Figs. 1861, 1862.

At a distance of $11 \frac{1}{4}$ miles from the sea, the line crosses a spur from some nills to the westward, at a place called Chalouf: the cutting here consisted partly of a bed of hard conglomerate, 8 or 10 ft . in thickness, below which were strata of sand and clay. The surface of the soil at this cutting was about 12 ft . above the salt-water level, so that the total depth of excavation is 40 ft . The slope of the side was 2 to 1 , and there was left a bench of about 12 ft . in width, 3 ft . above the surface of the water; while another bench, 12 ft . below the water, and 9 ft . wide, formed a base for the stonework employed to face the upper part of the slopes. The stone was procured in part from the excavation itself, and in part from quarries on the west shore of the Red Sea, a few miles south of Suez. The work here was carried on with a good deal of method. Inclined planes were cut in each bank, up which the wagons filled with spoil were hauled by steam-engines, and then drawn to spoil-banks at convenient distances. Pumps discharged the drainage water into a portion of an old salt-water canal, said to have been excavated by one of the Pharaols.


Soon after leaving this cutting the canal had to pass through the Bitter Lakes, the surface of the water, or soil, in which is nearly on a level with the Red Sea. Here the cutting did not exceed the ordinary section of the canal.

The distance between Suez and Ismailia is about 50 miles.
A little east of Ismaïlia the branch from the fresh-water canal comes to an end, and passenger boats drop by two single detached locks to the level of the Mediterranean, into a salt-water canal about the same size as the other. For some distance this was only a branch, but at about 2 miles from Ismailia the branch enters the line of the main canal, which turns sharp to the left or northwards, and passes through a heavy sand-bank from 40 ft . to 60 ft . above the level of the surface water in the canal.

The works here were well executed: the contractor cut down the face of the excavation, and loaded the spoil into wagons; trains of these wagons were constantly in motion, being drawn by locomotives obliquely up the slope of the hill, and thence to spoil-banks at some distance on the west side of the canal. Some dredging machines were also employed to deepen the channel; the
soil thus raised was discharged into wagons and hauled up the bank. The amount of work done here was very great, as the sand-bank or ridge extended about 5 miles.

To the north of the sand-bank the canal entered the beds of some lakes-the soil of which was but a few inches above the Mediterranean. Here, in places, the soil had evidently a disposition to slide into the canal; as the trifling load of an embankment, sufficiently high to cover the freshwater pipes, had forced the bank out, in spite of some piles and sheeting, which had been driven and fixed to support it. This sort of work extended the greater part of the remainder of the distance to Port Saïd, about 10 miles from which the canal entered Lake Menzaleh, a shallow sheet of salt water, through which the line was marked by slight embankments, distant apart the full width of the canal. At several points between the deepsand cutting and the shore of Lake Menzaleh, the canal was opened to its full width. Dredging machines were employed both to widen and deepen it, and a great amount of activity and much skill were developed at this place.

The dredging machines were well put together. They were worked by powerful engines, and a variety of expedients had to be devised for the purpose of adapting them to the work they had to perform in lifting the spoil from a great depth, and discharging it at a point above the engine.

The main work at Port Saïd, Fig. 1865, was the jetty, or breakwater, which protects the port against the action of the north-westerly or prevailing winds. This jetty is formed of blocks of concrete, rectangular in form, and weighing 20 tons. Experience has shown that a block of this size ( 10 cubic mètres) and of this weight is sufficiently massive to withstand the action of the heaviest sea. The concrete is composed of seasand, dredged from the harbour, and of good hydraulic lime procured from Marseilles, the proportion of lime to sand being about 350 lbs . of the former to a cubic metre of the latter, or about 1 to 13. Sea-water is used to mix the ingredients, which are well worked and amalgamated in mills, ten of which were driven by one steam-engine. The mixture was poured into cases, or frames, of wood, where it was allowed to remain four days; the cases were then removed, and the blocks subjected to the influence of an Egyptian sun for two months, by which time they became solid enough to withstand the action of the sea. Each mill turned out three blocks a day.

The contractor engaged to furnish the blocks, and to deposit them at appointed spots, for 400 francs a block.

The mass will, of course, stand at a much steeper slope than it would were the blocks of smaller dimensions, and there will be a large space between the blocks, which will eventually be filled with sand drifted in by the sea.

The Suez Canal was officially opened on the 17th of November, 1869. At that time fifty ships had passed from the Mediterranean to the Red Sca, or vice versâ. From the 1st to the 17th of February, 1870, nineteen vessels, or a little over a vessel a day, went through from one sea to the other. The successful completion of this great work places its engineer, M. de Lesseps, high among the engineers of our time. In France he holds a position similar to that held in America by W. J. McAlpine, or to the one that John Scott Russell should hold in England. See Barrage. Concrete Machine. Dredging Machines. Locks and Lock-gates. Rivers.

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CANDLES. Fr., Chandelles, Bougies; Ger., Kerze; Ital., Candele; Span., Velas.
In its natural state, fat of animals is always associated with cellular tissue and other foreign matters, which must be separated before it can be used as candle stock.

In the process called dry melting, much practised by small manufacturers, the rough suet is cut into coarse pieces and exposed to the action of a moderate heat. By the more recent processes, the fat is not exposed to heat till it has been subjected to certain mechanical and chemical appliances, for the purpose of destroying the tissues. The first-named method possesses this decided advantage, namely, that the residue or cracklings can be profitably used as food for hogs, fowls, and so on. There is, moreover, an economy in fuel, while the simplicity of the process commends itself to the notice of inexperienced manufacturers. Disadvantages, on the other hand, arise: an obnoxious smell emanates from the heating of rough tallow which has been collected and suffered to remain till it has become rancid, and the cellular tissues, blood, or other portions, advanced towards putrefaction. Fat from animals recently slaughtered does not, however, yield any very unpleasant effluvia. Another and more important disadvantage, in an economical point of view, is found in the smaller amount of fat obtained, as portions always remain with the cracklings when heated in this manner. The first care of the chandler should be to impress on the mind of his tallow merchant the importance of a more careful treatment of the rough suet than is generally observed by the butcher. The fat ought to be freed from the membranous and muscular parts, then cut into thin slices and hung up in a cool place, not heaped up while yet warm. By operating thus, the disagreeable odour existing before melting, and increased during the process to an unbearable degree, can, at least, be delayed for several days.

First, the fat is chopped, for which purpose cutting machines are often used similar to the straw-cutting table; sometimes a thin, sharp-edged mince-hatchet is employed, about $2 \frac{1}{2} \mathrm{ft}$. in length. This is held with both hands, and the fat, spread out on a beech block, is chopped into small pieces in all directions. A third instrument is a kind of stamp trough with muller, having a sharp blade in the form of an S, a contrivance frequently adopted for cutting beets. A more desirable and valuable instrument, however, is the ordinary rotary sausage-cutter. The fat is then placed in melting caldrons (hemispherical in form, and in this country made of cast iron), which are heated by open fire. These caldrons are covered with movable tin-plate hoods, so adjusted that, by means of pulleys, ropes, and counter-weights, they can be easily raised or lowered, whilst at the same time they serve to carry off the offensive vapours arising from the heated fat. Water is sometimes mixed with the fat in the caldrons, and this addition is specially beneficial when the fat has been long kept during the summer months, and thereby lost its natural moisture by evaporation. By gradually raising the temperature in the pan, the fat runs from the cells, and the whole is kept boiling from 1 to $1 \frac{1}{2}$ hour. The mixture of water with the fat-bubbles imparts to the liquid a milky appearance, but as soon as the water is volatilized the fat becomes clear. During the whole operation of melting and boiling, the ingredients must be constantly and thoroughly stirred in order to keep the fat and cracklings in incessant agitation, otherwise pieces of unmelted suet, coming in contact with the sides or bottom, would become scorched and acquire a brownish tint, of which the whole melting would necessarily partake. Scorched tallow is not very readily whitened. For separating the melted fat from the cracklings, it is ladled off from the caldron into a fine willow basket, or a copper box perforated at the bottom with innumerable small holes, set over large copper coolers, and allowed to remain undisturbed till all foreign matters have settled down. Before it congeals, it should be transferred into small wooden pails.

This operation is continued so long as the cracklings yield any fat; and during the process the heat must be maintained at a moderate temperature, to avoid scorching the materials. When the cracklings begin to harden they acquire a darkish tint, and hence are said to be browning. They are then pressed, and the fat thus obtained possesses somewhat of the brown colour of the cracklings, but not so much as to render it unfit for use as soap stock; it may, consequently, be mixed with that which has spontaneously separated while heating.

New Methods of Rendering.-The complaints of parties residing in the neighbourhood of candle and soap works, in consequence of the offensive effluvia disseminated by these establishments, have led to the invention of new apparatus, as well as to the introduction of new processes of rendering, until an entire reformation has resulted in the melting process, which we propose briefly to describe, long experience having demonstrated their utility.

No doubt the apparatus invented by d'Arcet, of Paris, in 1834, and introduced by the board of health of that city, has been tested in other places, where it did not interfere too much with the workmen's freedom of action, and the ready supervision of the melting process. As it is, moreover, applicable and may prove serviceable in other branches of manufacture, we here offer a few remarks relative to this invention.

One essential and valuable feature in his invention is his suggestion for conducting the rising vapours, consisting chiefly of hydrogen and carbon, through channels under the grate of the rendering pan, and using them as fuel. The pan is also covered with a strong iron plate, the front third of which can be lifted by means of a knuckle whenever it is necessary for stirring, filling, or emptying the kettle. D'Arcet was likewise the first who employed certain chemicals for the purpose of neutralizing or destroying the noisome effluvia arising from the pans. His propositions are found to be, as yet, the most valuable in use. In the process recommended by him, 50 parts,
by weight, of diluted acid (oil of vitriol) are first put into the kettle, then 1000 parts, in weight, of chopped fat are gradually added in four equal portions; and lastly, 150 parts of water, to which 5 parts, in weight, of sulphuric acid of $66^{\circ} \mathrm{B}$. have been previously added. The whole is next heated. Under the influence of the acid, which partly destroys, partly solves the membranes, the rendering of even greater amounts of fat is effected in $1 \frac{1}{4}$ to $2 \frac{1}{2}$ hours; two hours, however, are seldom required. The inventor's proposition of using acids was made when pans were heated by the direct action of the fire; but now, for various substantial reasons, steam is more generally employed. This, however, does not prevent the gases arising from the pans being thrown into the furnace and thereby aiding combustion. It is obvious, moreover, that in the boiler of d'Arcet, stirring, as well as filling or emptying the contents of the pan, cannot be accomplished so readily as in an open pan; nor can these processes be performed without opening the covers, when the noisome vapours escape into the room, to the annoyance of the operators. To obviate this, a contrivance similar to that used by distillers in the mashing process could be introduced with decided advantage of comfort, as well as of certainty, for keeping up the necessary motion, to prevent adhesions to the sides or bottom of the vessel, and consequent incidental scorching.

The same may be said in regard to the pan for boiling fats lately patented in this country by W. H. Pinner, who yet claims the conducting of the noxious vapours into the fire as a novelty.

Wilson's Process has first been described by Morfit in his 'Treatise on the Manufacture of Soap and Candles.' The chief feature of this process is to steam the rough suet for ten or fifteen hours in a perfectly tight tank, under a pressure of 50 lbs . to the square inch, or more when lard is being rendered. A higher pressure, according to Morfit, is not profitable, for, though expediting the process, it produces an inferior quality of fat. No chemicals are used. The apparatus consists of an upright cylindrical vessel, made of strong boiler-plates, tightly riveted together. Its diameter is about two and a half times less than its height, and its capacity amounts to 1200 to 1500 gallons. It has a false bottom or diaphragm, below this a pipe enters, which is connected with an ordinary steam-boiler. There is a man-hole at the top, through which the vessel is filled with the rough suet or lard to within about $2 \frac{1}{2} \mathrm{ft}$. of the top. By a safetyvalve, the pressure can be regulated. There are also some try-cocks, by which the state of the contents can be examined; if the quantity of condensed steam in the tank be too great, it will be indicated by the ejection of the fatty contents at the top one. There is, moreover, a regulating cock at the bottom for drawing off the condensed steam, as well as cocks in the side of the digester, by which the fatty materials can be drawn off. Through a hole made in the diaphragm, which can be shut and opened at will, the residual matters can be let out.

Fouche's Process is one of the most perfect. Fig. 1868 represents a vertical, and Fig. 1869 a horizontal section of the apparatus, as used by the inventor, after the line 1-2 in Fig. 1868. Fig. 1870 is a transverse section after the line 3-4 in the same figure. The vessel has a copper dome $\mathbf{B}$, fastened by rivets. In this dome is a hole C for introducing fat, having a cover, which may be lifted by a chain going over a pulley, and the margin of the cover may be fastened to the vessel by clamps. This cover has a hole for observing the inside, which can be shut by a valve fastened to the lever D. $E$ is a cap on the dome
 with the eduction-pipe for vapours, and PP is a safetyvalve, with a counter-weight R. There is, moreover, an outer valve for the passage of air, either when filling or emptying the vessel, as well as a box for a thermometer. The vapours escaping through $P$
(which may be opened by the faucet $O$ ), pass into $U$ for the purpose of being condensed there, or, when not condensed, for escaping through X. F is a worm, which fastencd to the stays G, Fig. 1869, lies on the bottom of the vessel. Through LL steam is introduced from a boiler, and through M passes back into the same boiler. H H is a small pipe entering into the vessel A , through which steam also passes into the vessel, mainly for the purpose of keeping the melted fat in agitation. $J$ is a tube, having a sieve at its upper end, and a movable crank below, by which it is fastened to the faucet Y. If the vessel is being emptied, the tube $\mathbf{J}$ is gradually let down until its upper part, with the sieve, reaches the bottom. The fat is then passed through J and Y, and through a fine sieve outside the vessel, which acts as a filter. In this, 1000 lbs. are first introduced with 80 lbs . of water; $2 \frac{4}{10} \mathrm{lbs}$. of sulphuric acid of $66^{\circ}$, previously mixed with 16 lbs . of water, are then added. Steam is next turned on, which, as described, passes from the generator through the worm, and must have a tension of three atmospheres, or a temperature of $255^{\circ} \mathrm{F}$. In the vessel, however, a tension of $1 \frac{1}{2}$ atmosphere is sufficient, and when this is reached, the safety-valve is no longer charged with weights. The vapours formed in the vessel are conducted through $\mathbf{X}$ into the hearth of the steam-boiler furnace, so that all the noxious odours (which, however, by the action of the sulphuric acid, are diminished, but not destroyed) are thus conveyed from the workingrooms.

Evrard's Process.-In its features, the apparatus used by this inventor very much resembles that of Wilson. The process, however, is based on the application of caustic lye, in the proportion of 25 gallons (each containing $\frac{1}{10}$ to $\frac{1}{7} \mathrm{lb}$. of solid caustic soda) to every 250 to 350 lbs . of rough tallow. It is the object of the application of the lye, as in d'Arcet's process, to dissolve the membranous parts, so that no preliminary mincing be necessary. For boiling the fat, steam is employed. As the alkaline lye is heavier than water, it will also, after the boiling is completed, more easily subside. It is then drawn off, and the fat left in the tank is again boiled with successive portions of fresh water, for the better separation of which, this compound is left for twenty-four hours in a warm liquid state before being drawn off into the coolers.

Stein's Process.-A mixture of slacked lime and small pieces of fresh-burnt charcoal is prepared, and spread upon a coarse cloth stretched over a hoop, of 2 in . in depth, and the circumference corresponding with the size of the pan. During the process of rendering, it is securely adjusted by suitable catches above the pan. The rising vapours from the latter, in necessarily passing this chemico-mechanical arrangement, are said to be entirely absorbed, so that thus all cause of complaint against tallow factories as health-destroying nuisances would be effectually removed.

Clarifying Tallow.-By mere melting and straining we do not obtain a fat entirely free from admixture of fine, undissolved substances. For separating these substances, therefore, it must be clarified. This is done by remelting it in water, either on free fire or by steam. Generally, no more water than 5 per cent. is taken, and stirred well with the fat till the mixture becomes emulsive. The whole is then allowed to rest, without further heating, till the water has separated, when the fat may be drawn off, or dipped off. Sometimes, in order to conceal the yellowish tint, a very little blue colour is added to the clear fat, consisting of indigo rubbed finely with some oil, of which a few drops are sufficient even for large quantities. The process of clarifying is occasionally repeated.

At the line of demarcation between the water and fat, a grey slimy substance is often perceptible, and the liquid itself is turbid. Instead of pure water, some tallow-melters take brine or solutions of alum, saltpetre, chloride of ammonium, or other salts. According to Dr. Orazio Lugo, these agents have no chemical action upon the fats, but simply induce a more rapid settling of the impurities and water, principally when strong agitation is used.

Hardening of Tallow by Capaccioni's Process.-In 1000 parts of melted tallow, 7 parts of sugar of lead, previously dissolved in water, are stirred, during which process the mass must be constantly agitated. After a few minutes the heat is diminished, and 15 parts of powdered incense, with 1 part of turpentine added, under constant stirring of the mixture. It is then left warm for several hours, or until the insoluble substances of the incense settle to the bottom. The hardening is produced by the sugar of lead, yielding a material similar to the stearic acid, while the incense is improving its odour; it is also said that by this treatment the guttering or running of the candles is entirely prevented.

Cassgrand's Process for Bleaching Wax.-By this process, the bleaching of the wax by the sun and air is not prevented, but much time saved. The inventor first melts the wax with steam, which together pass through long pipes, so that a large surface becomes exposed to the steam. After traversing the pipes, it is received into a pan with a double bottom, heated by steam; it is therein treated by water, left quiet for some time until its impurities are settled. It is then forced anew through the pipe together with the steam, washed a second time, and, if necessary, this process is repeated a third time. Probably water is absorbed by the wax, thus rendering it more easily bleached. The following is the arrangement of a bleachery :-

Stakes or posts are driven into the ground, and 2 ft . from the ground bag-clothes are stretched over them, or table-like frames are made from strips, and cloth stretched over the frames in the same manner as a sacking-bottom is stretched over a bedstead, care being taken to fasten the ends of the cords to the posts sufficiently firm as to prevent them loosening by the wind. This done, the wax ribbons are spread upon the cloth in a thin layer. It is important that the place selected for this arrangement be so that the sun's rays may have full play upon the exposed wax, but at the same time protected from the prevalent winds. The ribboned wax is daily turned over, in order that fresh portions of it may be affected by the sun; and should it not be sufficiently moistened by the dew or rain, soft water is poured over it. When it is not gradually becoming whiter, but still continues yellow upon the fracture, it is remelted, ribboned, and again bleached. The continuance of the bleaching process necessarily varies, depending, as it does, upon the weather; often one exposure to the sun and air suffices to bleach it, and no remelting is requisite. Four weeks are generally sufficient. The bleached wax is finally fused into cakes or square blocks,

## CANDLES.

previously moistening the moulds. As fast as the wax congeals, the cakes are thrown into a tub of clean, cold water, and then taken out and spread upon a pack-thread sieve for draining. Erentually, they are dried and packed in boxes for the market, the loss being from 2 to 8 per cent.

Wicks. - In the present day we designate the wicks as twisted and plaited; the former, loosely twisted, and the fibre presents the appearance of a spiral similar to the separate strands of a rope; the latter, now generally adopted for most kinds of candles, is made by interlacing and crossing the strands of the wicks in the same manner as plaiting straw of bonnets. Common wicks are simply an aggregation of several loosely-twisted threads forming one general cord of many fibres. This is effected by the ball winding machine, an apparatus of a very simple construction.

For cutting wicks, Sykes's Apparatus is in very general use, especially for tallow-candle wicks, which must be soaked with tallow at one end.

Fig. 1871 represents a vertical, and Fig. 1872 a horizontal view of it. cc are spools on which the wicks are wound. $b$ is a roller with grooves cut around it, by means of which the wicks are conveyed into the clamp d, represented in Fig. 1873 on a larger scale, and as seen from the side.

1873.


It consists of two wooden frames, which are made tapering from the middle towards the end. On each side there is a feather of steel attached, for the purpose of holding the frames, with a space between them, which may be diminished by sliding the feathered clamps $e e$ towards the middle, or increased by drawing them towards the end. Immediately behind the clamp there is a cutting apparatus, consisting of an immovable $f^{\prime}$ and a movable blade $f$, with a handle. $g$ is a small vessel filled with liquid fat (which may be kept from solidifying by steam), and a board $i$ lying on the lathe $h$.

The use of the apparatus is as follows :-The ends of the wicks, wound upon the spools $c c c$, are passed through the frame $d$, properly tightened by the clamps ee, so that all the wicks are kept firm. The knife $f$ of the cutting apparatus is then lifted out of the way; the frame, with the wicks enclosed, is drawn backwards to the vessel $g$, and the ends of the wicks dipped in the melted fat; this done, the fat-soaked ends are drawn farther back and placed under the weight I, which holds them firmly while the clamps are loosened on the frame, and this returned to its firstdescribed position and again tightened. The knife is next used, cutting all the wicks off at a stroke, then elevated, and the process repeated till a sufficient number of wicks are cut.

The thickness of the wicks, we may here remark, varies according to the diameter of the candles and the material of which they are made. The number of the cotton threads requisite to form a wick also varies according to their firmness. Scarcely two chandlers, however, observe the same rules in these respects. The yarn is composed of a slack-twisted cotton thread, No. 16 generally for plaited, and smaller, such as $8-12$, for common wicks.

Bolley has published the appended index relative to the thickness of wisks. The yarn employed is No. 16.

For tallow candles, 8 per lb., the wick contains 42 threads.

| $"$, | 7 | $"$ | 45 | $"$ |
| :--- | :--- | :--- | :--- | :--- |
| $"$ | 6 | $"$ | 50 | $"$ |
| $"$ | 5 | $"$ | $5 \tilde{5}$ | $"$ |
| $"$ | 4 | $"$ | 60 | $"$ |

These wicks, composed of ten, twelve, or even sixteen cords, are very loosely twisted, and form a kind of hollow tube.

For stearic candles, three-corded plaited wicks are generally used, smaller in size and of finer yarn. As, for instance-

| Stearic candles, 4 | 4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $"$ | 5 | per lb., the wicks consist of 108 threads. |  |  |
| $"$ | 6 | $-"$ | 96 | $"$ |
| $"$ | 8 | $"$ | 63 | $"$ |

Preparing Wicks.-This is done by the so-called wick-mordants, by means of which they are less combustible, especially those for stearic acid, and composite candles prepared. For this purpose, compounds composed of solutions of ammoniac salts, of bismuth, of borates, or boracic acid, are used. Some of the receipts given in the journals and books devoted to technology are good as far as the quality of the substances is concerned, but not as regards the quantity. The recommended solutions are generally too strong. This may be said most assuredly of the following (Smitt's) preparations for wax-candle wicks. He recommends 2 oz . of borax, 1 oz . of chloride of calcium, 1 oz . of nitrate of potash, 1 oz . of chloride of ammonium, to be dissolved in $\frac{3}{4}$ gallon of water, and afterwards dried.

A simple and cheap mordant for wicks consists in a sal-ammoniac solution of $2^{\circ}$ to $3^{\circ} \mathrm{B}$. This concentration is strong enough, and if a weaker one be used, a spark will remain on the wick after the candle has been blown out, and burning down to the fat, make relighting more difficult. Before moulding is performed, the wicks, having been saturated, are thoroughly dried in a tin box, surrounded by a jacket, in which steam is introduced. Instead of the sal-ammoniac, phosphate of ammonia is used in some factories. A very good mordant is also a solution of $2 \frac{4}{10} \mathrm{oz}$. boracic acid in 10 lbs. of water, with $\frac{1}{3}$ of an ounce of strong alcohol, and a few drops of sulphuric acid. Some mordants, we are aware, have become unpopular. The fault is in the nature of the crude cotton, which does not always readily become moistened; consequently, from not having completely imbibed the mordant, portions of the thread remain unsaturated, and are not equally combustible with the others. An admixture of alcohol will possibly remedy this defect, inasmuch as cotton is easier moistened in diluted spirit than in pure water.

Dips.-These candles are made by stringing a certain number of wicks upon a rod, and dipping them in melted tallow repeatedly. Though made in large quantities, they are only manufactured in comparatively small establishments. The process is very simple; it is as follows:-The clarified and remelted tallow is poured into a tightly-joined walnut or cherry trough, 3 ft . long by 2 ft . wide, and 10 to 12 in . wide at the top, gradually diminishing to 3 or 4 in . at the bottom. A handle is fixed on each end for its easy removal, and when not in use it is closed with a cover. The operator commences by stringing sixteen to eighteen wicks at equal intervals on a thin wooden rod, about $2 \frac{1}{2} \mathrm{ft}$. long, and sharpened at the ends. He then takes ten or twelve such rods and dips the wicks rapidly into the fluid suet in a vertical direction. This suet should be very liquid, in order that the wicks be soaked as uniformly as possible, after which the several rods are rested on the ledges of the trough, when, if any of the wicks be matted together, they are separated, and the rods so placed on a frame, having several cross-pieces, that the uncongealed tallow from the wicks may drop down, and while this is going on, which continues till the tallow is cooled and solidified, the operator is engaged in preparing another batch of rods. The fat in the trough, meanwhile, is so far cooled that in immersing the first dip again a thicker layer will adhere to the wicks. It is considered, we may observe, that when the suet solidifies at the sides of the vessel, the temperature is the most convenient for the object in view. It is, moreover, sometimes necessary to stir the ingredients to produce a uniform admixture, and in such cases much care should be taken so that no settlings be mingled with the mass, whilst by the addition of hot tallow any desired temperature may be obtained. The tallow on the wicks between each dipping becomes so gradually hardened, that at the third or fourth immersion new layers necessarily solidify; as a natural consequence of the method of dipping, the lower ends of the wicks become thicker than the upper, to remedy which the lower ends are again put into the melted fat for a few minutes, when the heat, as a matter of course, diminishes their dimensions. The process of dipping is continued until the candles acquire the requisite thickness. The conical spire at the upper end is formed by immersing deeper at the last dip, and if, eventually, the candles are too thick at the lower end, they are held over a slightly-heated folded copper sheet, so that the fat may melt, but not be wasted.

For the purpose of saving time, many mechanical arrangements have been devised and completed, one of the most useful and used of which, involving the least outlay and requiring only one operator, is the Edinburgh Candle-wheel, Fig. 1874.

The following is a description of it:-A strong vertical post A is mounted on pivots, resting on a block T, and attached at the top in a beam PP, so that it can revolve freely on its centre. In the upright post A, six mortices are cut at short distances from each other, and crossing one another at an angle of $60^{\circ}$. Each of these six mortices receives a bar D , which swings freely on a pin C , run through, the centre of the bar $\mathbf{E}$ and post A . At the extremity of each bar is suspended a frame $\mathbf{E}$, containing six rods, on each of which are hung eighteen wicks, making in all 1296 wicks on the wheel. As the bars B are all of the same length, and loaded with nearly the same weight, it is obvious that they will all naturally assume a horizontal position. In order, however, to prevent any oscillation of the machine when turning round, the levers are kept in a horizontal position by means of small
 chains R R, one end being fixed to the top of the upright shaft, and the other in a small block of wood M, which exactly fills the notch F. Notwithstanding its appearance, the machine is very casily turned round, and, when in motion, each
port, as it successively passes over the tallow-kettle H , in its water-bath, mounted on a furnace K , is gently pressed downwards by the handle S . By these means the wicks are regularly immersed in the tallow, and the square piece $M$ (when the handle $S$ is pressed down) is thrown out of the notch by the small lever $O$, inserted in the bar B. In order that, when the operator raises the port, the piece M may return to its proper position, a cord is attached to it, passing over the pulleys V V , and regulated by the weights W W . In the bars $\mathbf{D}$ several holes are bored, by means of which they may be heightened and lowered at will.

It may be readily perceived, therefore, with what expedition the whole operation is performed (and that too by one operator), the ports being not unnecessarily removed after each dip, and the process of congealing being much accelerated, as the candles are kept in constant motion through the air. In moderately cool weather not more than two hours are necessary for a single person to finish one wheel of candles of a common size, and that if six wheels are completed in one day, 7776 candles will be manufactured in that space of time by one workman.

Moulds.-For moulding, besides the common metal moulds (a mixture of tin and lead), moulds of glass are sometimes used. The former are slightly tapering tubes, varying in length and dimensions according to the size of the candle to be manufactured, and, when required, are arranged in regularly perforated wooden frames or stands, with the smaller end downwards, forming the upper or pointed part of the candle. At this smaller end, the wick, previously saturated in melted fat, is inserted, filling the aperture, and, passing up the centre, is fastened perpendicularly at the opposite, that is, the upper end of the tube, to which is attached a movable cover. The melted fat is then poured in, generally with a small can, but a tinned iron syphon is better. It is requisite that the tallow should completely fill the mould, that it should remain uncracked on cooling, and should be easily removable from the moulds. This can, however, only be obtained when the fat at the sides cools more quickly than that in the interior, and when the whole candle is rapidly cooled. A cool season is, for this reason, far better; but a certain condition of the tallow, namely, that which it possesses at a temperature very near its melting-point, is absolutely necessary. According to Knapp, candle-makers recognize the proper consistence of the tallow for moulding by the appearance of a scum upon the surface, which forms in hot weather between $111^{\circ}$ and $19^{\circ} \mathrm{F}$., in mild weather at $108^{\circ}$, and in cold about $104^{\circ}$. The tallow is usually melted by itself, sometimes, however, over a solution of alum. The candles are most easily removed from the mould the day after casting, to be cut and trimmed at the base.

Moulding.-Moulding by hand is a very tedious operation, and only practised in the smaller factories; in more extensive establishments, where economy of time and labour is a consideration, machinery is employed.

Kendall's Moulding Apparatus.-Fig. 1875 represents a vertical transverse section through one of the mould-frames, exhibiting the candles drawn from the moulds. Fig. 1876 represents a top view of a row of moulds, showing the clamp in place ready to centre the wicks. The moulds are mounted upon cars, for being carried from place to place as required, each capable of conveying several dozens, which are heated to about the temperature of the melted fat by running the car into an oven. The moulds thus heated are carried by cars to a caldron containing the melted fat, with which they are filled. The car is then attached to one of the empty trucks and allowed to remain till the candles are cooled, when it is moved to an apparatus, by means of which the candles are drawn and the moulds re-wicked, and again ready to be heated and filled.

To facilitate the transference of the moulds to different parts of the room, the cars on which they are mounted are carried about on trucks fitted with rails at right angles to the track on which they run; so that the car with the moulds can be carried forward or back by the truck, and run to the right or left on its own wheels upon lateral tracks at will.

In Fig. 1875, m $m$ represent moulds mounted on two horizontal boards $a$ and $b$ (in which round holes are cut) and tightly screwed at the upper end, around which a thin wooden frame is attached, three-fourths of which is firmly fastened, whilst the other one-fourth forms a slide. The lower end of the moulds rests os pieces of vulcanized india-rubber $o$, let into the cross bar e; each piece of india-rubber being pierced with a hole somewhat smaller than the wick, and as the wick is passed through this hole, the latter compresses it so tightly as to prevent the fat from leaking out. In like manner, the leakage is prevented between the
 bottom or tip $n$ by the pressure of the mould upon the india-rubber. The spools K hold the wicks firmly and centrically secured by clamps, as seen in Fig. 1876. On the ledge $c$, moreover, of the bottom $a$ there are four pins $i$, which tighten the clamps $j$. Fig. 1876, by means of small holes $g h$. On one side $\mathbf{F}$ of the clamp there are also toothed jaws, in which the wicks fit exactly, that is, they are thus kept vertical and in the centre of the moulds.

The construction of the clamp, Fig. 1876, is such that the arm working upon a joint ab $g$, and
being brought against the arm F, falls into a groove made in its length, so as to press and kink the wicks in said groove, and fasten them firmly there by means of the spring catch K. The object of this is, that in raising the candles from the moulds by this clamp they shall not slip nor move. As the candles are lifted out of the moulds (as in Fig. 1875), the wicks are drawn after them from the spools K, and are then clamped in position in the manner described. The wicks are next cut off above the lower clamp, the candles with the clamps removed, when, by sliding off the spring catch K , the spring S , between the jaws $t t$, causes the $\operatorname{arm} j \mathrm{~F}$ to separate and release the wicks.

Composite Candles.-Cérophane.-The mode for manufacturing this block is the following:-Melt together, over a water-bath, 100 parts of stearic acid, and 10 to 11 parts of bleached beeswax; but, to ensure success, the mixture must remain over the bath from 20 to 30 minutes, and without being stirred or agitated. At the end of that time, the fire is to be extinguished, and the fluid allowed to cool until a slight pellicle is formed on the surface, when it is cast direct into the moulds, previously heated to the same temperature, but with the precaution of avoiding stirring the mixture, as a disregard of it would cause opaqueness of the mixture, instead of transparency

Transparent Bastard Bougie, by Debitte, of Paris.-For 100 lbs . of stock take 90 lbs . of spermaceti, 5 lbs. purified suet of mutton, and 5 lbs. wax; melt each separately over a water-bath, and to the whole, when mixed together, add 2 oz . of alum and 2 oz . of bitartrate of potassa in fine powder, and while stirring constantly, raise the heat to $176^{\circ} \mathrm{F}$., then withdraw the fire and allow the mixture to cool to the temperature of $140^{\circ} \mathrm{F}$. When the impurities subside, the clear liquid must be drawn off into clean pans. For quality and good appearance, candles made of this cooled block are more than proportional to its cost. Morfit recommends to substitute plaited wicks for the foregoing mixture to the wicks generally used for composite candles, and to prepare them by previously soaking in a solution of 4 oz . borax, 1 oz . chlorate of potassa, 1 oz . nitrate of potassa, and 1 oz . salammoniac, in 3 quarts of water. After being thoroughly dried, they are ready for moulding.

1878.

Diaphane.-It is, like the block for cérophane, an invention of Boilot, and made by melting together, in a steam-jacket, from $2 \frac{1}{2}$ to $17 \frac{1}{2}$ lbs. of vegetable wax, $1 \frac{1}{2}$ to $10 \frac{1}{2}$ of pressed mutton tallow, and 22 to 46 lbs . of stearic acid. Both the latter and the vegetable wax are the hardening ingredients. By changing the proportions between the above limits, a more or less consistent mixture may be formed. As concerns the moulding, it is performed in the same manner as for stearic-acid candles.

Parlour Bougies, similar to Judd"s "Patent Candles."-Although not bougies, a name which, properly speaking, is only applicable to candles of wax alone, the similarity of these candles to those of wax has induced the aforenamed title for them. According to Morfit, their mode of manufacture is as follows:-Melt slowly, over a moderate fire, in a well-tinned copper kettle, 70 lbs. of pure spermaceti, and to it add piecemeal, and during constant stirring, 30 lbs . of best white wax. By increasing the proportion of wax to 50 lbs ., the resulting product is much more diaphanous; however, the bougies moulded of this mixture are not as durable as candles made exclusively of wax. They are tinted in different colours. For red, carmine or Brazil-wood, together with alum, are used. Yellow is given with gamboge, blue with indigo, and green with a mixture of yellow
and blue. Sometimes the bougies are perfumed with essences, so that in burning they mav diffuse an agreeable aroma.

A still more transparent and elegant bougie is made by adding only $6 \frac{1}{2} \mathrm{lbs}$. of wax to 100 lbs . of pure, dry sperm, and the candles made of the block formed in these proportions resemble very much the "patent candles" of Judd.

Composite Candles.-The block for these candles is made by adding a portion of hot-pressed cocoa-stearine to stearic acid of tallow. It is an excellent and economical mixture, in which the red, carbonaceous flame of the latter ingredient is improved in illuminating power by the white and more hydrogenated flame of the stearine.
"Belmont Sperm."-It consists merely of a mixed stock of hot-pressed stearic acid, from palm and cocoa-butters. Palmitic acid coloured by gamboge is called Belmont wax.

Harrison Gambo's Machine for manufacturing Candles, Figs. 1877, 1878.-B, Fig. 1878, is the receiving-chamber, surrounded by a steam-jacket B B, Fig. 1877, m M B', Fig. 1878. The tallow is forced from this receiving-chamber into supply-cylinders L', Fig. 1877, M QS, Fig. 1878, where it is acted upon by a plunger $N$, which forces the tallow into the space in front of a tubular plunger. This tubular plunger contains the candle-presser and the wick-tube $a$. The tallow is pressed into the mould $R$, then ejected from the mould, and the wick-tube drawn back; the wick being held by a clamp $d$, is severed by the drawn shears W . The levers $p p$ and the reacting springs repeat the motion. Upon leaving the mould R the candle enters a mould Y , which is surrounded by ice or cold water. From mould Y the candle is forced on to arms $h^{2}, h^{\prime 2}$, which are operated by levers $h$, and conveyed to an endless carrier $g$. The reels T T are used in combination with the $\operatorname{dog} \mathrm{U}$ for the purpose of limiting the let-off motion of the wick. $\mathrm{P}^{2}$ is a fixed support; E bevel gear; and CC'feed-screw.

CAOUTCHOUC. Fr., Caoutchouc ; Ger., Kautschur; Ital., Gomma elastica; Span., Cautchuc.
See Gutta-percha. India-rubber.
CAPSTAN. Fr., Cabestan; Ger., Spill (Schiffswinde); Ital., Argano; Span., Cubrestante
See Mechantcal Powers, Wheel and Axle.
CARDBOARD-CUTTING MACHINE. Fr., Machine à conper les cartons; Ger., Karten. schneidmaschine ; Ital., Macchina da tagliare cartoncini; Span., Guillotina.

See Paper Machinery.
CARDING ENGINE. Fr., Machine à carder; Ger., Kränpelmxschine; Ital., Macchina da cardare ; Span., Carda.

See Cotton Machinery.
CARPET-BEATING MACHINE. Fr., Machine à battre des tapis; Ger., Teppichreinigunys Machine ; Ital., Macchina du battere i tappeti; Span., Máquina para apalear.

The Carpet Beating and Cleaning Machine of G. P. Mitchell, Fig. 1879, is constructed with a shaft $K$ that drives a number of counter-shafts P, upon which endless ropes $e$ are arranged.

The carpet, carried and moved by the ropes $e$, meets with the whips $h$, worked by rock-shafts S, V, and thus the carpet is beaten; it is then passed between cylindrical brushes Q Q, which effect the cleaning and brushing.

The machine is fixed in a framework J, G, F, and may be temporarily placed in a field or an inclosure.

CARRIER. Fr., Moteur; Ger., Mitnehmer.

## See Lathe.



CARTRIDGE. Fr., Cartouche; Ger., Patrone; Ital., Cartuccia; Span., Cartucho.
See Ordnance. Small Arms.
CARVING MACHINERY. Fr., Machine à tailler en bois; Ger., Halzschnitzmaschine ; Ital., Macchina da intagliare ; Span., Máquina para tallar.

See Lathe.
CASE-HARDENING. Fr., Tremper à la voleé; Ger., Härten, Hartguss; Ital., Temperare a pacchetto; Span., Acerado por fuera.

See Anvil. Acger, p. 201. Iron. Steel.
CASEMATE. Fr., Casemute; Ger., Cusematte ; Ital., Casamatta; Span., Casamata.
See Fortification.
CASK MACHINERY. Fr., Machine à monter les futailles; Ger., Maschine zum binden der Fasser; Ital., Macchina da far barili; Span., Maquinaria para hacer barriles.

See Stave-making and Cask Machinery.
CASTING AND FOUNDING. Fr., Fondre ; Ger., Giessen ; Ital., Arte del fonditore; Span., Fundicion.

See Founding and Casting.
CATAPULT. Fr., Catapulte; Ger., Katapulte, Wurfmaschine; Ital., Catapulta; Span., Catapulta.
The engine used by the ancients, Fig. 1880, termed a Catapult, possesses several mechanical properties which deserve particular attention.

This engine, says Ramellus, writing in 1620, which we have borrow from the ancients, is of great service in defending a town from assault. Its use is to throw large quantities of stones, iron, balls filled with fire, and other nosious substances upon the enemy. The following is its mode of construction.

Upon a base or platform A B, Fig. 1880, are placed, in a vertical position, seven stout supports CDEE G H I; these supports are joined together and strengthened by horizontal pieces firmly fitted to them in the middle. The arrangement of the several smaller parts, such as wheels, screws, bolts, and so on, will be better understood from the figure. To the tops of the four supports F G H I are affixed, in the manner shown in the figure, three pieces K LM, connected at one extremity by a cross-bar and terminated at the other extremity by a kind of trough or spoon, of a sufficient capacity to contain a considerable quantity of stones or other matter which it is required to throw upon the enemy; the opposite end of each of these pieces is weighted to counterbalance the load of missiles. The pieces themselves being suspended by means of several double cords, great force may be communicated to them by twisting the cords; the force thus obtained is considerably increased by the pressure of the part NO, which is affixed to the three front supports C D E in the same manner as the pieces we have been considering. The part NO is also connected with the lever P by means of a cord attached to its lower extremity. This lever P is designed to throw balls or pots filled with fire, or other similar missiles.


The mode of preparing the engine for action may be thus described :-A rope is attached by a hook to a ring on the middle receptacle, the other end of which rope is passed round a drum $\mathbf{S}$; this drum is made to revolve by means of the perpetual screw T . The three receptacles are in this way brought down, and the lever $P$ is raised; the operation, by tightening the twisted cords, increasing the recoil. They are then fixed in this position by a bolt Z. The receptacles are now filled with missiles, and the engine is ready for action. When the bolt $Z$ is withdrawn, each of the pieces acted upon by the twisted cords, being liberated, recoils with great force. The ends of the levers carrying the missiles are brought violently against the ropes $Q$ and 12 , and the force of the concussion hurls the charge to a great distance.

The engine is removed from place to place by means of perpetual screws and toothed wheels fixed to the spindles of the screws and working into cavities in the two wheels 5 and 6, upon which the engine rests. Two men are thus sufficient to move and guide it. The five feet upon which the engine rests when stationary are firmly fixed into the ground with iron cramps; during its removal from one place to another these are turned up.

The cords should be made of the same material and in the same manner as the counter-bass strings of a violoncello. Sce Angular Motion. Inertia. Pendulum. Projectiles. Reaction. Velocity.

CA'Thetometer. Fr., Cathetomètre; Ger., Cathetometer; Ital., Catetometro; Span., Catetómetro.

See Matertals of Construction; Elasticity of Traction; Elasticity of Tortion; Elasticity of Flexure: Tenacity ; and Luctility.

Celient. Fr., Ciment; Ger., Cement; Ital., Cimento; Span., Cimento.
As the many varieties of cement which have during the last seventy years been more or less used for building purposes are now nearly superseded by the more valuable and better understood Portland and Roman cements, it is deemed unnecessary to describe them, more especially as they are now becoming, if not already, obsolete.

Portland and Roman cements, although frequently used for similar purposes, vary materially in their qualities. The former, rapidly superseding the latter, is an artificial preparation possessing many valuable properties which recent engineering works have been instrumental in developing.

Portland cement is made from an intimate and perfect admixture of chalk and clay, in varying proportions, according to their respective values-that is to say, the relative proportions of carbonate of lime in the chalk, and silica and alumina in the clay-the other chemical constituents being too insignificant for consideration when the suitable kinds of chalk and clay are selected. Generally speaking, a proportion of 20 per cent. of clay may be considered the right quantity, provided it does not contain more than 15 per cent. of ferruginous matter in iron oxides. 'The best selected materials used in making this cement in England contain the following ingredients :-


The method of preparation usually adopted is to combine these two materials in the most intimate and perfect manner. To effect this satisfactorily the whole is reduced to a soluble condition by the aid of water through the agency of different mechanical arrangements, and on the perfection of this process the future success of the manufacture depends; imperfect or irregular proportions of clay or chalk resulting in a cement disastrous to the manufacturer, and dangerous if used in works of construction by the engineer or architect. The mixture is passed into reservoirs, and the water of solution drained off by decantation. After the necessary interval of time has elapsed, the raw material, as it may now be called, is desiccated, and immediately thereafter placed in suitable kilns, where it is decarbonized, and then becomes what is technically termed clinker. From the kilns it is removed to the grinding and pulverizing mills, where the final process of reduction is effected, and the cement then becomes suitable for all the varied purposes to which it may be applied.

The system of manufacture thus briefly described is simple in character, and on due and intelligent watchfulness being exercised by the manufacturer satisfactory results are easily obtained. The process is necessarily a dilatory one, and considerable time must clapse before the raw material can be converted into a marketable commodity. Under ordinary circumstances, a period of from two to three months is required to make Portland cement. This difficulty has led some manufacturers to a consideration of the question of superseding the tedious operation of washing or mixing by that of the dry process. Hitherto all our attempts at improvement in that direction have not been attended with much success. Some progress has, however, been attained in Germany by the manufacture of Portland cement without the necessity of using large quantities of water for the mixture of the raw materials. Should this improvement be capable of adaptation in England, a considerable reduction in the price of the cement will necessarily follow.

By any system of cement manufacture the primary object is the thorough amalgamation, in the minutest form, of the raw materials, so as to realize to the fullest extent the maximum value of the combination in subsequent stages of the process. For even the most accurate proportions of chalk and clay will not accomplish the desired end unless the comminution of the various particles is perfect and complete. The wet system, as the English method may be called, aims at and indeed accomplishes the required desideratum in the simplest and least expensive manner. The dry, or German system, as it may be distinctly named, likewise succeeds in thoroughly amalgamating the raw materials (chalk and clay). While the application of the wet system is limited to the manipulation of chalk or soft calcareous earths, the dry system may embrace the manufacture of Portland cement wherever materials exist containing the required proportions of carbonate of lime and clay. Indeed, it may almost be considered as capable of unlimited or universal application, as there are but few localities deficient in the necessary quantities of these materials in one form or other.

An accurate compliance with the simple conditions as above described, invariably results in the production of a Portland cement possessing all the essential qualities desired by the engineer and architect. Much attention is now being pointedly directed to the question of a good Portland cement supply, and the necessary tests are of so searching a character as to preclude the possibility of any but the best cements being used where due vigilance is exercised by those entrusted with works of construction. The questionable reputation which has for so long a time been attached to. Portland cement was due as much to the professional indifference of the engineer or architect as the carelessness of the manufacturer; the one, probably impressed with a belief in the insignificance of the subject itself, and the other careless of the character attained by his commodity, so long as it brought profit to himself. Fortunately, the insistance of the engineers entrusted with the great drainage and embankment works of the metropolis, upon being supplied with Portland cement of undeniable quality, has resulted in the institution of tests, rendering the supply of faulty cement absolutely impossible where due and necessary vigilance is exercised. Experience acquired in those works has established beyond cavil that the most desirable cement is that which reaches or exceeds the following standard:-

1st. In weight not less than 110 lbs. an imperial bushel.

2nd. Exceeding fineness of powder, capable of being passed through a sieve of from 1600 to 3000 meshes to the square inch.

3rd. Equal to resist a tensile strain of 200 lbs . to the square inch, after being immersed for a period of seven days in water.

When these three conditions have been scrupulously complied with, there need be no hesitation in using Portland cement for any purpose of construction in water or the air.

Portland cement possesses in an eminent degree the valuable property of hydraulicity, or capacity of setting and hardening under water; and its many applications in the construction of harbours, docks, and breakwaters, have long since established its reputation for such purposes. It is now (1870) upwards of forty years since its first introduction, and during that period of time foreign engineers especially have devoted much attention to its properties and uses. Its appreciation in this country has not been so marked; but an awakening interest in the construction of concrete buildings is gradually leading to the question of its quality and supply.

When Portland cement is carefully and accurately manufactured, it should, on setting, be of a light grey colour, resembling Portland stone in appenrance; and very probably this peculiarity had some influence on giving it the name it now bears. Any departure from this colour indicates a faulty cement, and proves that either the proportions of the materials are irregular, or that its decarbonization was incomplete. It does not deteriorate by exposure to the air, and may be stored even for years if kept in a dry place. Various proportions of sand may be mixed with it for mortar ; sometimes as much as 8 parts of sand to 1 part of cement may be used if the cement is good in quality. Such a proportion, however, is not generally recommended; but one of 3 parts of sand to 1 part of cement will be found a good mixture for general building purposes. The cement when required for architectural embellishment should be worked neat, and for this purpose a quick-setting quality should be employed. The plasterer slould, before using it, make a sample test in the shape of a small brick, or circular pats, which must be put in water as soon as possible after it is worked up by the trowel. If after twenty-four hours' immersion there are no faulty indications developed in the shape of cracks, the cement may be used with safety. For concrete purposes, the proportions are sometimes 1 part of cement to 8 parts of shingle or gravel. Large concrete-blocks, weighing 330 tons, have been successfully made near Dublin with proportions of 1 part of cement to 6 parts of Liffey gravel. This is probably the most striking example of the application of concrete in engineering work.

Recent experiments have proved that Portland cement continues to harden or crystallize for a lengthened period. Briquettes made of neat cement (weighing 121 lbs. a bushel), after seven days' immersion in water, bore a tensile strain of nearly 400 lbs . to the square inch; and duplicate briquettes, after five months' similar treatment, sustained a weight of nearly 600 lbs . to the square inch, showing thereby an improvement of 50 per cent. in that time. Further experiments with cement bricks, to prove their capacity of resistance to compression, resulted as follows.

Bricks three months' old, $9^{\prime \prime}+4 \frac{1}{4}{ }^{\prime \prime} \times 2 \frac{33^{\prime \prime}}{}$ resisted fracture until a pressure of 65 tons was exerted by the agency of the hydraulic press. The weight was applied to the brick on its bed, having a surface of $38 \frac{1}{4} \mathrm{sq}$. in. Similar bricks, under like conditions, at six months old, withstood a pressure of 92 tons. The third experiment, with a brick nine months old, resulted in 102 tons, showing upwards of 50 per cent. improved resistive value in a period of six months.

In the same series of experiments other building materials of acknowledged excellence were - operated upon, and the following values were obtained :-

> Exposed Surface.

| Oldham red bricks.. <br> Medway gault clay bricks |  |  | . | .. |  | $39 \cdot 33 \text { sq. in. }$ |  | 40 tons. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $40 \cdot 50$ | " | 17 | , |
| , ${ }^{\text {a }}$ pressed | - |  |  |  |  |  | $40 \cdot 50$ | ", | 48 | ", |
| Stafford blue brick | \% |  |  |  |  | $27 \cdot 90$ | " | 50 | " |
| Fireclay |  |  |  |  |  | $34 \cdot 85$ | ", | 65 | ", |
| Wortley blue " |  |  |  |  |  | $34 \cdot 76$ | " | 72 | " |
| Portland stone |  |  |  |  |  | $39 \cdot 94$ | " | 47 | " |
| Bromley Fall stone |  |  |  |  |  | $39 \cdot 94$ | ", | 91 | " |
| Yorkshire Landing | - | . |  |  |  | $38 \cdot 28$ | ", | 96 | " |

Showing thereby that at nine months old a neat Portland cement brick exceeded in resistive value the best-known building materials of this country. It is quite possible also that a greater age may yet indicate an increase of strength, an advantage our experience does not lead us to expect from bricks or stones. Induration continues in the case of Portland cement to an extent beyond the limits of our present knowledge.

In another series of experiments it was found that cement, weighing 106 lbs . a bushel, resisted a tensile strain of 210 lbs . to the square inch, and when the weight of the cement was 130 lbs a bushel it required an exertion of 406 lbs. an inch to cause rupture-showing that by an increase in weight of 24 lbs. a bushel the strength of the cement was nearly doubled.

Heavy cement is necessarily slow-setting in character ; but these experiments and the most eminent engineering practice indisputably prove that without weight you cannot obtain great strength

The analysis of a good average sample of Portland cement should be-

| Calcium | . | .. | . | .. | 62 | Alumina | . | .. | . | .. | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Silica | .. | .. | .. | .. | .. | 23 | Oxide of iron | .. | .. | .. | 4 |

To give full effect to the required conditions necessary to obtain a good Portland cement for building purposes, the use of a testing machine is indispensable ; and on works of ordinary extent the engineer or architect should insist on its being applied to every delivery of cement sent on to the works.

About ten years ago, and indeed since then, many attempts have been made to introduce what were ostentatiously termed natural Portland cements, obtained from the blue lias lime formation of the West of England. Their indiscriminate use led to much dissatisfaction and loss, from their unfitness as a substitute for Portland cement, properly so called. Th $\Theta$ charm of cheapness for a time obtained for them a place amongst cements, and the comparative ignorance which then prevailed as to the essential qualities of a good cement enabled the manufacturers of these lias limes to hold a position in the market which a more advanced knowledge of the subject wonld now prevent.

The Boulogne-sur-Mer natural Portland cement differs considerably from the above, being made from a deposit of argillo-calcareous earth containing from 19 to 25 per eent. of clay; the silica and alumina of which is variable. When more than one-twentieth of sand is present in its composition it has to be rejected as unsuitable. This clay or earth is found in the inferior cretaceous formation.

The objection to all kinds of natural cements is the uncertainty in the value of their constituent parts. Their ever-varying ingredients render them unsuitable for building purposes when any degree of excellence or strength is required. No such obstacle exists in the use of sound Portland cement, and that is the reason why it lays claim to and maintains its superiority over all other kiuds of cements. The manufacturers can, when duly impressed with the necessity of accuracy in manipulating the materials from which the cement is made, unerringly ensure the required results in quality and strength; the moderate limitation of which is only controlled by the cost.

In the above remarks it is not intended to convey the impression that there is any difficulty in making Portland cement of light specific gravity and colour. Portland cement can be made from 90 to 140 lbs . weight a bushel ; but the value of the oue is of course much less than the other; so also is its strength and quality-the light cement setting more speedily than the heavy cement. The colour also varies from a light buff to the deepest grey.

Notwithstanding the point of excellence to which Portland cement has attained, especially under the accurate and intelligent manipulation of Henry Reid, there is still much room for improvement in its manufacture. Its continually increasing application for all kinds and forms of construction points it out as a valuable agent for substituting concrete houses for the worthless brick buildings of this country. It is to be hoped that changes in the processes of manufacture will eventually result in reducing the price of an article which at present bears a very disproportionate relation to the value of the simple and inexpensive materials from which it is prepared.

Roman Cement.-This cement, although gradually giving way to its more energetic rival Portland cement, possesses nevertheless many good qualities. It differs from Portland, being a natural cement; and in consequence of there being great varieties of stones from which it is made, much care is necessary in ascertaining their exact quality before using it for building purposes. The origin of the name arose probably from its resemblance in colour and general characteristics to the mortar found in Roman remains in England. It was first introduced by Parker in 1796, who obtained a patent for manufacturing cement-named by him Roman-from the septaria nodules of the London clay found along the shores of the Isle of Sheppy, in Kent. Subsequently, Frost and other makers introduced Roman cement made from similar materials found in the clay formations of other districts in England. The term Roman is now generally applied to all natural cements of a dark brownish colour, and it is this distinguishing peculiarity which so markedly 'separates it from the Portland cement, which is of a grey colour. The Roman cement stone is found in many parts of England, North Wales, the Isle of Wight, and in Scotland. Several districts in France also furnish considerable quantities both from the clay and argillacecus limestone formations. There are very extensive deposits in the United States of America, from which is manufactured a cement resembling that of this country both in colour and other peculiarities. There can be no doubt that in many districts of England and Scotland this cement stone abounds; and it is only by reason of the apathy and indifference of engineers and architects on the subject that so little progress has been made towards its greater development.

The Roman cements generally sold in London are the Harwich, Isle of Sheppy, Southend, Isle of Wight, and Medina; in the North of England, the Yorkshire, Atkinson, or Mingrave; in Liverpool, the Holywell, Flintshire, North Wales: and in Scotland, the Calderwood. In America it is extensively made in the states of Illinois, Ohio, Maryland, Virginia, and New York. Some years ago large quantities of these American cements were shipped to Australia and other places, and called Portland cements; but they differed so much from that article that their mauufacturers finally abandoned their pretensions, after great loss had been incurred by the merchants who had shipped them.

In general, these cement stones contain about 60 per cent. of carbonate of lime, or magnesia, with from 30 to 40 per cent. of clay. The oxides of iron and magnesia which they contain are yariable. Few of them have alumina or soda, and it is doubtless from the absence of these ingredients that they are so inferior to the artificial Portland. The clay is highly ferruginous in character, which to a great extent accounts for the rapidity with which they set; and to this cause also may be due their want of permanently indurative capacity.

The following analysis of well-known Roman cement stones will better explain the relative quantities of the ingredients:-


The American cement stones have more alumina and less carbonate of lime than those of this country. It is a curious circumstance that, notwithstanding their excellence and abundance, Roman cement is still sent from London to New York.

A remarkable difference between the Roman and Portland cement is the necessity for applying opposite tests in selecting the best sorts when manufactured. Portland is best when heaviest: up to 140 lbs . weight a bushel if possible. On the other hand, Roman is judged by its lightness ; and when it weighs about 75 lbs. a bushel it is best. These two different properties tell, however, in a most marked manner when we come to consider which is the better cement to keep for a lengthened period without deterioration. Portland cement possesses this valuable property in a high degree, and Roman cement can lay but little claim to this excellent quality.

The stone principally used in London is that obtained from the coast near Harwich, and is delivered in London by barges at a cost, according to quality, of from $6 s$. to $8 s$. a ton.

The process of manufacture is most simple, and the only care required is to prevent its being vitrified when burnt. The great aim is to have the cements of the lowest weight; this can only be attained by a maximum amount of decarbonization in the kiln. The stones lose in weight by this process about 30 per cent. without any appreciable loss in bulk. The form of kiln used does not differ much from the commonest kind of lime-kiln. It is usually from 20 to 25 ft . high, from 9 to 12 ft . wide at top, and from 7 to 8 ft . wide at the bottom. Each kiln has four eyes, or drawing-holes, to take away the stone when burnt; it is seldom necessary to have them all in use at the same time. The stone is broken to a uniform size of from 2 to 3 in ., and placed in the kilns in layers; each layer having the necessary amount of fine coal placed on it, for the purpose of decarbonizing or burning the stone. The fuel required for this purpose varies, as the stone or coal is seldom regular in quality. The experienced burner will easily regulate the necessary quantity. When a kiln is once lighted up, it may be kept burning for many months without any attention beyond the regular daily withdrawal of the burnt stone and the addition of the fresh materials so that the kiln may always be kept full. The best and most profitable kiln is one with a 70 or 80 tons capacity. Smaller kilns, however, may be used in localities where the demand for cement is limited. It is advisable to adjust the size of the kiln so that it may always be burning ; such an arrangement is the most economical, as you thereby save the waste incurred in relighting after each charge has been drawn. The burnt stone may be kept for a considerable time before being ground, and it is better, therefore, if necessary, to allow it to remain in this state than to keep the powder, as in that condition it speedily and permanently deteriorates.

The grinding or pulverization of the cement should be carefully attended to. The point of reduction is seldom, in this country at least, carried beyond the No. 30 gauge; but in America the engineers insist on having it ground so that only 8 per cent. will be rejected by an 80 -gauge sieve ( 6000 meshes to the square inch). English engineers and architects have not been so exacting in their stipulations for fine grinding, and a No. 40 (1600) gauge may be considered as the maximum of pulverization of English Roman cement.

There can be no doubt that progress in the use of Portland cement was much retarded by interested Roman cement manufacturers, in consequence of the ease with which the latter could be made. Indeed, it was and is so simple in character that the most ignorant are almost capable of preparing it for the market. The selection of the stone itself is not difficult; and when the most ordinary care was bestowed on the processes of burning and grinding, the makers run but little risk in carrying on what doubtless was, until very lately, a very profitable trade. Roman cement can be prepared for the market in a few days. That adrantage, from a manufacturer's point of view at least, was considerable; whereas the conversion of the raw materials into Portland cement occupied a period exceeding two months, according to the seasons. Again, the manufacture of Roman cement requires but little, if any, scientific or advanced technical knowledge. On the other hand, Portland cement making is attended with much risk and anxiety, and cannot be successfully conducted without a perfect knowledge of the raw materials and their accurate manipulation. For these cogent reasons, therefore, the ascendency of Roman cement was maintained for a much longer time than its merits really deserved. It was not until the professional mind was thoroughly awakened to its shortcomings and failings that it was, in important works at least, superseded by Portland cement, with good reason, as the following tests will prove.

It should be premised, to account for the variability of the results in the following Tables, that Roman cement, as already observed, is prejudicially influenced when in contact with the air and the moisture which it contains. Its highest value, therefore, can only be realized when fresh burnt and finely ground. The cement submitted to experiments was obtained from four makers of established reputation.

Age of Bricis.
Tensile Strain per sq. in. in lbs.


Attempts were made during the time these experiments were being tried to extend their usefulness by ascertaining the value of the cement when mixed with various proportions of sand. The results, however, were found to be so low and unsatisfactory that the further search for information in that direction was abandoned, as not being likely to lead to any useful result.

Notwithstanding the many disadvantages attending the use of Roman cement, it must be borne in mind that it is to be found in many countries and localities where it might be readily used with advantage when Portland cement or hydraulic lines could not be obtained. It is now fre-
quently used in protecting the joints of Portland cement, while setting, from the injurious action of the sea or running water in harbour and similar works. Smeaton, when building the Eddystone Lighthouse, long before the introduction of Roman cement, used plaster of Paris, in protecting the joints of his composite mortar, made from puzzolana and blue lias lime, from the violent action of the sea.

Roman cement cannot bear a greater admixture of sand than 1 to 1 . For concrete, however, in foundations at least, it may be used up to 5 or 6 of gravel to 1 of cement. In the Thames Tunnel neat cement was used for the arches, 1 of sand to 1 of cement for the foundations, and $\frac{1}{2}$ sand to 1 of cement for the piers. When used for internal plastering, it requires painting to hide the unsightly reticulated appearance of its surface after having been exposed for some time to atmospherio action. Much unsightly plastering is executed in London, from a supposed economical application of Roman and Portland cements. The wall to be plastered is first coated with a layer of Roman cement, which, when dried, is covered with a thin coating of Portland to give a finish and appearance to the whole. This mode of using the two cements is highly objectionable, and can only proceed from a want of knowledge of the peculiarities of the cements, which differ so materialiy in their properties. It is difficult to put two layers of the same cement on each other. The joints between two coats of cement are seldom perfect.

Cement manufacturing is an important branch of our industry. Some years ago, and before Portland cement had established its position, the English Government had their attention called to the scarcity of Harwich cement stone caused by its being dredged and taken away in large quantities by foreigners. The matter appeared of sufficient importance to attract the consideration of the late Sir Robert Peel, who intended levying a tax on the stone taken out of the country. On representation being made to him, however, that this country contained inexhaustible beds of raw materials which, when operated upon, produced an artificial cement much superior to the Roman, he abandoned his intended taxation.

It is remarkable that the three perhaps most celebrated engineering works of modern times led to improvements in mortars and cements. Smeaton, while (1756) experimenting on mortars for the Eddystone Lighthouse, discovered the value of puzzolana in combination with blue lias lime in imparting great setting energy to the mixture. Indeed, at that time he established the value of clay in conjunction with the carbonate of lime, which more than eighty years afterwards led to the inyestigation which ultimately resulted in the discovery of Portland cement. The construction of the Menai Bridge by Telford set at rest all doubts as to the value of Aberthaw lime for hydraulic purposes. The Thames Tunnel, again, furnished conclusive evidence of the great value of Roman cement, which was almost entirely used in its construction by the Brunels.

Portland Cement.-Particulars specified by the War Office, England.-Portland cement is to be of the best quality, ground extremely fine, weighing not less than 100 lbs . a striked bushel (filled into a bushel measure as lightly as possible), and capable of maintaining a breaking weight of 450 lbs. seven days after being made into a mould of the form and dimensions given in Figs. 1881 to 1883; the specimen being immersed in water as soon as it has set, and so left during the interval of seven days.

Cement placed in the testing-mould, Fig. 1883, to be experimented upon, must be in good condition, and thoroughly mixed on a clean floor, and applied as soon as mixed.

The cement and sand have to be carefully measured in proper measures, which are kept for the purpose.

Hydraulic Cements, natural and artificial.-The term hydraulic cement is generally used in distinction to hydraulic lime. The former, containing a larger proportion of silica and alumina and a smaller proportion of carbonate of lime than the latter, does not slake, and sets generally in a few minutes even under water. Hydraulic limes, on the other hand, slake thoroughly and harden slowly under water. Some limestones exist which, when completely calcined, yield hydraulic lime; but when imperfectly calcined, yield cement. Other limestones, such as chalk, when imperfectly calcined, or too much calcined, yield fairly hydraulic limes; while, if they be calcined merely up to the point when all the carbonic acid is driven off, and no further, they yield a lime which never solidifies under water. Other limestrnes yield on calcination a result which can ne ther be termed lime nor cement, owing to its slaking very imperfectly, and not retaining the hardness which it quickly takes when first 1 laced under water.

The natural hydraulic cement best known in England, of which we have spoken, is that which is most absurdly termed Roman cement, or, more sensibly, Parker's cement. The stone is found in the form of nodules in the island of Sheppy. The composition of these nodules is almost the same as that of the Boulogne pebbles, from which a similar cement is made. Before being burnt, the stone is of a fine close grain, of a rather pasty appearance; the surfaces of fractures being greasy to the touch. It sticks easily to the tongue: its dust, when scraped with the point of a knife, is a greyish white. During the calcination the stone loses about one-third of its weight,
and the colour becomes of a brown tinge, differing with the stones from which the cement is made. It becomes soft to the touch, and leaves upon the fingers a very fine dust; it sticks very decidedly to the tongue. Wheu taken out of the kiln it absorbs water with much difficulty. It is usually burnt in conical kilns, and pulverized by the manufacturer, and then sold in well-closed casks. In Russia, America, India, and elsewhere, similar natural cements have been met with; but, as they are comparatively rare and expensive, much attention has been bestowed on finding or inventing a substitute for them. Carbonate of magnesia alone has been found to yield, on calcination, an excellent hydraulic cement. When mixed with $1 \frac{1}{2}$ times its bulk of sand, it makes a beautiful hard plaster or stucco.

Vicat was the first to point out the method of forming an artificial hydraulic cement by the mixture of lime and unburnt clay; and General Pasley afterwards, in a most elaborate series of experiments, proved that a hydraulic cement might be formed equal to the best obtained from natural sources

Pasley's experiments were made principally with chalk lime and the blue alluvial clay of the river Medway, near Chatham. The result he arrived at was, that a mixture of 4 parts by weight of pure chalk, perfectly dry, with $5 \cdot 5$ parts, also by weight, of alluvial clay, fresh from the Medway, or of 10 parts of the former with $13 \frac{3}{4}$ of the latter, would produce the strongest artificial cement that could be made by any combination of these two ingredients.

From the experiments of Vicat and Pasley on making artificial cements, it would appear that the best mixture for making cement consists, before burning, of

$$
\begin{aligned}
& \text { Two ingredients of carbonate of lime .. .. .. .. .. .. } 50.5 \times 2=101.0 \\
& \text { One ingredient of clay, of which the probable composition is- } \\
& \text { One equivalent of alumina .. .. .. .. .. .. .. .. } 51.4 \\
& \text { Six ditto silica .. .. .. .. .. .. .. .. .. .. .. } 93 \cdot 0 \quad=144.4 \\
& 245 \cdot 4
\end{aligned}
$$

So that the composition in 100 parts is :-

| Carbonate of lime | .. | .. | .. | .. | 41 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Clay .. | .. | .. | .. | .. | .. | .. |
| 9 |  |  |  |  |  |  |

The weight of the bluo clay he found to be $90 \mathrm{lbs} .$, and of the dry chalk powder 40 lbs a cubic foot.

His metnod of proceeding was as follows:-The clay was weighed when fresh from the river (taken from about 18 in . below the surface), and was never dug unless required at once; it being found that even twenty-four hours' exposure to the atmosphere injured it. 'The chalk was not weighed until well dried and pounded, owing to its extraordinary retentiveness of moisture. The chalk was then mixed with water into a thick paste. The chalk and clay were then each separately divided into portions or lumps as nearly as possible equal, and put alternately into a pug mill of the ordinary description, Figs. 1884 to 1888, where they were most thoroughly and inti-

mately mixed. The raw cement thus formed was then made up into balls of about $2 \frac{1}{2} \mathrm{in}$. in diameter, and placed in the kiln alternately with about equal layers of fuel-a layer of fuel always being at the top and bottom. The fuel used was coke, in preference to coal; and, in the small furnace or kiln used by Pasley, three hours was found to be about the average time required for burning the cement. As the calcined cement was drawn from the bottom of the kiln, fresh
cement could he put in at the top. The balls, on being raked out, could be tested by applying to them diluted hydrochloric acid. If sufficiently burned, no effervescence ensued; but if they effervesced, they were put into the top of the kiln again to be reburnt. The calcined cement balls, since they would not slake like ordinary lime, were then ground to impalpable powder, and stored for use, so that they should not be exposed to the atmosphere. The average out-turn was about $9 \frac{1}{2}$ measures of calcined out of 10 measures of raw cement.

The proportions just named formed the best artificial cements. Pasley, after due investigation, found that any given weight of well-burned chalk lime, and consequently of any other pure quick-lime, fresh from the kiln, combined with twice its own weight of blue clay, fresh from the river, will form an excellent water cement; observing, however, that the quick-lime, after being weighed, must be slaked with excess of water into a thinnish paste, and allowed to remain in that state about twentyfour hours before it is mixed with the clay.

These proportions by weight are nearly equivalent to a mixture of 5 measures chalk powder to $2 \frac{1}{4}$ measures of blue clay. Since 555 grains of quick-lime are the produce of 1000 grains dry chalk, which, in a state of powder, will fill 5 measures; of which 1110 grains of clay will fill $2 \frac{1}{4}$ measures.

Portland cement is formed very much from the ingredients used by Pasley, but with different proportions. It takes its name from its likeness in colour to Portland stone; but it is in no way connected with it. The ingredients are chalk and the mud of the river Medway, in the neighbourhood of which it is chiefly manufactured. The process is as follows:-The chalk used contains about $7 \frac{1}{2}$ per cent. of clay. The mud contains about 70 per cent. of alumina and iron to 30 per cent. silica. Eight or nine parts of chalk with two parts of mud are passed through a crushing mill, into which is let a supply of water, which, running off at the side opposite that at which it enters, carries off the crushed particles into a large vat, $60^{\prime} \times 40^{\prime} \times 3^{\prime}$. Here the sediment settles down, and becomes tolerably solid, while the surface water is run off or evaporates. The mixture is now spread out on a drying floor to a depth of about 6 in., and is dried by coking ovens below it. These ovens, at the same time as they dry the raw cement, are converting coal into coke; and this being accomplished, both cement and coke are taken to the kiln, where they are spread in alternate layers, and burnt at a very high temperature. The cement, when burnt, is in a state of incipient vitrification: indeed, it should be over-burnt; as the particles should be just beginning to run together. This excessive burning is a distinctive feature of its manufacture. It is now crushed between two iron wheels revolving on one another, and ground between
 two sandstone millstones; and then the cement is complete, and is carefully packed in 3-bushel casks. The danger of the use of this cement is that it is apt to swell in the joints of masonry after being applied; but it is admirably adapted for buildings exposed to the action of water, and for external plastering, as it sets very fast and attains great hardness, and does not allow of the formation of vegetation, as the natural cements do.

Coignet's agglomerated concrete consists of a proportion of sand, a smaller quantity, perhaps $\frac{1}{4}$, of lime, and a minimum quantity of Portland cement, say $\frac{1}{15}, \frac{1}{20}$, or $\frac{1}{30}$. Instead of mixing these ingredients with mortar, so as to form a mortar of ordinary consistency, a very small quantity only of water is added, and the mass is made to undergo a trituration of greater or less duration by means of special apparatus. By this trituration, notwithstanding the feeble quantity of water, a pulverulent mass is obtained, which acquires by a more prolonged trituration the consistency of a firm plastic paste. It is then ready for the mould, into which it is introduced in successive and very thin layers, which are submitted to a powerful ramming process. This operation of compression effects such a complete agglomeration, that each cubic mètre and a-half of sand and lime is made to occupy the space of only 1 mètre; and the lime, whose particles are thus mechanically brought into contact, becomes indurated with astonishing rapidity and intensity. A few days, in some cases hours, suffice for the mass to acquire the character of hard stone.

As the work of one day, with this concrete, unites perfectly with that of the preceding, it is plain that a mass of this species of masonry can be augmented indefinitely, so that a house, bridge, reservoir, or any other construction, can be made to form a monolith.

The most important building executed in agglomerated concreie is the church of Vesinet, situated in a park of the same name, near Paris. It is of Gothic style, and the entire masonry forms a single block. The division of the naves is formed by piers of grouped colonnettes in cast iron; these, together with the cast-iron responds against the lateral walls, support the weight of the roof, which is of wrought iron. The concrete of which the walling is composed contains : riversand, 3 parts; earthy and ferruginous sand, from Vesinet, 1 part; slacked lime, in powder, from Argenteuil, 1 part ; heavy cement (ciment lourde), from Paris, $\frac{1}{4}$ part; cement forming $\frac{1}{21}$ part of the whole.

Another advantage arising from the suppression of water, the energetic trituration, and the forced aggregation of the particles of lime, in these concretes, is, that a slight difierence in quality
of the lime affects but little the result obtained; therefore the nearest hydraulic lime at hand will serve the purpose; whereas, in other concretes, the best lime must be obtained, in order to secure a good mortar. In fact, M. Coignet presented to the Society of Civil Engineers, in Paris, specimens capable of receiving a high polish, composed of the worst description of lime, even ordinary quicklime, bad and marly sands, fine sea-sand in impalpable powder unfit for common mortar.

With regard to the fact that, in masonry, mortars and concretes, when well made, are progressively and continually becoming harder, while cements do not acquire an increasing induration proportional to the energy of their original hardening or setting;-this may be explained by considering that cements contain very little free lime or oxide of calcium, being almost entirely composed of silicates formed by torrification at a high degree of temperature. Now, as pure lime has the property of absorbing, in order to pass into the state of carbonate, a quantity, nearly equal in weight to its own, of carbonic acid, it is plain that the more perfect a cement is-its perfection consisting in having but a trace of free lime-the less it will have the power of absorbing carbonic acid gas; whilst on the other hand the more a mortar contains of free lime, the more powerful and energetic will be the absorption, and the more the weight will be increased-that is to say, a cubic mètre of mortar containing 400 lbs. to 600 lbs . of free lime can absorb nearly 400 lbs . to 600 lbs . of carbonic acid, while a cement will absorb little or none.

This action of the carbonic acid of the air has a more striking effect upon agglomerated mortars and concretes than upon ordinary ones. It is easily understood that if to a compressed concrete like Coignet's, so dense that a cubic mètre contains 15 hectolitres of ingredients, with the particles so intimately blended by trituration, we add a new substance, carbonic acid, of equal weight with that of the pure lime, the increase of density of the whole mass will be great. Thus, the betons agglomarated, composed of 5 parts of lime in powder, weighing 55 kilogrammes the hectolitre, and $\frac{1}{2}$ part or $\frac{1}{12}$ of cement, yield in a few days an artificial stone resisting more than 20 kilogrammes of crushing force per square centimètre; while substances, composed of 1 part of the best cement and 2 parts of sand, cannot withstand 10 kilogrammes of the same applied force. For lime mortars or concretes to harden into the state of artificial stone it requires much time.

Among the latest theories put forth by French engineers as to the cause of the setting of hydraulic cements, we may mention those laid before the Academy of Sciences, in Paris, by M. Fremy, last June. Vicat established that the hydraulicity of a cement is due to a composition formed when limestone is calcined in presence of argil. He states that this composition is a double silicate of alumina and lime, which, in hydrating, causes the setting of hydraulic cements. MM. Rivot and Chatonay, in their important work on Cements, state that the calciuation of an argillaceous limestone produces aluminate of lime, the formula for which is $\mathrm{Al}^{2} \mathrm{O}^{3}, 3 \mathrm{CaO}$, and also a silicate of lime, represented by $\mathrm{SiO}^{3}, \mathrm{CaO}$; these two salts, in contact with water, produce the two hydrates

$$
\mathrm{Al}^{2} \mathrm{O}^{3}, 3 \mathrm{CaO}, 6 \mathrm{HO}, \quad \mathrm{SiO}^{3}, 3 \mathrm{CaO}, 6 \mathrm{HO},
$$

which become the cause of the hardening in question.
In both these two theories the hydraulicity of the cements is ascribed to a simple action of hydration, like the setting of plaster of Paris; but by M. Fremy another cause is assigned. Ho made extensive experimental researches on the properties and mutual action of four substances which, according to Vicat, Rivot, and Chatonay, constitute hydraulic cements:-1, silicate of lime ; 2 , silicate of alumina and lime; 3, aluminate of lime; 4, caustic lime, or oxide of calcium. The experiments are detailed in the memoir, which concludes with the following statement:-That the hardening of hydraulic cements is not due to the hydration of the silicate of lime, or that of the double silicate of alumina and lime; these salts do not form any combination with the water: that the setting of hydraulic cement is the result of two different chemical actions,- -1 , the hydration of the aluminates of lime; 2 , the reaction of the hydrate of lime upon the silicate of lime, and upon the silicate of alumina and lime, which act in this case as puzzolana. The calcination of an argillaceous limestone cannot yield a good cement except when the proportions of argil and lime are such that these can be formed. In the first place, an aluminate of lime represented by one of these formulæ,- $-\mathrm{Al}^{2} \mathrm{O}^{3}$, $\mathrm{CaO}: \mathrm{Al}^{2} \mathrm{O}^{3}, 2 \mathrm{CaO} ; \mathrm{Al}^{2} \mathrm{O}^{3}, 3 \mathrm{CaO}$ : in the second place, a simple or multiple silicate of lime, represented approximately by one of these formulæ,- $\mathrm{SiO}^{3}, 2 \mathrm{CaO} ; \mathrm{SiO}^{3}, 3 \mathrm{CaO}$; and in the third place, pure lime capable of acting on the preceding puzzolanic silicates.

In the course of the memoir, M. Fremy states that when aluminates of lime represented by $\mathrm{Al}^{2} \mathrm{O}^{3}, \mathrm{CaO} ; \mathrm{Al}^{2} \mathrm{O}^{3}, 2 \mathrm{CaO} ; \mathrm{Al}^{2} \mathrm{O}^{3}, 3 \mathrm{CaO}$, are reduced to a fine powder, and slightly wetted, they solidify almost instantaneously, and produce hydrates which acquire in water a considerable degree of hardness. They have, moreover, the property of agglomerating inert substances, such as quartz, sand, and so on.
M. Fremy mixed aluminate $\mathrm{Al}^{2} \mathrm{O}^{3}, 2 \mathrm{CaO}$, with 50,60 , and even 80 per cent. of sand, and obtained a pulverulent mass which in water acquired the hardness and solidity of the best stone. The interest attached to these mixtures of aluminates of lime and siliceous substances is the more important in a practical point of view when we consider that blocks are required to be produced capable of resisting atmospheric influences and the powerful effects of sea-water. M. Fremy considers that the solution of the problem of constructions resisting the ravages of the sea lies probably in the employment of concretes which are formed nearly wholly of siliceous substances united together by a feeble proportion of aluminate of lime. On this point M. Fremy cites the indications given by M. Coignet on the subject of agglomeration of cements, saying that he has observed their importance himself.

Portland cements, according to M. Fremy, have no good quality unless they are produced at a very high temperature. The aluminates of lime have this character, also, of not being able to solidify under water unless they have been exposed to an intense heat; thus they seem to be the principal agents in hydraulic cements of rapid setting.

Perhaps the inferiority of some of the Portland cements of commerce may be traced to want of sufficient intensity of heat in burning. M. Fremy made the aluminates of lime, for his experiments,
with a blast furnace. See Asphalte. Concrete Machine. Foundations. Kilns. Lime and Mortar. Ovens.

Recapitulation.-Roman Cement.-Parner's Analysis.-One part of common clay to $2 \frac{1}{2}$ parts of chalk, set very quick.

Concrete.-Eight parts of pebble, or pieces of brick, about the size of an egg, to 4 parts of scrap river-sand, and 1 part of lime, mixed with water and grouted in, makes a good concrete.

Lime Mortar.-One part of river-sand to 2 parts of powdered lime mixed with fresh water.
Hydraulic Mortar.-One part of pounded brick powder to 2 parts of powdered lime mixed with fresh water. This mortar must be laid very thick between the bricks, and the latter well soaked in water before laid.

Hydraulic Concrete, by Treussart.-Thirty parts of hydraulic lime, measured in bulk before slaked; 30 parts of sand, 20 parts of gravel, and 40 parts broken stone-a hard lime-stone. This concrete diminishes about one-fifth in volume after manipulation.

Asphalte Composition for Street Pavement, by Colonel Emy.-2 $2 \frac{1}{2}$ pints (wine measure) of pure mineral pitch, 11 lbs. of bitumen, 17 pints of powdered stonedust, wood-ashes, or minion.

Cement for Stone and Brick Work.-Two parts ashes, 3 of clay, and 1 of sand, when mixed with oil, will resist the weather equal to marble.

Brown Mortar.-One part lime, 2 of sand, and a small quaritity of hair.
Hydraulic Mortar.-Three parts of lime, 4 puzzolana, 1 smithey ashes, 2 of sand, and 4 parts of rolled stone or shingle.

Cements for Cast Iron.-Two oz. sal-ammoniac, 1 oz . sulphur, and 16 oz . of borings or filings of cast iron, to be mixed well in a mortar, and kept dry. When required for use, take 1 part of this powder to 20 parts of clear iron borings or filings, mixed thoroughly in a mortar; make the mixture into a stiff paste with a little water, and then it is ready for use. A little fine grindstone sand improves the cement.

Or, 1 oz. of sal-ammoniac to 1 cwt. of iron borings. No heat allowed to it.
The cubic contents of the joint in inches, divided by five, is the weight of dry borings in pounds (avoirdupois) required to make cement to fill the joint nearly.

Works on Cements:-B. Higgins, 'On Calcareous Cements and Quick Lime,' 8vo, 1780. Treussart, 'Mémoire sur les Mortiers Hydrauliques,' 4to, Paris, 1829. L. J. Vicat, 'On Calcareous Mortars and Cements,' 8vo, 1837. Pasley, 'On Limes, Calcareous Cements, and Mortars,' 8vo, 1847. W. A. Becker, ' Pratische Anleitung zur anwendung der Cemente,' folio, Berlin, 1861-68. J. G. Austin, 'On Limes and Cements,' post 8vo, cloth, 1862. Gillmore (Q. A.), 'Practical Treatise on Limes, Cements, and Mortars,' 8 vo , New York, 1864. Burnell, 'On Limes, Cements, and Mortars,' post 8vo, 1868. H. Reid, 'Practical Treatise on the Manufacture of Portland Cement,' 8vo, 1868. Dr. W. Michaelis, 'Die Hydraulischen Mörtel,' 8vo, Leipzig, 1869. R. D. Charleville, 'Traité sur l'Art de faire de bons Mortiers,' 8vo, Paris.

CENTRE-BIT. Fr., Mèche anglaise ; Ger., Centrumbohrer; Ital., Saetta a centro; Span., Berbiquí.
See Augers. Hand-Tools.
CENTRE OF GRAVITY. Fr., Centre de gravité; Ger., Schwerpunct ; Ital., Centro di gravità. Span., Centro de gravedad.

See Gravity. Centre of Oscillation, see Oscillation. Centre of Percussion, see Percussion. Centre of Gyration, see Angular Motion, p. 103.

CENTRIFUGAL PUMP. Fr., Pompe à force centrifuge; Ger., Centrifugalpumpe; Ital., Tromba centrifuge; Span., Bomba centrífuga.

See Pumps.
CHAFF-CUTTER. Fr., Coupe-paille, Hachoir; Ger., Häcksellade; Ital., Trinciapaglia; Span., Máquina para cortar paja, \&c.

The cutting device in F. B. Hunt's chaff or straw cutter, Figs. 1889 and 1890, deserves particular attention. The machine in which this device is employed consists of a spirally-arranged

knife secured to two arms upon a shaft in front of a feed-box, so that as the shaft is rotated the knife works closely over the outer edge of a metallic bed-plate attached to the feed-box. The upper roller is so arranged as to adjust itself to the varying thickness of the layer of straw or other substance pressed under it, the roller raising or falling in the arc of a circle, so as to be always at the same distance from the shaft of the cutter, and the substance to be cut will be retained in a proper position on the bed of the machine. Attached to the frame of the upper feed-roller is a guard-board or plate, which is caused to rise or fall with the upper feed-roller, so as to prevent the substance being cut from passing over the top of the roller.

In referring to Figs. 1889, 1890, it will be observed that the combination, of the cutting-bar or plate $\mathbf{E}$ of the single knife $\mathbf{D}$ and the arms $c, c$, attached to the driving-shaft $\mathbf{C}$, operate to effect an
ollique or drawing cut. The fly-wheel $\mathrm{E}^{\prime}$ is so attached to its shaft C that the wheel is admitted to slip on the shaft in case the motion of the cutter is arrested by any foreign substance, thus the kuife or cutter is preserved. The bar $c b$ is connected directly to the shaft $n$, shown in section, Fig. $1890 ; n$ is the shaft of the lower feed-roller $G$, which is connected to the shaft $p$ of the upper feedroller H by an arm $i$. The bar $c b$, Fig. 1889, has pinions $b d$ attached to it, which gear with the pinions $a^{\prime} c^{\prime}$ of the feed-roller shafts $n, p$, exhibited in section, Fig. 1890. The guide-board or guideplate $u$ is attached to the plate $t$ of the frame of the upper feed-roller $\mathbf{H} ; u$ extends down at the back of H to a level with $p$ the shaft of H .

CHAIN PUMP. Fr., Pompe à chapelet; Ger., Kettenpumpe; Ital., Noria; Span., Bomba de cadena. See Pumps.

## CHECK-VALVE. Fr., Soupape d'arrêt; Ger., Absperrventil.

See Valves.
CHEESE PRESS. Fr., Machine à comprimer le fromage; Ger., Käsepresse; Ital., Torchio da formaggio; Span., Prensa de queso.

Dick's Anti-friction Cheese Press.-In Fig. 1891 is represented the method of applying Dick's anti-friction power to the pressing of cheese. By the use of this press the whey is entirely removed, and half the labour usually required in the manufacture of cheese is saved, it being ready for market or transportation as it comes from the press, without risk or loss to the purchaser and the consequent vexation so frequent in the case of the method originally pursued. The pressing may be carried to any extent deemed requisite without danger to the press, the working portions of the press being made of iron, and capable of sustaining the force that may be applied.

Attached to this press is the platform-scale; so that the cheese can be accurately weighed before it is removed from the press-a matter of great convenience and importance to the manufacturer and vendor of these articles, as well as to the purchaser, who can depend upon the weight marked upon it as strictly accurate. In fact, the advantages of this press, above others, must be at first view apparent, and the power such that it will come into, general use.

CHEESE VAT. Fr., Eclisse à fromage; Ger., Käsenapf; Ital., Tino da formaggio; Span., Quesera.

The objects to be attained by the use of a cheese vat are: a separation of the caseous or cheesy particles from the whey or watery particles of the milk, and the proper comminution of the latter preparatory to pressing. P. Colvin's cheese vat, Figs. 1892 to 1895, is intended to produce these results effectually, rapidly, and economically.

Fig. 1892 is a front view of the machine in perspective; Fig. 1893, an end view; Fig. 1894, a stirring-frame; and Fig. 1895, a cutting-frame; both these last operated by a crank. The vat A is semi-cylindrical and double-walled,

water being containcd between the shells. Under the vat, and attached thereto, is a furuace $\mathbf{B}$ for heating the water, the smoke from which escapes by the pipes C .

The degree of heat admitted to the water is regulated by a sliding damper D, Fig. 1893. A coil of circulating pipes is affixed to the outer shell of the vat, connected with the water-space at the centre and ends of the vat, thus equalizing the heat in the water-space. Convenient spouts or cocks are attached for drawing off the whey, the water from the water-space, and discharging the curd.

To aid in this, one end of the machine is set on eccentrics E. For keeping the curd separate during the operations of scalding, salting, and cooling, a stirringframe, Fig. 1894, turned by a crank, is used. This stirring-frame consists of curved paddles, and does the work usually performed by hand with a paddle; it is seen placed in the vat in Fig. 1892. The cutting-frame, Fig. 1895, seen in place in the machine, Fig. 1893, cuts the curds into small blocks by the longitudinal and transverse cutters on the rotating frame. This not only cuts the curd, but by its sweep cleans it from the inner surface of the vat. Either of these revolving frames may be lifted instantaneously from the vat, as the shafts bear at one end on a fixed pin, and at the other rest in an open box. At the discharge end of the vat, Fig. 1893, is a semicircular recess separated from the vat proper by a strainerplate sliding in rertical grooves in the inner shell of the vat, and can be withdrawn vertically when the curd is to be discharged into the hoop of the press.
W. Ralph's cheese vat, Fig. 1896, has a false bottom D, so constructed that the water heated by the furnace E does not come in contact with the bottom of the inner vat B until the water has been in contact with the ends and sides of the vat and imparted a portion of its heat to the same. Hollow pipes F , arranged with a valve $n$ and damper $P$, extend from the
 false bottom to the heated water below, and open at the top under hollow supports $g$ upon which the vat rests.

CHEVAUX DE FRISE. Fr., Chevaux de frise ; Ger., Spanische oder friesische Reiter; Ital., Cavalletto di frisa; Span., Caballo de frisa.

## See Fortification.

Chimney. Fr., Cheminée; Ger., Schornstein; Ital., Camino; Span., Chimenea.
The appellation Chimney Stalk is usually applied to a lofty chimney; such stalks are erected for steam-engines, or employed to create a draught of air through furnaces.

1. Heating apparatus vary in form and arrangement with the nature of the effect to be produced, but in general they all consist of three distinct parts: the furnace or place in which the heat is generated, the place in which the heat is utilized, and the chimney.

The function of chimneys is twofold:-
1st. To discharge at a great height in the atmosphere, the heated air often charged with smoke, which would be noisome to animal life if ejected at a small altitude.

2nd. To cause a sufficient flow of air through the furnace to maintain combustion.
2. Motion of Heated Air in Vertical Tubes.- When a mass of air is at a temperature above that of the surrounding air, it has a tendency to rise in virtue of a force equal to the excess of the weight of the air displaced over its own weight. This is a particular case of the principle of Archimedes.

The same thing happens when the heated air is containe in a vertical tube A B, Fig. 1897, open at both ends. If we represent by P the pressure of the atmosphere in kilogrammes at the height of the point A, upon a surface equal to the section of the tube, and by $p$ and $p^{\prime}$, the weight of two columns of air having the volume of the tube under the atmospheric pressure, one at the temperature of the external air, the other at that of the heated air, it is evident that the pressure at the point B , exerted in an upward direction, will be $\mathbf{P}+p$, and that the pressure at the same point, exerted in the contrary direction, will be $\mathrm{P}+p^{\prime}$. Therefore the column of heated air will have a tendency to rise in virtue of the pressure $p-p^{\prime}$. It follows from this, that if the pipe A B be fixed at the bottom to a horizontal pipe B C, heated externally, Fig. 1898, the external air will enter constantly through the orifice C, and will escape through the orifice A. This flow of air tekes place in virtue of the excess of the atmospheric pressure at the point C over the internal pressure at the bottom of the pipe A B.
3. To determine the velocity of ingress of the external air at the point $C$, supposing the section
of the tube constant, we must remember that the flow of a liquid or of a gas, the resistance depending on the form and the dimensions of the tube apart, is represented, from the point of view of velocity, by the formula $v=\sqrt{2 g \mathrm{P}}$, in which P denotes the height of a column of fluid subjected to the pressure, and which would hold this pressure in equilibrio, however it might be produced. In the case in question, P is evidently the height of a column of external air, and its value is readily found.

Denoting the height of the chimney from the centre of the section C, Fig. 1898, by H, the external temperature by $\theta$, that of the air inside the chimney by $t$, and the pressure of the atmosphere at the point $A$, in air at $\theta^{\circ}$, by M, the pressure of the point C , from the outside inwards, measured by a column of air at $\theta^{\circ}$, will be $\mathrm{M}+\mathrm{H}$, and the pressure in the contrary direction, measured in the same way, will be

$$
\mathrm{M}+\mathrm{H}(1+a \theta) \div(1+a t) .
$$

Consequently, we shall have as the excess of the former pressure over the latter,

$$
\mathrm{M}+\mathrm{H}-\left(\mathrm{M}+\frac{\mathrm{H}(1+a \theta)}{1+a t}\right)=\frac{\mathrm{H} a(t-\theta)}{1+a t}
$$

and therefore,

$$
\begin{equation*}
v=\sqrt{\frac{2 g \mathrm{H} a(t-\theta)}{1+a t}} . \tag{A}
\end{equation*}
$$

4. It may be useful to remark that the pressure at any height in the column of heated air is equal to that at the point C. If we consider a horizontal section of the chimney at a height $H^{\prime}$ from the top, the upward
 pressure which it will support, reckoned in air at $\theta$, will be $\mathrm{M}+\mathrm{H}-\left(\mathrm{H}-\mathrm{H}^{\prime}\right) \frac{1+a \theta}{1+a t}$; the pressure in the contrary direction will be $\mathrm{M}+\mathrm{H}^{\prime} \frac{1+a \theta}{1+a t}$; and subtracting one from the other we find, as before, $\mathrm{H} a \frac{(t-\theta)}{1+a t}$.
5. For the rate of flow of the hot air, as the velocities are in inverse proportion to the densities, we shall have

$$
\begin{equation*}
v^{\prime}=v \frac{1+a t}{1+a \theta} ; \text { whence } v^{\prime}=\sqrt{\frac{2 g \mathbf{H} a(t-\theta)(1+a t)}{(1+a \theta)^{2}}} \tag{B}
\end{equation*}
$$

6. The formulæ [A] and [B] may be put into a more convenient form for practice, by substituting for the term $\sqrt{2 g a}$ its value; we shall then have

$$
v=0.268 \sqrt{\frac{\overline{H(t-\theta)}}{1+a t}} ; \text { and } v^{\prime}=\frac{0.268}{1+a \theta} \sqrt{\overline{H(t-\theta)}(\overline{1+a t)}} .
$$

The formula [A] is alone important, for the useful effect of chimneys always consists in their ability to create a draught.
7. We have supposed that the density of the air depended only upon its temperature. This is not strictly true, for the density of the air at the same temperature decreases with the increase of altitude; but it is obvious that in the highest chimneys, the variations of density resulting from variations of height are quite imperceptible. For the altitude $0^{\text {m }} \cdot 76$ of the barometer, the pressure of the atmosphere is represented by a column of water of $0^{\mathrm{m}} \cdot 76 \times 13 \cdot 6=10^{\mathrm{m}} \cdot 336$, and by a
 density of the air at a height of 20 mètres, for example, would be to that of the air at the surface of the earth in the ratio of 7930 to 7950 , that is, in that of 1 to $1 \cdot 0025$, a variation of no importance whatever.
8. We have supposed the vertical tube cylindrical and completely open at the ends; but it is evident that the position and form of the pipe have no influence on the force produced by the ingress of external air, when the temperature of the pipe is everywhere equal to $t, \mathrm{H}$ representing the difference of level between the two ends of the column of heated air. If the air, in ascending the pipe, met with variations of temperature, we should have to take for $t$ the mean temperature.
9. The formula [A] gives the velocity or rate of ingress of the external air, all resistance apart; but there is always a loss of force attributable to friction and the form of the tube. If the chimney, instead of being cylindrical, varied in section and direction, the weight or force which would cause the inward flow of cold air would be still the same; but the actual rate of ingress would be found by taking into account the resistance, by means of the general formula given. We will return to this question later.
10. To determine how the rate of ingress V of the external air varies with the increase of $t$, according to the formula [A], suppose $\theta=0$; this formula becomes $v=\sqrt{2 g \mathrm{H}} \sqrt{\frac{a t}{1+a t}}$. The first factor of the value of $v$ represents the velocity acquired by a body falling from the height

H; and, as the second factor is always less than unity, the rate of ingress is never more than a fraetion of this velocity. Taking successively for $t 50^{\circ}, 100^{\circ}, 150^{\circ}, 200^{\circ}, 250^{\circ}, 300^{\circ}, 350^{\circ}, 400^{\circ}, 500^{\circ}$, $1000^{\circ}, 1500^{\circ}, 2000^{\circ}$, we find as the values of the second factor, $0 \cdot 39,0.51,0 \cdot 57,0 \cdot 64,0 \cdot 68,0 \cdot 71$, $0.74,0.76,0.80,0.88,0.91,0.93$. These numbers divided by the first, 0.39 , give the ratios: $1,1 \cdot 31,1 \cdot 51,1 \cdot 65,1 \cdot 76,1 \cdot 83,1 \cdot 90,1 \cdot 96,2 \cdot 06,2 \cdot 25,2 \cdot 34,2 \cdot 38$. Thus the rate of ingress of the cold air increases constantly with $t$, but this increase has a limit; for as $t$ increases, the ratio $\frac{a t}{1+a t}$, which is equal to $\frac{a}{\frac{1}{t}+a}$, constantly approaches unity, and consequently, the rate of ingress of the cold air, from $t=50^{\circ}$ to $t=\infty$, varies in the ratio of $0 \cdot 39$ to 1 , or in that of 1 to $2 \cdot 56$.

It follows from this that the rate of ingress of the cold air increasing very slowly with the temperature, the useful effect of the chimney costs in fuel an amount proportionate to the temperature within the chimney.

The maximum rate of ingress of the outside air being $\sqrt{2 g \mathrm{H}}$, for chimneys of $5 \mathrm{~m}, 10^{\mathrm{m}}, 20^{\mathrm{m}}$, $30^{\mathrm{m}}, 40^{\mathrm{m}}$, it is equal to $9 \mathrm{~m} \cdot 9,14^{\mathrm{m}} \cdot 0,19^{\mathrm{m} \cdot 8}, 24^{\mathrm{m} \cdot 3,} 28^{\mathrm{m} \cdot 0}$, and for an excess of temperature of $300^{\circ}$, the rate would be only 0.71 of the maximum. The velocity is besides considerably reduced by the resistance experienced by the heated air when in motion, as we shall see later.
11. We have supposed that the air, entering the vertical pipe, underwent, while being heatea, no other change of density than that resulting from the change of temperature. But usually the heat is caused by combustion, the nature of the gas is changed, and it is necessary to see what the formulæ [A] and [B] become when the tabular density of the gas is equal to $\delta$, instead of being equal to unity.

In this case, the height of the inner column, reduced to $\theta^{\circ}$, and to the density of the outer air, will be $\frac{H \delta(1+a \theta)}{1+a t}$, and the difference of the two columns, or the force, will become

$$
\mathrm{H}-\frac{\mathrm{H} \delta(1+a \theta)}{1+a t}=\frac{\mathrm{H}[1-\delta+a(t-\delta 0)]}{1+a t} ; \text { or, } \frac{\mathrm{H}(1-\delta+a t)}{1+a t}
$$

by rejecting the term $\delta \alpha \theta$.
If, for example, the whole of the oxygen were transformed into carbonic acid, the density of this latter gas being $1 \cdot 529$, and that of ozote 0.976 , the density of the mixture would be

$$
0.21 \times 1.529+0.79 \times 0.976=1.091
$$

if only half the oxygen were transformed into carbonic acid, which is the ordinary case, the density would be only $1 \cdot 04$. Neglecting $\theta$ in the expression of the force, it would become in the two cases, $\frac{\mathrm{H}(a t-0.091)}{1+a t}$; and $\frac{\mathrm{H}(a t-0.04)}{1+a t}$ : and these would be the same if the values of $t$ were diminished by $25^{\circ}$ in the first case, and by $11^{\circ}$ in the second. These variations are of small importance, especially when $t$ is considerable. Besides, as the gases which are evolved in the furnaces always contain a certain quantity of steam obtained from the water held by the combustibles, and as the density of steam is equal to $0 \cdot 621$, its pressure diminishes the effect of the carbonic acid. Thus we may admit, as an approximation amply sufficient in practice, that the effect produced by a column composed of air and gases, resulting from combustion, is the same as if the column were of pure air.
12. Motion of Heated Air in a Pipe consisting of several Vertical Tubes successively passed through.-If the heated air traverses successively, ascending and descending, several vertical tubes in which the temperature is not the same, the force or pressure at the beginning of the
pipe will depend at once on the heights of the tubes and the temperatures of the heated air. We will endeavour to determine it in a general manner.
13. Let us, in the first place, consider a chimney A B, Fig. 1899, extending horizontally in the direction BC , and descending again vertically in the direction CD, so as to form a syphon.

If we represent by $t$ and $t^{\prime}$ the temperatures of the air in the portions AB and CD, by $\theta$ that of the external air, by $p$ and $p^{\prime}$ the pressures of the atmosphere at the points $\mathbf{A}$ and D reckoned in air at $\theta^{\circ}$, under the normal pressure, and by $m$ and $m^{\prime}$ the heights of the columns of air at $\theta^{\circ}$, equivalent to the columns of heated air AB and CD, the force in air at $\theta^{\circ}$, which will produce the rush of cold air at the point A, will evidently be equal to $p-m+m^{\prime}-p^{\prime}$; and as the pressure of the external air at the point $\mathbf{D}$ exceeds that at the point A by a column of external air equal to $H^{\prime}-H$, we shall have as the force in air at $\theta^{\circ}$, at the point $A$,

$$
p-\frac{\mathrm{H}(1+a \theta)}{1+a t}+\frac{\mathrm{H}^{\prime}(1+a \theta)}{1+a t^{\prime}}-\left(p+\mathrm{H}^{\prime}-\mathrm{H}\right)=\frac{\mathrm{H} a(t-\theta)}{1+a t}-\frac{\mathrm{H}^{\prime} a\left(t^{\prime}-\theta\right)}{1+a t^{\prime}} .
$$

Thus the force is equal to the difference of the corresponding forces at the two branches supposed alone. The flow will evidently take place in the direction AB, as we had supposed, if the preceding expression be positive. Taking the external air at $\theta^{\circ}$, and rejecting the terms $\mathbf{H} a^{2} t t^{\prime}$ and $\mathrm{H}^{\prime} a^{2} t t^{\prime}$, which are always small, seeing that $a^{2}=0 \cdot 000134$, the value of the force in external air becomes $\frac{a\left(\mathrm{H} t-\mathrm{H}^{\prime} t^{\prime}\right)}{1+a\left(t+t^{\prime}\right)}$, an expression that is positive when

$\mathrm{H} t$ is greater than $\mathbf{H}^{\prime} t^{\prime}$, a circumstance which may always be obtained, whatever $\mathrm{H}^{\prime}$ may be, by diminishing $t^{\prime}$. This may be effected by cooling the air in the tubes BC and CD. Thus the
gases evolved in the furnace may always be discharged at any height, oven beneath the level of the furnace.
14. If there were a third column EF, Fig. 1900, denoting its height by $\mathbf{H}^{\prime \prime}$, and the mean temperature of the air in it by $t^{\prime \prime}$, by $m^{\prime \prime}$ the height of a column of air at $\theta^{\circ}$ producing the same pressure, and retaining the preceding notation, the force at the point A would be $p-m+m^{\prime}$ $-m^{\prime \prime}-p^{\prime}$; but we have $p^{\prime}+\mathrm{H}^{\prime \prime}-\mathrm{H}^{\prime}+\mathrm{H}=p$, and therefore the force at the point A in air at $\theta^{\circ}$ will be

$$
\begin{gathered}
p-\begin{array}{c}
\mathrm{H}(1+a \theta) \\
1+a t
\end{array}+\frac{\mathrm{H}^{\prime}(1+a \theta)}{1+a t^{\prime}}-\frac{\mathrm{H}^{\prime \prime}(1+a \theta)}{1+a t^{\prime \prime}}-p-\left(\mathrm{H}^{\prime \prime}-\mathbf{H}^{\prime}+\mathbf{H}\right) \\
=\frac{\mathbf{H} a(t-\theta)}{1+a t}-\frac{\mathbf{H}^{\prime} a\left(t^{\prime}-\theta\right)}{1+a t^{\prime}}+\frac{\mathrm{H}^{\prime \prime} a\left(t^{\prime \prime}-\theta\right)}{1+a t^{\prime \prime}}
\end{gathered}
$$

The flow will take place in the direction AF, when this expression is positive. Rejecting, as before, the terms containing $a^{2}$, and $a$ fortiori $a^{3}$, and supposing $\theta=0$, the preceding expression is reduced to $\frac{a\left(\mathrm{H} t-\mathrm{H}^{\prime} t^{\prime}+\mathrm{H}^{\prime \prime} t^{\prime \prime}\right)}{1+a\left(t+t^{\prime}+t^{\prime \prime}\right)}$, an expression which will be positive when $\mathrm{H} t+\mathrm{H}^{\prime \prime} t^{\prime \prime}$ is greater than $\mathrm{H}^{\prime} t^{\prime}$.

It would be easy from this to find the formula for any number of tubes.
15. Let us now suppose the pipe to have the form of a syphon reversed, Fig. 1901; retaining the same notation, the force at the point A , in air at $\theta^{\circ}$, will be $p+m-m^{\prime}+p^{\prime}$; or

$$
p+\frac{\mathrm{H}(1+a \theta)}{1+a t}-\frac{\mathrm{H}^{\prime}(1+a \theta)}{1+a t^{\prime}}-p+\mathrm{H}^{\prime}-\mathbf{H}=-\frac{\mathrm{H} a(t-\theta)}{1+a t}+\frac{\mathrm{H}^{\prime} a\left(t^{\prime}-\theta\right)}{1+a t^{\prime}}
$$





As in the preceding case, the effect produced is equal to the difference of the effects which the two branches would produce alone. Rejecting the terms containing $a^{2}$, and making $\theta=0$, the value of the force is reduced to $-\frac{a\left(\mathbf{H} t-\mathbf{H}^{\prime} t^{\prime}\right)}{1+a\left(t+t^{\prime}\right)}$, a value which will be positive when $\mathbf{H}^{\prime} t^{\prime}$ is greater than $\mathrm{H} t$; this condition may always be satisfied by increasing $\mathrm{H}^{\prime}$, and by preventing the cooling of the air in the horizontal portion BC.
16. If the pipe were formed of three tubes, Fig. 1902, the force at the point A, in air at $\theta^{\circ}$, would evidently be $p+m-m^{\prime}+m^{\prime \prime}-p^{\prime}$; or

$$
\begin{gathered}
p+\frac{\mathrm{H}(1+a \theta)}{1+a t}-\frac{\mathrm{H}^{\prime}(1+a \theta)}{1+a t^{\prime}}+\frac{\mathrm{H}^{\prime \prime}(1+a \theta)}{1+a t^{\prime \prime}}-\left(p+\mathrm{H}-\mathrm{H}^{\prime}+\mathrm{H}^{\prime \prime}\right) \\
=-\frac{\mathrm{H} a(t-\theta)}{1+a t}+\frac{\mathrm{H}^{\prime} a\left(t^{\prime}-\theta\right)}{1+a t^{\prime}}-\frac{\mathrm{H}^{\prime \prime} a\left(t^{\prime \prime}-\theta\right)}{1+a t^{\prime \prime}}
\end{gathered}
$$

or on the hypothesis $\theta=0^{\circ}, \frac{a\left(-\mathrm{H} t+\mathrm{H}^{\prime} t^{\prime}-\mathrm{H}^{\prime \prime} t^{\prime \prime}\right)}{1+a\left(t+t^{\prime}+t^{\prime \prime}\right)}$.
The force for any number of tubes may be readily calculated in the same way.
In general, the effect produced is equal to the sum of the effects produced separately by the tubes in which the heated air rises, diminished by the sum of the effects produced by the tubes in which the heated air descends.
17. It is of importance to remark that in the case in which the heated gas begins by descending, supposing the expression of the force to be positive, motion will not take place until the system of syphons has been put into action by heating some of the branches in which the air ascends. It is

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evident that this precaution will not be needed in the case in which the heated air ascends in the first column.
18. Motion of Heated Air in a Pipe consisting, throughout a certain length, of several Tertical Tubes treversed simultaneously.-Let us consider, in the first place, a vertical pipe, Fig. 1903, consisting, throughout a certain length, of two equal and parallel branches. If both contain air at an equal temperature, everything else being equal in the two branches, the ascending heated air will traverse them with the same velocity; but however little difference there may be in the diameters, the resistance, or the causes of cooling, this equality of velocity, should there be any difference, will not exist; and if the section of each of them is equal to the section of the end tubes, the motion will take place through one only, that one which offers the least resistance. This phenomenon will evidently occur, whatever the number of the tubes may be.
19. If the pipe were only enlarged throughout a portion of its length, Fig. 1904, the current of heated air could not necessarily fill the enlarged portion of the pipe; if it filled it at first, the cooling of the sides would soon cause a decrease of temperature from the centre to the circumference; there would occur, in the elementary veins and in the same direction, variations of velocity which would go on increasing, so that the vein of heated air would soon traverse the enlarged space, without much increasing in section.

1904.

1905.

1906.

20. Suppose now that the heated air drawn by the chimney descends a pipe consisting of two parallel branches, Fig. 1905. In this case, the heated air will separate itself equally in the two branches, and the equality of velocity will be maintained in spite of the inequalities of cooling which the air may undergo. Indeed, in each branch, the force which produces the motion is equal to the difference of the pressures in the chimney and in the pipe under consideration, supposing them isolated; and, consequently, if in one of the branches the cooling was greater than in the other, the velocity there would become greater. The same thing would occur in the case of any number of parallel tubes.
21. If the pipe through which the heated air descends were enlarged, Fig. 1906, the elementary veins in the enlarged portion would assume and retain the same temperature, for the same reason as in the preceding case.

These facts, which have been many times proved by experience, are of great importance in the construction of heating apparatuses, as we shall see later.
22. It must be remarked that in all the preceding considerations relative to the velocity of the gases in chimneys, we have left out of the question the resistance experienced by the air when in motion. But as this resistance is always considerable, there will be a greater or less reduction of the velocity. When the descending pipe is single, friction takes place against the surface; therefore, on this account alone, the velocity would there be less than in the other points of the section; but the cooling which the heated air there undergoes tends to diminish the difference of velocity which would naturally arise in the veins, and this difference will be still more diminished by the gradual transmission of the resistance and the temperature. If the descending pipes were formed of several tubes having the same section and the same length, the same phenomena would occur, and the heated air would have the same velocity in each of them. But if the tubes have different sections, the velocity will be modified by the cooling which tends to increase it and by the friction which tends to diminish it. The friction varies in proportion to the square of the velocity, whilst the cooling is proportional to the diameter and depends also upon the velocity; it is, therefore, not

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easy to foresee and to calculate in a general manner what will happen. The cooling is, however, small in general, and the influence of friction will in almost all cases exceed it, so that the actual velocity will decrease with the diameter of the pipes, a theoretical result that is in perfect accordance with the results of experience. The difference of the velocities will evidently decrease with the velocities themselves.
23. Former Theory with respect to the Drawing of Chimneys.-Hitherto it has been admitted that the force in air at $\theta^{\circ}, \mathrm{H} a(t-\theta) \div(1+a t)$ was applied directly to the heated air, and the result of this was, that the force in air at $t^{\circ}$ being $\mathrm{H} a(t-\theta) \div(1+a \theta)$, the velocity of the heated air was
$v_{1}^{\prime}=\sqrt{\frac{2 g \mathrm{H} a(t-\theta)}{1+a \theta}}$; and the rate of ingress of the cold air was

$$
v_{1}=v_{1}^{\prime} \frac{1+a \theta}{1+a t}=\sqrt{\frac{2 g \mathrm{H} a\left(t-\frac{\theta)(1+a \theta)}{(1+a t)^{2}}\right.}{(1)}, ~}
$$

formulæ which differ widely from the formulæ [A] and [B], for we have evidently

$$
v_{1}^{\prime}=v^{\prime} \sqrt{\frac{1+a \theta}{1+a t}} ; \text { and } v_{1}=v \sqrt{\frac{1+a \theta}{1+a t}}
$$

which give for $v_{1}^{\prime}$ and $v_{1}$ velocities less than $v^{\prime}$ and $v$. Besides this, the value of $v_{1}$ offers a circumstance that is not found in the value of $v$. If $\theta$ remaining constant, the value of $t$ be progressively increased, $v_{1}$ increases at first, reaches a maximum for $t=\frac{1}{a}+2 \theta=274+2 \theta$, and afterwards decreases indefinitely.
24. But the force in air at $\theta$ acts directly upon the cold air which is entering the chimney and not upon the heated air, and it is the velocity of the cold air that is afterwards transmitted to the heated air. Thus the formulæ which we have just given, for a considerable time admitted by all who have studied the question and by Péclet in the second edition of his work, do not represent the phenomena as they actually are. This discovery, due to Péclet, resulted from a more careful examination of the facts and from several experiments which we here relate, taken from Péclet's 'Traité de la Chaleur.'
25. Taking the case of a horizontal pipe through which air coming from a gasometer flows, let L and D denote the length and the diameter of the pipe; $\mathbf{P}$ the excess of pressure in the gasometer over the pressure of the atmosphere; $p$ the force corresponding to the velocity of flow at the end of the pipe, both expressed in air at the external temperature $\theta^{\circ}$; and $A$ the coefficient of the loss of force at the orifice. The pipe being at the external temperature, we shall have

$$
\begin{equation*}
\mathbf{P}-p=\left(\mathbf{A}+\frac{\mathbf{K} \mathbf{L}}{\mathbf{D}}\right) p, \text { and } \mathrm{V}=\sqrt{\frac{2 g \mathbf{P}}{1+\mathbf{A}+\frac{\mathbf{K L}}{\mathbf{D}}}} \tag{x}
\end{equation*}
$$

Suppose now the pipe heated so as to raise its temperature to $t^{\circ}$; admitting that the flow of heated air takes place by the force in heated air, this force will be $\mathrm{P} \frac{(1+a t)}{1+a \theta}$, and the velocity of flow $v_{1}$ of the cold air from the gasometer will be

$$
\begin{equation*}
v_{1}=\frac{1+a \theta}{1+a t} \sqrt{\frac{2 g \mathbf{P}(1+a t)}{1+a \theta} \cdot \frac{1}{1+\mathbf{A}+\frac{\mathrm{KL}}{\mathbf{D}}\left(\frac{1+a t}{1+a \theta}\right)^{2}}} \tag{b}
\end{equation*}
$$

and as, by placing the whole under the radical sign, the first factor becomes $2 g \mathrm{P} \frac{1+a \theta}{1+a t}$, we see that the ratio of V to $v_{1}$ is greater than the square root of $\frac{1+a}{1+a \theta}$.

In the new theory, we have as the velocity $v$ of the cold air,

$$
\begin{equation*}
v=\sqrt{\frac{2 g \mathrm{P}}{1+\mathbf{A}+\frac{\mathrm{KL}}{\mathbf{D}}\left(\frac{1+a t}{1+a \theta}\right)^{2}}} \tag{c}
\end{equation*}
$$

and the ratio of V to $r$ decreases but slowly, in proportion as $t$ increases, and only by the increase of resistance due to friction, and which is caused by the elevation of the temperature.
26. To determine the influence of the heating of the air in the pipe A B, Péclet had recourse to the arrangement shown in Fig. 1907. A B is a copper pipe 1 mètre in length and $0^{\mathrm{m} \cdot 01}$ in diameter, communicating with a gasometer; throughout a portion of its length it was enclosed by another concentric tube CD; the space between the two tubes was closed at the ends by cocks, and steam introduced by the
 pipe $a$, and let out by the pipe $b$. When the pipe A B was not heated, a certain volume of air flowed through in 612"; and when the pipe was heated, the same volume of air, under the same pressure and at the same temperature, flowed through in 618".
27. To obtain a higher temperature, Péclet adopted the arrangement shown in Fig. 1908. A B


$0^{\mathrm{m}} \cdot 90$ in length and $0 \mathrm{~m} \cdot 011$ in diameter; beneath the iron tube was placed a stove D E of nearly the same length, for the purpose of holding burning charcoal. For this arrangement, the equations $[a],[b]$, and [c] were replaced by the following:

$$
\begin{gather*}
\mathrm{V}=\sqrt{\frac{2 g \mathrm{P}}{1+\mathrm{A}-\mathrm{B}+\frac{\mathrm{K} l}{d}+\frac{\mathrm{K}}{\mathrm{~L}} \cdot \frac{d^{4}}{\mathrm{D}^{4}}}} \\
v_{1}=\frac{1+a \theta}{1+a t} \sqrt{\frac{2 g \mathrm{P}(1+a t)}{1+a \theta} \cdot \frac{1}{1+\mathrm{A}-\mathrm{B}+\cdot \frac{\mathrm{K} l}{d}+\frac{\mathrm{K} \mathrm{~L}}{\mathrm{D}} \cdot \frac{d^{4}}{\mathrm{D}^{4}}\left(\frac{1+a t}{1+a \theta}\right)^{2}}} \\
v=\sqrt{\frac{2 g \mathrm{P}}{1+\mathrm{A}-\mathrm{B}+\frac{\mathrm{K} l}{d}+\frac{\mathrm{K} \mathrm{~L}}{\mathrm{D}} \cdot \frac{d^{4}}{\mathrm{D}^{4}}\left(\frac{1+{ }^{\prime} a t}{1+a \theta}\right)}}
\end{gather*}
$$

in which V represents the velocity of flow in the small tube when the air is not heated; $v_{1}$ and $v$, the velocities of the cold air when the air is heated in the large tube, for the old and new theories; $l, \mathrm{~L}, d, \mathrm{D}$, are the lengths and the diameters of the tubes A B and $\mathrm{BC} ; \mathrm{A}$ and B are the coefficients of the variations of force at the orifice, and at the point in which the tube is suddenly enlarged.

In this case, as in the preceding, the ratio of V to $v_{1}$ is greater than the square root of $\frac{1+a t}{1+a \theta}$, whilst the ratio of V to $v$ differs but little from unity.

The iron tube being at the external temperature, a certain volume of air flowed through in $620^{\prime \prime}$; and when it was heated to a dark red, the same volume of air, in the same circumstances, flowed through in $650^{\prime \prime}$. The iron tube having been replaced by another of $0^{\mathrm{m} \cdot} 013$ in diameter and $1^{\mathrm{m} \cdot} 16$ in length, the same volume of cold air flowed through in $615^{\prime \prime}$; and when the tube was heated red hot, the time of the flow, in the same circumstances, rose to $648^{\prime \prime}$. The external temperature was at $14^{\circ}$.

These experiments do not enable us to determine exactly to which of the two formule [b] and $[c]$ or $\left[b^{\prime}\right]$ and $\left[c^{\prime}\right]$ they conform most, because it was impossible to measure the temperature of the air on issuing, and because the increase of temperature took place successively, which prevented the calculation of the resistance in the tube AB. It is, however, plainly seen that the equations $[b]$ and $\left[b^{\prime}\right]$ are incompatible with the equations [ $\left.a\right]$ and $\left[a^{\prime}\right]$.
28. In the first experiment, the ratio of the velocities of the cold air, when the second tube is cold and when it is heated, is equal to $\frac{618}{612}=1 \cdot 0099$; and, according to the formulæ $[a]$ and $[b]$, supposing the air at $100^{\circ}$, this ratio should be greater than the square root of $\frac{1 \cdot 366}{1 \cdot 0366}$, which is equal to $1 \cdot 1471$. Neglecting the increase of resistance in the tube A B, which increase augments the ratio in question, we should have to admit for formula [b], in order to make it agree with experience, $t=15^{\circ} \cdot 7$; but this is impossible, for the temperature must have been near $100^{\circ}$. In the second experiment, the ratio of the velocities in question was equal to $\frac{650}{620}=1 \cdot 0483$; and, according to formula [ $b^{\prime}$ ], supposing only the air at $400^{\circ}$, this ratio should exceed the square root of $\frac{2 \cdot 464}{1 \cdot 05124}$, which is equal to $1 \cdot 53$. Neglecting the increase of resistance in the tube A B, to make formula [ $b^{\prime}$ ] agree with experience, we should have to suppose $t=42^{\circ} \cdot 4$. But this is impossible, for we find in the register of the experiments these words: The air issuing is burning; a thermometer held in it rapidly rises to $150^{\circ}$. And it could not have been otherwise, for the iron tube was red hot. In the third experiment, the ratio of the velocities was $\frac{648}{615}=1 \cdot 0536$; and, according to formula [b'], supposing only the air at $600^{\circ}$, this ratio should exceed the square root of $\frac{3 \cdot 196}{1 \cdot 0512}$, which is equal to $1 \cdot 743$. Neglecting as before the increase of resistance in the tube $A B$, to make formula [ $b^{\prime}$ ] agree with experience, we should have to suppose $t=45^{\circ} \cdot 5$. This again is impossible, for the air issued hotter than in the preceding experiment.

The discrepancies wnich we have found between the results of the formulæ [b] and [ $b^{\prime}$ ] and those
of experience, are too great to be attributed to errors of observation. For example, for the first experiment, neglecting the resistance due to the increase of friction, the time of the flow should have been $612 \times 1 \cdot 1471=900^{\prime \prime}$, instead of $618^{\prime \prime}$; that is, greater by $900-618=282^{\prime \prime}$; and this difference should have been much greater for the others; now, each experiment was repeated twice with the same result each time.
29. It follows evidently from all these experiments that the formulæ [ $b$ ] and $\left[b^{\prime}\right]$ cannot be admitted; that is, when air flows in a horizontal tube in virtue of a pressure, and the air at a certain distance is heated, the velocity of the cold air cannot be deduced from that of the heated air by taking as the force of this flow of cold air, the force in heated air. But if we suppose that the external force is applied to the cold air, the velocity will undergo only a small diminution resulting from the increased resistance of the heated air, and this agrees with the experiments.

That which we have just affirmed must evidently be true, whatever the nature of the pressure exerted on the cold air on its entrance into the tube may be; therefore, if it is vertical and heated externally, the pressure resulting from the two columns of air of the same height, one at the external temperature $\theta^{\circ}$, and the other at the mean temperature $t$, is the force which acts directly on the cold air entering the chimney; this force should be measured in air at $\theta^{\circ}$, and it is this velocity which is afterwards transmitted to the heated air, as we have already explained (3).
30. Former Experiments on the Flow of Air in Chimneys.-Many years ago Péclet made a long series of experiments on the flow of air in chimneys. He employed pipes of sheet iron, cast iron, and earthenware, of various heights and sections. The apparatus was arranged in the following manner: the stove was large and partly covered with charcoal for the purpose of rendering the resistance of the fire-place nearly nul; the chimney was above the stove. The temperature of the air in the chimney was shown by two thermometers placed, one at the bottom, the other at the top. As anemometers were unknown in those days, Péclet determined the velocity of the air by introducing rapidly through the opening in the ash-pan under the stove, a piece of burning tow which had been previously dipped in turpentine and fixed to the end of an iron rod. Withdrawing immediately the burning tow, a small quantity of smoke was produced in the fire-place and carried upwards by the heated air. By observing the time when the smoke was produced and the time when it appeared at the top of the chimney, the velocity of the current of air in the chimney was ascertained. The experiments were repeated with the chimney more or less closed at the top, and at the bottom by diaphragms formed of a sheet of iron pierced with a circular orifice. The method of observation was too imperfect to give exact results, yet these experiments enabled the experimenter to ascertain some important facts of which we will speak later.
31. General Considerations on Factory Chimneys.-In order to study in a complete manner the phenomena which occur in chimneys, it is necessary to have a clear notion of the general arrangement of heating apparatuses. As we have said already, they consist of the furnace, the space in which the heat is utilized, and the chimney. Furnaces are usually formed of cast-iron bars placed in a horizontal plane, or slightly inclined, separated from each other by small intervals, and upon which the fuel is placed. Beneath is a space communicating freely with the air, and known by the name of the " ash-pit."

The space comprised between the furnace and the chimney, and where a portion of the heat produced passes into the body which it is wished to heat, has forms and dimensions varying with the nature of the effect to be produced.

Chimneys are always vertical tubes or channels, constructed of brick or sheet iron, and designed to discharge the burned air, and to create a draught through the furnace, which draught is necessary to support combustion.
32. The inward rush of external air caused by the temperature of the burned air and by the height of the chimney is called the draught. The draught of a chimuey, as we have calculated it, is always diminished, in a very large proportion, by the resistance of the bars, and so on, of the furnace, the sudden or continuous changes of section and direction, and by friction.

The phenomena which occur in heating apparatuses are very complicated and very varying, chiefly with the condition of the furnace; but we may determine the influence of the various circumstances which modify the draught. We will suppose, in the first place, that the circuit has everywhere the same section, from the opening of the ash-pit to the top of the chimney. This hypothesis differs but little from ordinary facts; even to the sum of the free orifices of the furnace-bars a surface is given differing but little from the horizontal section of the chimney. Denoting, then, the section of the air-channel which conducts the air under the furnace-bars by S , the rate of ingress of the cold air by $v, \mathrm{~S} v$ will represent the volume of cold air entering per second. Putting $G$ for the resistance of the furnace, $C$ for that of the circuit preceding the chimney, and H and D for the height and diameter of the chimney, the friction in the chimney will be $\mathrm{K} \mathrm{L}(1+a t)^{2} \div \mathrm{D}(1+a \theta)^{2}$, and would be exactly the same if the section of the chimney were the square circumscribed about the circle. Applying here the laws relative to the motion of compressed gases, we shall have, supposing the temperature outside equal to $0^{\circ}$

$$
\mathrm{P}-p=(\mathrm{G}+\mathrm{C}) p+\frac{\mathrm{KH}}{\mathrm{D}}(1+a t)^{2} p ;
$$

whence

$$
v=\sqrt{\frac{2 g \mathrm{H} a t}{(1+a t)\left[1+\mathrm{GC}+\frac{\mathrm{KH}}{\mathrm{D}}(1+a t)^{2}\right]}} .
$$

33. The examination of this formula leads to several consequences of great importance in practice.
34. When the values of $G$ and $C$ are very great relatively to the friction in the chimney, which is almost always the case, the draught is proportional to the square root of the height.
35. If we supposed $G$ and $C$ nul, which would happen only in the case in which the furnace is placed at the bottom of the chimney, the grating occupying only a small portion of the section, and the air experiencing no resistance on entering the chimney, the draught would vary very little with the height of the chimney; it would be nearly independent of it when $K H \div D$ is very great relatively to 1 . These are circumstances which occur rarely in practice, but they were present in the experiments to which we have alluded (30). The height of the chimney had hardly any influence upon the draught, because the air experienced hardly any resistance.
36. Suppose again that, nothing being changed in the dimensions of the apparatus, the temperature $t$ varies. If $G$ and $C$ be supposed constant, the friction in the chimney being in general very small relatively to $G+C$, the draught, will vary nearly proportionally to the square root of $\frac{t}{1+a t}$, that is, in the same ratio as the theoretical draught. Thus, in the supposition which we have made, the ratio of the actual to the theoretical draught would be a constant number, depending only on the dimensions of the apparatus. But, in an apparatus in operation, $t$ can rise only by the increase of the velocity of the current, and this increase can take place only by the diminution of the resistance, which, in the case supposed, can result only from the condition of the furnace or from the raising of the register; the draught in that case increases in a greater ratio than in the supposition of $G$ and $C$ constant, only there is at the same time a certain increase in the value of C , resulting from the increased velocity. In like manner, the value of $t$ could decrease only by the increase of resistance in the grating, or by the lowering of the register, and the diminution of draught would then be more rapid than if the resistance did not vary.

34 . Let us now see what would happen if the circuit were interrupted by a diaphragm. This is a circumstance that occurs in every heating apparatus, for all are provided with registers for the purpose of regulating the draught or of stopping it altogether during the interruptions of work. We will first suppose that the diaphragm is placed in that portion of the air-channel which conducts the external air beneath the grating. Denoting the orifice in the diaphragm by $d$ its diameter, we shall have

$$
\begin{gathered}
\mathrm{P}-p=(\mathrm{G}+\mathrm{C}) p+\frac{\mathrm{K} \mathrm{H}}{\mathrm{D}}(1+a t)^{2} p+\left(\frac{\mathrm{D}^{4}}{d^{4}}-1\right) p ; \\
\text { whence } v=\sqrt{\frac{2 g \mathrm{H} a t}{1+a t}} \sqrt{\frac{1}{1+\mathrm{G}+\mathrm{C}+\frac{\mathrm{K} \mathrm{H}}{\mathrm{D}} \frac{1}{(1+a t)^{2}+\frac{\mathrm{D}^{4}}{d^{4}}-1}}}
\end{gathered}
$$

neglecting the loss at the entrance into and the gain at the issue from the orifice, which are both very small relatively to the denominator of the second radical in the value of $v$.

It follows from this formula that the influence of the diaphragm becomes smaller as the total resistance of the circuit becomes greater. In large steam generators, the actual rate of ingress is less than a fifth of the theoretical rate. Supposing it equal to this fraction, we shall have nearly

$$
v=\sqrt{\frac{2 g \mathrm{H} a t}{1+a t}} \sqrt{\frac{1}{25+\frac{\mathrm{D}}{d^{4}}-1}} .
$$

If we suppose that the ratio of the sections of the chimney and of the orifice $\frac{\mathrm{D}^{2}}{d^{2}}$ becomes successively, $2,4,6,8,10,20,40,60,80,100$; the values of the second radical will be $0 \cdot 189,0 \cdot 158$, $0 \cdot 130,0 \cdot 1066,0 \cdot 09,0 \cdot 048,0 \cdot 025,0 \cdot 0166,0 \cdot 0125,0 \cdot 01$. The value of the radical without the diaphragm is $0 \cdot 20$, and by the influence of the diaphragm it is reduced in the proportion, $0 \cdot 945$, $0 \cdot 75,0 \cdot 64,0 \cdot 53,0 \cdot 45,0 \cdot 24,0 \cdot 12,0 \cdot 083,0 \cdot 062,0 \cdot 059$. Thus, as it was easy to see by inspection from the formula, diaphragms diminish the draught in a proportion much smaller than the ratio of the sections; a diaphragm which reduces the section to a tenth reduces the draught by only the half. This small influence is due to the retardation of the velocity by the diaphragms, and, consequently, the diminution of resistance in the rest of the circuit.
35. The diaphragm reducing the rate of flow in a proportion less than the ratio of its section to that of the chimney, it follows that the velocity of the air in the orifice of the diaphragm must increase in proportion as its surface decreases. In the general case, the velocity of the air in the diaphragm is :

$$
v=\sqrt{\frac{\overline{2 g \mathrm{H} \bar{a} t}}{1+a t} \cdot \frac{\mathrm{D}^{2}}{d^{2}} \cdot \sqrt{1+\mathrm{G}+\mathrm{C}+\frac{\mathrm{KH}}{\mathrm{D}}(1+a t)^{2}+\frac{\mathrm{D}^{4}}{d^{4}}-1}} ;
$$

and in the particular case which we have examined, we have

$$
v^{\prime}=\sqrt{\frac{2 g \mathrm{Hat}}{1+a t}} \sqrt{\frac{\frac{\mathrm{D}^{4}}{d^{4}}}{25+\frac{\mathrm{D}^{4}}{d^{4}}-1}}
$$

Taking as the ratio of the sections $\frac{\mathrm{D}^{2}}{d^{2}}$ the same numbers as before, we find as the value of the second radical, $0.378,0.632,0.77,0.85,0.90,0.97,0.992,0.996,0.998,0.999$ and as the value
of this radical without the diaphragm is 0.20 , the velocities in the diaphragm with respect to the velocity in the circuit are, $1 \cdot 89,3 \cdot 16,3 \cdot 87,4 \cdot 26,4 \cdot 49,4 \cdot 85,4 \cdot 96,4 \cdot 98,4 \cdot 99,4 \cdot 99$.

Thus this ratio approaches 5 as the diameter decreases, because the actual velocity without the diaphragm is $0 \cdot 2=\frac{1}{5}$ of the velocity due to the force, and, the diaphragm retarding the velocity in the chimney, diminishes the friction. It is evident that if the velocity of the current of air were a fraction $\frac{1}{m}$ of the velocity V due to the force, the ratio in question would continually approach $m$ as the diameter of the diaphragm diminished.
36. If there were in that portion of the air-channel which conducts the air to the ash-pit several diaphragms at a sufficient distance apart to allow the vein of air, after having traversed each of them, to fill the space in which they are placed before meeting with the next, the velocity would become

$$
\left.v=\sqrt{\frac{2 g \mathrm{H} a t}{1+a t}} \sqrt{\left.1+\mathrm{G}+\mathrm{C}+\frac{\mathrm{KH}}{\mathrm{D}}+a t\right)^{2}+m\left(\frac{\mathrm{D}^{4}}{d^{4}}-1\right.}\right),
$$

and if $\frac{\mathrm{D}^{4}}{d^{4}}$ were very great relatively to the terms which precede, for the same value of $\frac{\mathrm{D}^{2}}{d^{2}}$ the value of $v$ would vary in an inverse proportion to the square root of $m$.
37. We have supposed that the diaphragms were placed in the current of cold air before it reached the grating of the furnace; let us now suppose them placed in that portion of the circuit traversed by the heated air. As the velocities are inversely as the densities, we shall have in general

$$
\begin{gathered}
\mathrm{P}-p=(\mathrm{G}+\mathrm{C}) p+\frac{\mathrm{KH}}{\mathrm{D}}(1+a t)^{2} p+m\left(\frac{\mathrm{D}^{4}}{d^{4}}-1\right)(1+a t)^{2} p \\
\text { whence } v=\sqrt{\frac{2 g \mathrm{H} a t}{1+a t}} \sqrt{\frac{1}{1+\mathrm{G}+\mathrm{C}+\frac{\mathrm{KH}}{\mathbf{D}}(1+a t)^{2}+m\left(\frac{\mathrm{D}^{4}}{d^{4}}-1\right)(1+a t)^{2}}} .
\end{gathered}
$$

Thus diaphragms placed in the current of heated air reduce the draught more than when they are placed before the ash-pit, because the velocity of the heated air is greater than that of the cold air.
38. Experiments made by M. Combes confirm the consequences which we have drawn from the general formula. It was found that by closing the orifice for the ingress of the air into the ash-pit of a generator, by means of a sheet of iron pierced with holes, the velocity of the air traversing the aperture, measured by an anemometer, continued to increase with the decrease of the size of the holes. The following are the details of this experiment.

The chimney was 20 metres in height; its section at the bottom being 0.383 square mètre, and at the top 0.196 square mètre. The surface of the furnace-grating was 0.6525 , and the sum of the spaces between the bars $0 \cdot 163$ square metre. The mouth of the ash-pit was covered with a sheet
 and altogether a surface equal to the whole open surface between the bars.

The velocity was first ascertained by means of an anemometer with all the holes open, immediately after the fire had been made up, and again after it had been stirred. Two experiments made in succession gave as the velocities $0^{\mathrm{m}} \cdot 5275,1^{\mathrm{m}} \cdot 68,0^{\mathrm{m}} \cdot 80,1^{\mathrm{m}} \cdot 15$; mean, $1^{\mathrm{m} \cdot 17 \text {. With }}$ two of the holes closed, under the same circumstances, the velocities were $1 \cdot 607,1 \cdot 99,1 \cdot 89,2 \cdot 05$; mean, $1^{\mathrm{m}}$-88.

When four of the holes were closed, the velocities under the same circumstances were 3.51 , $4 \cdot 05,2 \cdot 68,3 \cdot 46$; mean $3^{\mathrm{m} \cdot 42}$.

In these three series of experiments, the surfaces for the ingress of the air were $0.168 \times 0.65=$ $0 \cdot 1092 ; 0 \cdot 0728$ and $0 \cdot 0364$ square metre; and the mean velocities $1 \cdot 17,1 \cdot 88$, and $3 \mathrm{~m} \cdot 42$. Thus the velocities increase rapidly with the decrease of the sections of the orifices; for orifices in the proportion of 3,2 , and 1 , the velocities increased $1 \cdot 26,1 \cdot 65,3$.
39. Lateral Pressure in Chimneys.-Suppose a chimney placed beyond a furnace of any form, and an opening made in it at a certain height; it is evident that the external air will enter with a velocity greater in proportion to the nearness of the aperture to the ground. At the top, this velocity is nul. This rush of external air would occur if the aperture were in any part of the heating apparatus. Therefore care should always be taken in building to avoid an ingress of external air through fissures in the masonry, because they always occasion a loss of draught and often diminish the useful effect of the fuel.
40. It follows necessarily from what we have said, that inside the furnace and chimney there is a negative pressure, that is, smaller than that of the atmosphere, and that this negative pressure continues to increase from the ash-pit to the bottom of the chimney, and decreases in like manner to the top of the chimney where it is nul.
41. Effects produced by the Meeting of Currents.-When several pipes enter the same channel the currents of air project themselves beyond the orifices. and in certain circumstances they may by their mutual action, modify the velocities of the air in the pipes. If, for example, two pipes entered a third exactly opposite each other, the third pipe being perpendicular, the influence of the currents would be nul if they had the same velocity; for the effect would be the same as if the currents flowed a arainst a fixed plane placed between them. But if the velocities were unequal, the current having the greater velocity would diminish that of the other, and close more or less, the orifice through which it flowed. A vast number of phenomena leave no doubt on this fact;

## CHIMNEY.

besides, these currents must act upon each other in nearly the same manner as currents of water, and it is known, from the experiments of Savart, that when two currents of the same section act in contrary directions and one of them has a very small excess of velocity over the other, the latter is driven back to the mouth of the vase, and the flow ceases completely. The effects resulting from this collision may be prevented by placing in the pipe a diaphragm P, as shown in Fig. 1909.
42. Phenomena of the same kind would occur if the two pipes were at right angles to each other, Fig. 1910, with the additional complication caused by the effect of the lateral pressure. But these effects may be wholly avoided by means of the diaphragm $P$.


If the pipe were enlarged, as in Fig. 1911, the narrowing of the pipe in front of mouth of the lateral pipe would have the effect of a diaphragm; the influence of the collision of the currents might be neglected, but that of the diminution of pressure, due to the enlargement of the pipe, might be very great.
43. In the case of a current of heated air flowing horizontally into a vertical chimney, it may happen that the draught is completely stopped, though the section of the chimney is larger than that of the current of heated air, when the velocity of the latter is very great, because in that case it will close the chimney like a plug. This is a fact which Péclet has had occasion to remark several times, and especially in the case of a chimney which he had built in a soda manufactory, and which formed a portion of a condensing apparatus. This chimney was $13^{\mathrm{m}} \cdot 30$ in height, and had a section of about 0.75 square mètre; the smoke-channel entered it horizontally, Fig. 1912. When the fire in the furnace was kindled, the draught existed: it increased for some time, and then gradually decreased, till finally it ceased. Péclet soon discovered the cause of this singular phenomenon, and removed it by a vertical partition A placed in the chimney so that the heated air did not come in contact with the gases of the chimney until it had taken the same upward direction.

Thus, it is necessary to take the greatest care, in all draught chimneys which receive currents of air perpendicularly or variously inclined, to direct them according to the axis of the chimney before they come in contact.
44. Loss of Heat occasioned by Chimneys.-The
 quantity of heat lost by chimneys is very considerable, because the burned air is allowed to escape at very high temperatures, always higher than that of the heated body. This quantity varies with the temperature of ihe air which escapes, with the volume of cold air entering the furnace, and with the weight
of water contained by the fuel, or produced by its combustion, for the gases are almost alvays evolved at too high a temperature to allow the condensation of the steam. This loss may be computed approximatively, however, by comparing the temperature of the burued air in the chimney with that which it would have if all the heat produced were employed to heat it. In this calculation it may be assumed that the burned gases possess the calorific capacity of air.

For example, in steam generators the burned air is discharged at about $300^{\circ}$, and on an average 18 cubic mètres of air is employed per kilogramme of coal, or $18 \times 1 \cdot 3=23^{\mathrm{k}} \cdot 4$; taking $0 \cdot 24$ as the calorific capacity of the gases produced, the temperature to which the gases would be raised by the 8000 calories or units of heat, resulting from combustion, would be equal to 8000 $\div(23 \cdot 4 \times 0.24)=1425^{\circ}$; consequently, the loss is equal to $300 \div 1425=0.21$. This loss would evidently be doubled or trebled if twice or three times the quantity of air were employed, and it would be reduced to $0 \cdot 1$ if only that volume of air were employed which is rigorously necessary to combustion. Under ordinary circumstances, the loss must be reckoned at not less than $0 \cdot 25$, on account of the steam produced.

In furnaces for melting metals and for the production of coal-gas, the burned air is allowed to escape at a much higher temperature, on account of the high temperature of the substances heated, and the loss of heat is much greater. The loss often amounts to 0.80 or $0 \cdot 90$, even when the volume of air which escapes combustion is very small; but the heat thus lost may, in general, be utilized, at least in a great measure. The utilization of this lost heat is a question of the highest importance in factories, for the cost of fuel is nearly always a considerable item in the cost of the productions. We will return to this subject later.
45. Arrangements for utilizing the whole of the Hcat produced by a Furnace.-In the apparatus generally employed, the chimney is placed beyond the furnace, which causes, as we have seen, a loss of heat that may be avoided by certain arrangements.

Let us suppose that above or beyond a furnace, well enclosed, there is a chimney of 3 or 4 metres in height, of considerable thickness, and constructed of badly-conducting material. The burned air in this chimney will have a very high temperature, and will acquire an ascending velocity much greater than that necessary to support combustion. Suppose, again, that above this chimney, in its continuation or at the side of it, is the boiler or body to be heated: the course of the smoke may be lengthened, so as to cool it completely or almost completely; and on leaving the heating surfaces it may be allowed to escape immediately into the air, or conducted to a chimney, the use of which is merely to discharge it at a convenient height in the atmosphere.

A similar arrangement is adopted in certain kinds of glass-works; the height of the draught chimney is equal to the distance from the furnace-grating to the roof of the furnace, and the burned air may give up all its heat in the arch if the latter is of sufficient length.
46. At first sight, this method seems applicable only when the bodies to be heated are to be raised to a temperature exceeding but little the ordinary temperature, since the burned air cannot be ejected at a temperature lower than that of these bodies. But by moving the bodies to be heated in a direction contrary to the motion of the heated air, it is evident that, in almost all cases, nearly the whole of the heat of this air may be utilized, and the temperature reduced to that of the atmosphere.
47. This method, which consists in placing the chimney in front of the heating surface, has a disadvantage which we ought to point out. The heating surface must be larger than in the ordinary arrangement, because it is not heated directly by the radiation of the fuel ; notwithstanding this disadvantage, however, there are many cases, as we shall see later, in which this method may be adopted with profit.
48. The apparatus may be arranged so as to produce the draught through the heating surface. The arrangement consists in placing the body to be heated in the chimney. If the chaunels of circulation are narrow, and the heating surface large, the burned air will reach the top of the chimney at a temperature but little above that of the heated body, although the draught be very strong, because the ascending velocity will depend on the mean temperature of the burned air in the channels of circulation, and the temperature at the bottom will be that of the furnace. This arrangement is seen in lime-kilus.
49. Dimensions of Factory Chimneys.-The work of a chimney consists, as we have said, in drawing into the furnace the volume of air necessary to combustion. The weight of fuel to be burned an hour is always given; the height of the chimney is, in general, determined by particular conditions, but the section depends on the volume of air employed in consuming each kilogramme of fuel, on the mean temperature which the air will have in the chimney, on the loss of force occasioned by friction, on the changes of section and direction, and on the resistance of the furnace-grating. The phenomena occurring in the draught of a heating apparatus are so complicated, that we cannot hope, by means of simple theoretical considerations, to calculate exactly and for every case the section which should be given to a chimney to produce a given effect. These calculations would be rendered more inexact by the facts that we do not know beforehand the temperature of the air in the chimney, the mean temperature of the air which circulates around the body to be heated, nor the resistance of the grating, and that these three elements, which it would be necessary to know before calculating anything, vary with the condition of the fuel in the furnace, and with its thickness upon the grating. Thus, in every case, we must consider the results of experience to ascertain what section should be given to a chimney; but it is important to remark that there will always be an advantage in giving to chimneys an excess of section, because this produces an excess of draught which may be necessary in certain cases, and which may always be regulated by means of a register.
50. For fixed steam generators, arranged in the usual way, Péclet discovered, by bringing together a large amount of information, and by some experiments, that for chimneys having 10,20 , and 30 mètres, containing air at $300^{\circ}$, witn gratings, the open spaces of which are equal to the section of the chimney, and upon which is burned 1 kilogramme of coal an hour per square deci-
mètre, circumstances of ordinary occurrence, the rate of ingress of the cold air is about $0.18 v$, $0 \cdot 17 v, 0 \cdot 16 v, v$ being the theoretical rate.

For the heights of 10,20 , and 30 mètres, which we have supposed, and for a mean temperature of $300^{\circ}$ in the chimney, the rates of ingress of the cold air, deduced from the formula $\mathrm{V}=\sqrt{\frac{2 g \mathrm{H} a t}{1+a t}}$, are $10.13,14.33,17.55$ mètres, and, consequently, the actual rates are $0.18 \times 10 \cdot 13=1 \cdot 82,0.17 \times 14 \cdot 33=2 \cdot 44,0 \cdot 16 \times 17 \cdot 55=2 \cdot 80$. The volumes of air drawn an hour per square decimètre of section being $v \times 0 \cdot 01 \times 3600$, will be for the three heights, $65 \cdot 52$, $87 \cdot 84,100 \cdot 8$ cubic mètres; and assuming that only the half of the air is transformed into carbonio acid, and, consequently, that the volume of air required to burn 1 kilogramme of coal is equal to 18 cubic mètres, the weight of coal an hour per square decimètre of section will be represented by the preceding numbers divided by 18, that is equal to $3 \cdot 42,4 \cdot 71,5 \cdot 50$ kilogrammes. These numbers differ but little from those adopted by the most experienced engineers; by employing them we may be sure of having a considerable excess of draught, but no disadvantage can result from this, as we have shown above.

The proportion of the section of the chimney to the consumption of fuel'necessarily supposes that the resistance remains constant, an hypothesis that may be admitted; for the resistance caused by the grating and by changes in the direction of the current, undergoes but little variation, and that occasioned by friction has, in general, but little influence on the total resistance.
51. According to the foregoing, denoting the sum of the resistances experienced by the air in its course by $R$, we have in the three cases considered,

$$
v=0.18 \mathrm{~V}=\mathrm{V} \sqrt{\frac{1}{1+\mathrm{R}}} ; v=0.17 \mathrm{~V}=\mathrm{V} \sqrt{\frac{1}{1+\mathrm{R}}} ; v=0.16 \mathrm{~V}=\mathrm{V} \sqrt{\frac{1}{1+\mathrm{R}}}
$$

and the values of R are $29 \cdot 98,33 \cdot 49,38 \cdot 29$. These numbers represent the sum of the resistances due to friction, to changes of direction, and to the grating.

Comparing the dimensions of a large number of generators, we find that, for the three heights of chimney given above, the values of $\frac{\mathrm{KL}}{\mathrm{D}}$ are equal to $1 \cdot 5,2 \cdot 37,3 \cdot 57$ : as there are usually eight changes of direction at right angles, assuming that these changes take place in a continuous manner, the corresponding losses would be represented by 4; and considering the velocity in the chimney as double that of ingress, which is not far from the truth, the sum of these two kinds of resistances would be $5 \cdot 5 \times 4,6 \cdot 37 \times 4,7 \cdot 57 \times 4$, or $22 \cdot 0,25 \cdot 48,30 \cdot 28$; the resistance of the furnace would then be represented by 8 . But these calculations must be considered as only rough approximations, sufficient to give an idea of the value of the different kinds of resistance which take place.
52. Sections of the Chimncys of Generators for different kinds of Fuel.-Let us consider two generators having the same form and dimensions, the furnaces of which are supplied with different kinds of fuel, containing only carbon and fixed matter. It is evident that, if for each of them the same portion of air escaped combustion, the phenomena would be the same; the only difference would be that resulting from the unequal quantities of radiated heat, and from the unequal resistance of the furnace; but these differences would be of small importance, and could occasion but little in the weight of carbon burnt a square decimetre of chimney. But if one of the kinds contain water or produce it, the sections of the chimneys, for the same weight of fuel, will no longer be the same. In this case, we may admit, as an approximation sufficient for practice, that the sections of the chimneys are proportional to the volumes of the gases to which they are to give a passage for the combustion of the same weight of the different kinds of fuel. If we denote by S the section of chimney necessary to consume a weight P of coal, by $\mathrm{S}^{\prime}$ the section of chimney corresponding to the combustion of a weight $P$ of another kind of fuel, by $V$ and $V^{\prime}$ the volumes of the gases which are evolved by the combustion of a kilogramme of the two kinds, we shall have $\mathrm{S}^{\prime}=\frac{\mathrm{S} \times \mathrm{V}^{\prime}}{\mathrm{V}}$; from this we obtain the following results :

$$
\begin{aligned}
& \text { Dry wood .. .. .. .. .. .. } \mathrm{S}^{\prime}=\mathrm{S} \times \frac{10 \cdot 08}{17 \cdot 28}=0.59 \times \mathrm{S} \text {. } \\
& \text { Wood with } 0.20 \text { of water } \quad . \quad \text {.. } \quad . \quad \mathrm{S}^{\prime}=\mathrm{S} \times \frac{7 \cdot 42}{17 \cdot 28}=0.43 \times \mathrm{S} \text {. } \\
& \text { Dry turf with } 0.05 \text { of ashes } \quad . \quad \quad . \quad \mathrm{S}^{\prime}=\mathrm{S} \times \frac{12.01}{17.28}=0.69 \times \mathrm{S} \text {. } \\
& \text { Turf with } 0.20 \text { of water } \quad . \quad \quad . \quad \quad . . \quad S^{\prime}=S \times \frac{8.78}{17.28}=0.51 \times S \text {. } \\
& \text { Charcoal .. .. .. .. .. .. } \mathrm{S}^{\prime}=\mathrm{S} \times \frac{15 \cdot 28}{17 \cdot 28}=0.88 \times \text { S. } \\
& \text { Coke with } 0.02 \text { of ashes } \quad . \quad \quad . \quad \quad . . \quad S^{\prime}=\mathrm{S} \times \frac{17 \cdot 40}{17 \cdot 28}=1 \cdot 00 \times \mathrm{S} \text {. } \\
& \text { Coke with } 0.15 \text { of ashes } \quad . \quad \text {.. } \quad . \quad \mathrm{S}^{\prime}=\mathrm{S} \times \frac{15 \cdot 10}{17 \cdot 28}=0.87 \times \mathrm{S} \text {. }
\end{aligned}
$$

These numbers necessarily suppose that the resistance of the gratings is the same for all kinds of fuel; this is not strictly true, for all kinds of fuel do not encrust the bars so much as most
kinds of coal; but, as we shall see later, smaller surfaces of grating are employed for coke, wood, charcoal, and turf, so that the resistance of the gratings ought not to differ much. They suppose also that the other resistances do not change with the section, for it is on this condition that the volumes of air are proportional to the sections; but when the forms of the generators are alike, there is the same number of changes of direction, and the differences of section have only a small influence on the total resistance. But these numbers must be considered as approximative values, representing, however, sections producing an excess of draught.
53. It is important to examine what would happen if the chimney only or the whole of the circuit had a section greater than that resulting from the preceding considerations.

Suppose, in the first place, that a larger section is given to the chimney alone; the draught will increase on account of the expansion which the heated air will undergo on entering it, and on account of the diminution of friction. But in general this increase of draught will be small, and, if the chimney is too large, the velocity of the heated air may be diminished to such a degree, that the draught will be modified by the influence of the wind. It might even happen, if the section of the chimney were much too large, that the current of heated air did not completely fill it, and in that case downward currents of air would be caused which would greatly diminish the draught. To remedy this, it would be necessary to reduce the section of the chimney by a register placed in the upper portion.

If, on the contrary, the section of the chimney were diminished so as to make it much smaller than that of the gatherings, there would be a loss of force which could be compensated only by an increase of temperature of the heated air. Even this compensation could not be effected without difficulty and would be confined within narrow limits.
54. If the gatherings, the furnace-grating, and the chimney had a section much larger than those we have indicated, it is evident that, for a constant consumption of fuel obtained by closing the register in a greater or less degree, the resistance would diminish in proportion as the section increased, and finally we should obtain as the rates of ingress and egress of the gases the theoretical rates. To effect this, it would not be necessary to make the diameter of the air-channel very large; for if it were only five times the diameter calculated, the velocities would be twentyfive times smaller, and every kind of resistance $25 \times 25=625$ times less. This would be, there fore, a certain means of suppressing every kind of resistance; but in that case, to protect the draught from the influence of the wind, the register would have to be placed at the top of the chmmey. This arrangement would largely increase the cost of construction and the loss of heat; for this reason it is never adopted.
55. Various Methods that have been proposed to determine the Section of Chimneys.-Montgolfier was the first who endeavoured to determine the section of a chimney, taking as starting-points its height, the volume of air necessary to combustion, and the temperature of the burned air. But he did not consider either the friction of the air against the sides or the resistance of the grating. The section thus determined was much too small.

Clement, in his lectures at the Conservatoire, gave Montgolfier's method; but he took only the fifth of the velocity calculated, which gave too large a section. Tredgold, in his 'Treatise on the Steam Engine,' gives a complicated method founded upon singular suppositions. He starts from the theoretical velocity, on the supposition that for boilers the temperature of the smoke is equal to that of the steam, and he multiplies the velocity obtained by 0.65 , which represents the flow of air through orifices in a thin material.

According to M. Darcet, chimneys should be 10 mètres in height, and have a section such that each square decimètre may correspond to a consumption of 3 to $3 \cdot 3$ kilogrammes of coal an hour; the surface of the furnace-grating should be three times greater than the section of the chimney. These results differ but little from those we have indicated.
56. Influence of the cooling of the outer surface of Chimneys upon the Draught.-It might be thought that in isolated chimneys, traversed by air at a temperature of about $300^{\circ}$, the quantity of heat emitted by the surface being considerable, the air undergoes a great lowering of temperature, which must diminish the draught; but this is not the case. The quantity of heat carried away by the heated air is always very great relatively to that lost through the surface of the chimney, and the cooling of the air has no appreciable effect. Let us, by way of example, consider a chumney of 20 mètres in height, $0^{\mathrm{m}} \cdot 5$ in diameter, and 0.196 square mètre in section, containing air at $300^{\circ}$. The theoretical rate of ingress of the cold air through a channel having the section of the chimney will be $11^{\mathrm{m}} \cdot 93$, the actual rate $11 \cdot 93 \times 0 \cdot 165=1 \mathrm{~m} \cdot 968$, the volume of air entering an hour, $1 \cdot 968 \times 3600=7084$ cubic mètres, the weight of which is $7084 \times 1^{\mathrm{k}} \cdot 3=9209$ kilogrammes and the quantity of heat carried away an hour about $9209 \times 300 \div 4=553175$ units of heat Now, according to the formulæ of the cooling, which we shall see later, the quantity of heat emitted an hour, a lineal mètre, under these conditions, is 1587 ; for the 20 mètres, therefore, the loss will be $1587 \times 20=31740$. The ratio of 31740 to 553175 is $0 \cdot 057$. Thus the loss of heat through the chimney is not $\frac{1}{600}$ of the heat carried away by the air; consequently, the temperature of the air will not be lowered by 0.06 -that is, it will remain above $282^{\circ}$, and the draught will not be affected in an appreciable degree.
57. For an iron chimney of the same dimensions, the quantity of heat emitted would be, accordiun to the formulæ, 8037 a lineal metre, and 160740 for the 20 metres. The ratio of the heat lust to the heat carried away by the air would then be $160740 \div 553175=0 \cdot 29$. Thus, the temperature would be reduced by $87^{\circ}$, and the heated air would escape at about $213^{\circ}$, which corresponds (10) to a diminution of $0 \cdot 1$, nearly, in the draught.
58. Calculation of ihe Diameter of Chimneys in the general case.-All that we have said respecting the section of chimneys, designed to produce the combustion of a given weight of fuel, is applicable ouly to chimneys of fixed generators in ordinary conditions, when the temperature of the air is about $3 \cap 0^{\circ}$, and when about 1 kiiogramme of coal is burned an hour and on a square decimetre of surface of grating. But, if the circumstances were different, the sections recommended would not
be suitable. As it is of great importance to be able to calculate them, at least within a certain degree of approximation, in every case that may occur, Péclet has endeavoured to find a simple method of accomplishing it.
59. We will suppose that the consumption of coal a square decimètre of grating is always about 1 kilogramme an hour. We have seen (51) that, for generators having chimneys of 10, 20, and 30 mitres in height, the resistance is about equal to 8.

Supposing the section of the air-channel constant and square, and denoting its length by L, the side by D , the number of changes in direction at right angles by N , we shall have as the rate of ingress of the cold air,

$$
\begin{equation*}
v^{2}=\frac{2 g \mathrm{H} a t}{1+a t} \cdot \frac{1}{1+8+\left(\frac{\mathrm{KL}}{\mathrm{D}}+\mathrm{N}\right)(1+a t)^{2}} \tag{1}
\end{equation*}
$$

But denoting by V the volume of cold air required a second, which volume may be easily deduced from the weight and nature of the fuel to be consumed in the same time, and by $S$ the section of the conduit, we shall have

$$
\begin{equation*}
\mathrm{V}=\mathrm{S} v ; \text { and } v^{2}=\frac{\mathrm{V}^{2}}{\mathrm{D}^{4}} \tag{2}
\end{equation*}
$$

Equating the values of $v^{2}$ in the equations [1] and [2],

$$
\mathrm{V}^{2}=\frac{2 g \mathrm{H} a t}{1+a t} \cdot \frac{\mathrm{D}^{4}}{9 \mathrm{D}+(\mathrm{KL}+\mathrm{ND})(1+a t)^{2}}
$$

if $t$ were known, putting in the place of $\mathrm{V}, g, \mathrm{H}, a, \mathrm{~N}$, their values, the last equation would assume the form

$$
\begin{equation*}
\mathrm{D}^{5}=\mathrm{A}+\mathrm{BD} \tag{3}
\end{equation*}
$$

an equation that may be solved approximatively, by neglecting at first A or B , substituting the value of D , thus found, for D in the second member of equation [3], and successively for D the last value obtained, until two consecutive values differ only by a quanvity smaller than the approximation required.

If the temperature $t$ were not known, we should have to assign to it a certain value found by experiments on similar apparatus. We will remark here that great accuracy in the value of $t$ is of no importance; for, as we have seen (10), the draught varies only in the ratio of 57 to 71 , for $t=159^{\circ}$ and $t=300^{\circ}$.
60. As an example of these calculations, suppose it be required to burn 50 kilogrammes of coal an hour; let $\mathrm{L}=40$ mètres, $t 150^{\circ}$, and let there be ten changes of direction at right. angles, we shall have $\mathrm{V}=50 \times 18 \div 3600=0 \cdot 25 ; 2 g \mathrm{H} a t=161 \cdot 40 ; 1+a t=1 \cdot 55 ; \mathrm{KL}=0 \cdot 96$; and the formula [3] becomes $\mathrm{D}^{5}=0 \cdot 00057+0.0114 \mathrm{D}$. Neglecting, at first, the first term, and operating by successive approximations, we find for D the values $0 \cdot 325,0 \cdot 335,0 \cdot 337,0 \cdot 338,0 \cdot 338$. Thus the second substitution gives the value of D within a centimètre. The section being about 13 square decimètres, the consumption of fuel the square decimètre would be about 3 kilogrammes.
61. We have supposed that the resistance of the furnace-grating was constant and equal to 8 ; this will be the case when the consumption of coal the square decimetre and by the hour, is 1 kilogramme. If this consumption were less, the velocity of the air traversing the grating would be smaller, as well as the resistance, and consequently the section of the chimney calculated in the manner described would be too large. In the contrary case, it would of course be too small. In every case, it may be admitted that the resistance of the grating is proportional to the square of the velocity of the air traversing it; thus, denoting by $n$ the number of kilogrammes of coal burned an hour the square decimètre, the resistance would be equal to $8 n^{2}$.
62. The results of these calculations must be considered as only approximative, because they rest upon hypotheses relative to the resistance of the grating and to the temperature of the air in the chimney, and because the numbers admitted may therefore be considerably from the truth.
63. The preceding remarks suppose not only that the air-channel retains its section throughout, but that it is not divided into several branches traversed simultaneously by the gases from the furnace. For if it were so, the sum of the resistances in the partial channels would be greater than that which the air would meet with in a single channel having the same section. When the air traverses simultaneously a large number of equal pipes, as in locomotive engines, the resistance is represented by $\mathrm{K} l \mathrm{~S}^{2} \div d \mathrm{~S}_{1}^{2}$, or by $\mathrm{K} l \mathrm{D}^{4} \div n^{2} d^{5}$, S denoting the section of the whole channel, $\mathrm{S}_{1}$ that of the tubes, D the diameter of the channel, $d$ that of the tubes, and $n$ the number of the tubes.
64. If in a long circuit, from a generator or any other apparatus, there happened to be a number of tubes, we could not, without modification, employ the method indicated (59), to calculate the diameter of the chimney, because this method supposes the channel single throughout its length and constant in section. It might, however, be used tentatively. In fact, the resistance of a number of tubes is equivalent to that of a single pipe of a diameter D , the L of which is given by the equation

$$
\begin{equation*}
\frac{\mathrm{K} l \mathrm{D}^{4}}{n^{2} d^{5}}=\frac{\mathrm{K} \mathrm{~L}}{\mathrm{D}} ; \text { whence } \mathrm{L}=\frac{l \mathrm{D}^{5}}{n^{2} d^{5}} \tag{a}
\end{equation*}
$$

Taking first a certain approximative value of $D$, we deduce from it the value of $L$ by means of the equation $\lceil a]$; the equation [3] (53) will give a new value of $D$, which will enable us to find a new value of L , and so on, till a constant value of L is found, and consequently of D . Besides, as in general we need only a rough approximation, and as in generators with tubes the course of the smoke is always very short, the increase of resistance in the tubes is nearly compensated by the
diminution of length of circuit, and by the increase of the sum of the sections of the tubes, so that we may take as the section of the chimney that indicated in (59).
65. Chimneys common to several Furnaces.-In most large factories there is only one chimney for all the furnaces. Two advantages are derived from this arrangement-1, a saving in the cost of construction; 2, uniformity of draught, which does not exist in a chimney communicating with only one furnace. The saving is obvious, for one chimney costs less to build than several, supposing the section of the single chimney equal to the sum of the sections of the others. With regard to the second advantage, it must be remarked that, in a furnace having a special chimney, the draught is very variable; it is small immediately after the fire is made up, and increases in proportion as combustion becomes more active. It is diminished in a high degree by opening the furnace-doors. But if several furnaces communicate with a common chimney, and care be taken to make up the fires successively, a mean draught will exist in the chimney, which will increase in regularity with the number of the furnaces.

A common chimney has usually a section equal to the sum of the sections of the partial chimneys that would correspond with each furnace. The section thus obtained is certainly too great, because the resistance is much smaller than the sum of the resistances in the partial chimneys which it replaces. But this excess of draught, which may be modified at will, offers no inconvenience.
66. Chimneys in which the Smoke, or the Gases soluble in Water, are precipitated or absorbed by Injections of Water.-It has been proposed to precipitate the smoke and to dissolve the soluble gases, which, in certain circumstances, are found mingled with the gases from the furnace, by dropping water under the form of very fine rain into horizontal pipes traversed by the gases before reaching the chimney; but the gases being almost completely cooled, it became necessary to reheat them to produce the draught.

Hedley, an ironmaster at Newcastle-on-Tyne, invented the plan of making the smoke pass through a series of vertical pipes, ascending and descending, and into each of the pipes in which the smoke descended he dropped a small quantity of water. In spite of the cooling caused thereby, the draught was very strong, even in pipes of 3 or 4 metres in height. By this means, all the matters carried off by the gases may be stopped, and especially the soot which floats upon the heated air. This arrangement has been successfully applied to a locomotive of the Sunderland and Durham Railway. Although this mode of condensation has long been known, it has never been generally adopted, because it is complicated and necessitates a certain amount of labour to raise the water required for the purpose of condensation, and because this water, if not clean, would often be a source of embarrassment. But this arrangement would possess great advantages when in the gases which escape from the furnace there are, independently of those involved by combustion, noisome gases and vapours, such as those evolved in the manufacture of soda, or solid matters carried off by the current and which it is profitable to arrest, as in the case of lead or zinc works.
67. Draught Chimneys.-All chimneys are, strictly speaking, draught chimneys; but we denote in a more special manner by this name chimneys constructed for the purpose of rendering certain situations wholesome by ventilation. In these, the phenomena that happen are much more simple than in factory chimneys. Indeed, the air drawn through them, being usually only slightly heated, is not all found to pass through the furnace-grating, which occulies only a small portion of the section of the chimney. Sometimes even the furnace is placed laterally, so that the section of the chimney is not sensibly changed in the vicinity of the grating; the resistance of the latter may, in this case, be wholly neglected, and the results of calculation are then very accurate. When a grating is wholly covered with fuel, and the whole of the air drawn by the chimney is forced to pass through it, if there is no appreciable loss of heat by radiation, the temperature of the air beyond the grating is about $1200^{\circ}$, and the volume of air drawn per kilogramme of coal is 18 cubic mètres. For draught chimneys, on the contrary, the air is rarely heated above $20^{\circ}$; the heat which it absorbs the cubic metre is $1^{\mathrm{k}} \cdot 3 \times 20 \times 0 \cdot 25=6.5$, and, consequently, the volume of air drawn by 1 kilogramme of coal is $8000 \div 6 \cdot 5=1230$ cubic mètres.
68. To begin with the simplest case, let us consider a draught chimney of a height $H$ and a diameter $\mathbf{D}$, communicating in its lower portion with a cylindrical channel of a length $l$ and a diameter $d$ smaller than D ; denoting the temperatures of the air in the chimney and the air in the channel or pipe by $t$ and $\theta$, the negative pressure at the bottom of the chimney will be $\mathrm{H} a(t-\theta) \div 1 a t$; denoting it by P , and calling $p$ the force corresponding to the effective velocity, we shall have $\mathbf{P}-p=\frac{\mathrm{K} l}{d} p+\frac{\mathbf{K} \mathbf{H}}{\mathbf{D}} \cdot \frac{d^{4}}{\mathrm{D}^{4}} p+p+(\mathbf{A}-\mathbf{B}) p$. The first two terms of the second equation represent the resistance occasioned by friction in the pipe and in the chimney; the third, the loss of force due to the sudden change of direction; the last, the loss of force at the entrance of the pipe and the increase of force at the entrance of the chimney. But in general the conducting-pipes are long enough to render their resistance very great relatively to the value of $\mathbf{A}-\mathbf{B}$. Thus the second member of the equation may be reduced to the first three terms, which gives as the rate of ingress,

$$
\begin{equation*}
v=\sqrt{\frac{2 g \mathrm{H} a(t-\theta)}{1+a t}} \sqrt{\frac{1}{\frac{\mathrm{~K} l}{d}+\frac{\mathrm{KH}}{\mathrm{D}} \cdot \frac{d^{4}}{\mathrm{D}^{4}}+2}} . \tag{1}
\end{equation*}
$$

69. Suppose, for example, a square chimney communicating with a pipe having also a square section; taking $\mathrm{H}=30$ mètres, $\mathrm{D}=1$ mètre, $d=0^{\mathrm{m}} 5, l=500$ mètres, $\theta=15^{\circ}, t=35^{\circ}$, the formula becomes

$$
v=\sqrt{\frac{19 \cdot 62 \times 30 \times 0.00366 \times 20}{1+0.00366 \times 35}} \sqrt{\frac{1}{21+0.72+2}}=6 \cdot 18 \times 0.193=1^{\mathrm{m} \cdot 19}
$$

The section of the pipe being 0.25 square mitre, the volume of air drawn per second will be $1.19 \times 0.25=0.297$ cubic mètre; the expenditure of heat will be about $1069 \times 1^{\mathrm{k}} \cdot 3 \times 20 \times$ $0 \cdot 24=6670$ an hour, a quantity of heat only slightly less than that produced by the combustion of 1 kilogramme of coal.
70. Let us now consider the most general case. Suppose a chimney, still prismatic, commanicating with a pipe turning in all directions, the several parts of which are at different temperatures. The variations of temperature will act in two ways: 1st, by modifying the force which occasions the flow; 2nd, by causing variations of velocity, and, consequently, variations in the resistance due to friction. But, as the influence of the variations of temperature upon the velocity, and, consequently, upon the friction is very small, we may suppose that the velocity varies only inversely as the sections. Then, denoting the force and the total sum of the resistances by P and R , the velocity of flow will be given by the formula $v=\sqrt{\frac{2 g \mathrm{P}}{1 \times \mathrm{R}}}$.

The value of R may be found by the methods indicated when treating of the flow of compressed gases.
71. Construction of Chimneys.-As the height and the section of chimneys may be determined by the preceding considerations, we have now only to examine the nature of the materials, the thickness, and the general arrangements to be adopted.
72. Factory Chimneys.-Factory chimneys always stand alone; they are constructed of bricks or of iron, and with either of these materials they produce the same effects, because the cooling which the air undergoes in iron chimneys has no appreciable influence upon the draught (57).
73. The best form of section, from the point of view of diminishing the resistance, is that which, for a given surface, has the least perimeter or circumference. This is, of course, the circular form, and next the polygonal with a large number of sides. Iron chimneys have always the circular form. In brick chimneys, the section is circular, square, or octagonal. Round chimneys are generally preferred; they require, for an equal section, less material.
74. Let us now examine the form of the vertical section passing through the axis. When chimneys are of small height, they are made prismatic internally, and the walls have a greater thickness at the base than at the summit, Figs. 1913, 1914. But when they are of great height, they have always the form of the pyramid inside and outside, Figs. 1915, 1916, to increase the base on which they stand, while reducing, as much as possible, the cube of the masonry. It is impossible to calculate the interior and exterior diminution of section, because the calculation would depend upon too large a number of unknown elements. We shall confine ourselves to giving that which has been found sufficient in practice.
75. In large factory chimneys, the internal dimi-
 and the external diminution varies from 0.025 to $0 \cdot 035$. The thickness of the masonry at the top is from $0 \cdot 11$ to $0 \cdot 22$, the breadth or length of an ordinary brick. Thus, if we denote the interior diameter at the top of a chimney by $d$, its exterior diameter by $d^{\prime}$, and the interior and exterior diameters at the bottom of the chimney by $\mathbf{D}$ and $\mathbf{D}^{\prime}$, we shall have $d^{\prime}=d+0: 22$, or $d^{\prime}=d+0 \cdot 44, \mathrm{D}=d=2 \mathrm{H} m, \mathrm{D}^{\prime}=d^{\prime}+2 \mathrm{H} m^{\prime}, m$ being included between 0.012 and 0.018 , and $m^{\prime}$ between $0 \cdot 025$ and 0.035 .

Suppose, for example, a chimney of 20 mètres in height and 0.60 in interior diameter at the top; the interior diameter $D$ at the bottom will be, taking $m=0 \cdot 014,1^{\mathrm{m}} \cdot 16$; the exterior diameter $d^{\prime}$ at the top will be $0.60+0 \cdot 22=0 \cdot 82$; and the exterior diameter $\mathrm{D}^{\prime}$ at the bottom, taking $m=0 \cdot 03$, will be $2^{\text {n }} 02$.

The chimney of the tobacco manufactory at Paris, which is constructed to burn 700 kilogrammes of coal an hour, which corresponds to more than 150 horsepower, is 29 mètres in height; internally it is $1^{\mathrm{m}} \cdot 03$ in diameter at the top, $2^{\mathrm{m}} \cdot 15$ at the base; and externally, at the top, $1^{\mathrm{m} \cdot} \cdot 30$, and at the base $3^{\mathrm{m} \cdot 45}$. The diminution is, in this case, both externally and internally, too great.
76. If it were required to construct conical surfaces internally and externally, the execution of the work would be rather difficult, and we should be obliged to cut a large number of the bricks, which would entail considerable expense; besides, bricks do not resist so
 well when they are broken as when they are whole, because their external crust has much greater tenacity than their inner portions.

Conical or prismatic chimneys may be constructed by a series of cylinders or prisms, which would occasion sudden retreats, both on the inside and on the outside; but the arrangement shown
in Fig. 1916 is generally preferred. The chimney is conical or pyramidal on the outside, and the thickness of the masonry varies by sudden leaps on the inside; the retreats occur every 11 centi mètres, the breadth of a brick.
77. Brick chimneys are usually from 20 to 30 mètres in height; sometimes, but rarely, 40 mètres. M. Grouvelle mentions a chimney at Manchester that is 125 mètres in height, $7 \cdot 50$ in exterior diameter at the base, and $2 \cdot 70$ at the top; $4,000,000$ bricks were employed in its construction.
78. Brick chimneys are usually placed upon pris-
matic socles, pierced on opposite sides with two apertures; one is designed to receive the channel which conducts the smoke to the chimney; the other, which is usually closed up by a thin brick wall, serves to admit the workman when the chimney needs sweeping or repairs. To facilitate this labour, the chimney is furnished with horizontal bars placed $0^{\text {m }} \cdot 50$ apart, forming a kind of ladder.
79. It is of the highest importance that a chimney should stand upon a firm foundation, for the sinking often takes place unequally; and, when this is the case, the chimney either falls, or stands very insecurely. This is a point to which builders do not pay sufficient attention.
80. In chimneys intended to receive air at a very high temperature, as those of reverberatory furnaces, fire-bricks must be employed, and the interstices between them filled with brick clay. In chimneys which are intended to receive vapours at a temperature rarely exceeding $300^{\circ}$, as in the case of steamboilers, ordinary bricks may be used, bound together by mortar containing siliceous sand; it would be well, however, to employ fire-bricks for the casing of the inner portion at the bottom of the chimney. Plaster should never be used when the temperature of the heated air exceeds $100^{\circ}$, because at this temperature it begins to lose the hygrometric water which it contains, and, consequently, its tenacity.
81. Tall chimneys, when standing alone, may be built without exterior scaffolding, if their diameter is considerable; the workman in this case raises himself upon supports fixed on the inside of the chimney. A good workman accustomed to this kind of work, assisted by a boy, may, in this way, in about a fortnight, raise a pyramidal chimney of 13 or 14 mètres in height, 2 mètres and 1 mètre in exterior and interior diameter at the base, and 0.80 and 0.60 in exterior and interior diameter at the top.
82. Chimneys usually terminate in a portion having a greater diameter, and resembling the chapiter of a column; this portion of the chimney is designed only as ornament. The chapter is often of bricks like the rest of the chimney; sometimes it is of cut stone.
83. Brick chapiters, if they were not protected, would allow the rain to penetrate the masonry; to prevent this they are covered with plate iron, which extends over the outside and inside by 0.10 to 0.15 mètre.
84. Fig. 1917 represents a section of the arrangement adopted for large iron chimneys. The chimney itself is bolted to an iron socle or base, fixed upon a mass of masonry by four stout bolts passing through the masonry. When these chimneys are very high, they are tied externally to the ground or to some neighbouring buildings. To prevent the rusting of the metal, the
 chimney is covered with coal-tar, which will bear a high temperature without alteration.
85. Tall factory chimneys attract the lightning, both by their great height and by the great conducting power of the soot which covers their inner surface; it is, therefore, necessary to protect them by means of a conductor. Figs. 1918 and 1919 show the usual arrangements in such cases. In the first, four rods, riveted to the iron plate at the top of the chimney, support the conductor; one of the rods is produced horizontally and fixed at its extremity to the conducting-wire. The second represents a lightning conductor upon an iron chimney. The conductor is supported by
three rods riveted to the edge of the chimney, which serves instead of a conducting-wire, and the chain which establishes communication with the ground is fixed to its lower portion.

86. When single chimneys are very high, and are designed to receive air at a high temperature, it is necessary to strengthen them. We see an instance of this strengthening in the square section chimneys of puddling furnaces. When chimneys are conical, iron hoops are built into the masonry.

In the rectangular chimneys of steamboilers, flat iron bars having a kind of hook at each end of sufficient dimensions to take in two or three rows of bricks are built into the masonry in alternate directions, as shown in Fig. 1920.
87. Figs. 1921 to 1923 represent a large factory chimney of a circular section, built according to the plans of MM. Thomas and Laurens. This chimney is 40 metres in height; the diameter is at the base $3 \cdot 35$ and at the top $2^{\mathrm{m} \cdot 02}$; therefore the diminution of section is on the inside 0.016 a mètre: the exterior diameter at the base is $5 \cdot 65$, and at the top $2 \cdot 52$; consequently the external diminution is 0.030 a mètre. The chimney is formed of five cylinders having each a height of about 8 mètres, and conical both on the inside and on the outside. The thickness of the masonry is 3 bricks for the first portion, and is successively reduced to $2 \frac{1}{2}$ bricks, 2 bricks, $1 \frac{1}{2}$ and 1 , for the other portions. Thirty-eight iron hoops $h h^{\prime}, 2 i^{\prime}$, are built into the

## CHIMNEY.

masonry. The first portion of the chimney has an inner casing of fire-bricks. The top is covered with an iron plate $a b$, composed of four pieces bolted together.

The engineers above alluded to, often place in their chimneys at the points $c c^{\prime}, d d^{\prime}, e c^{\prime}, f f f^{\prime}, g g^{\prime}$, the base of each of the cylinders, bands of coloured bricks; and the spaces between those bands are filled with bricks of several colours, worked into various patterns for ornamental purposes.
88. Registers.-In every heating apparatus, whatever its dimensions and purpose may be, it is requisite to place, either at the top or at the bottom of the chimney, an iron plate, by means of which the draught may be diminished, and the chimney closed entirely during the suspension of work in the furnace. Registers are of great service in apparatuses which are only occasionally in operation, because, the current of air being intercepted, the furnace cools very slowly.
89. They may be arranged in a great variety of ways. In apparatuses of small dimensinns, having iron chimneys, the opening in the chimney is regulated by a disc, Fig. 1924, turning about an axis traversing the opposite sides of the pipe, and worked from the outside.
90. This arrangement might with equal advantage be adopted for larger chimneys; but as the friction of the axis in its bearings would not alone be sufficient to keep the disc in its position, we should have to affix to it, on the outside of the chimney, a rod or handle perpendicular to its direction, by which it might be fixed as shown in Fig. 1925. The dise should be of cast iron, as wrought iron is more liable to rust, and should have considerable thickness to prevent warping. When the air-channel is horizontal, the arrangement shown in Fig. 1926 may be adopted. The register, in this case, is a cast-iron plate $b$, turning about an axis upon a pivot $c$, and which is moved in an iron frame by means of a crank $a$.

1925.

91. Horizontal, sliding trap-doors are sometimes employed; but in large chimneys it is more usual to employ those which move vertically, and which are held by a counterpoise. Fig. 1927 is a section of this arrangement through the length of the air-channel. $R$ is a cast-iron plate sliding in grooves, also of cast iron, fixed in the masonry. It is held by a chain which passes over a pulley P , and is attached to a weight M .
92. This last arrangement is the one generally adopted; it has, however, one great objection, namely, a considerable space between the edges of the plate and the grooves of the iron frame in which it moves; this allows a rush of cold air into the chimney, whereby the draught is considerably diminished in the furnace. This rush of cold air may be prevented when the register is lowered by constructing it with projecting arms at the top, made to fall into a groove filled with sand.
93. When the temperature of the heated air exceeds $500^{\circ}$ or $600^{\circ}$, the contrivances which we have been considering cannot be employed, because the iron would be quickly warped or rusted. The best method of regulating the draught of a chimney in such cases, is to place the register at the top of the chimney, where the disc, or plate, being always exposed on one side to the air, becomes heated in a very much smaller degree. This arrangement is shown in Fig. 1928; c is a cast-iron dise affixed to a vertical rod, connected by a joint to the end of the lever $a b$ turning about the point $d$; a chain $g$ is attached to the extremity $a$, by means of which the distance of the disc from the top of the chimney is regulated. This arrangement has the disadvantage of requiring a workman to ascend to the top of the chimney when the apparatus is got out of order.

See Anemoneter. Barometer. Boiler. Fuel. Pyrometer. Thermometer. Ventilation. CHISEL. Fr., Ciseau, Burin; Ger., Meissel; Ital., Scurpello; Span., Escoplo.
See Hand-Tools.
CHOCOLATE MACHINE. Fr., Moulin à chocolat; GEr., Chocoladenmühle; Ital., Macchina da cioccolata; Span., Molino de chocolute.

See Mills.
CHRONOGRAPH. Fr., Chronograph; Ger., Kronograph; Ital., Cronografo. See Gunpowder.
CHUCK. Fr., Mandrin; Ger., Futter, Patrone ; Ital., Coppaia.
A chuck is contrived to fit the mandrel of a turning-lathe, and to hold the material to be turned. See Lathes.

CHURN. Fr., Baratte; Ger., Butterfasz ; Ital., Zangola; Span., Mantequera.
L. Bacon's churn, Figs. 1929, 1930, has two reciprocating dashers F, H, with fingers $h h, i$, these dashers being worked by a double-throw crank D. The dasher-rods $c c$ have two guides $b b$, one through the cover of the churn and the other through the upper end; and connecting-rods $a, a$, attach the dasher-rods to the crank. The machine is enclosed in the case A, C, B.
1929.


In J. Harrison Doolittle's churn, Figs. 1931, 1932, the dasher B $b$ C is turned by a driving journal $\mathrm{D} d$. The dasher is removable, and when replaced in the churn rests in the grooved recess $a$, where it is secured by a tapering dove-tail key E.

A movable horizontal brake L $l$, Fig. 1932, resting in a cavity M and on a ledge N, thoroughly agitates the cream, the brake being fastened with the pin O .

CIDER MILLL. Fr., Moulin à pommes; Ger., Apfelpresse; Ital., Torchio da sidro; Span., Molino y prensa para hacer sidra.

See Mills.
CIRCULAR SAW. Fr., Scie circulaire; Ger., Kreissage ; Ital., Sega circoïare; Span., Sicrra circular.

See Saws.
CIRCUMFERENTER. Fr., Graphomètre; Ger., Graphometer; Ital., Grafometro a bussola; Span., Grafómetro.

See Surveying.
CLACK-VALVE. Fr., Soupape à charnière; Ger., Ventilklappe; Ital., Valvola a ganghero; Span., Válvula de cuero.

## See Pumps.

CLAMIP. Fr., Emboîture; Ger., Hirnleiste; Ital., Tuirluppo; Span., Cepo.
See Mechanical Movements.
CLEAT. Fr., Taquet; Ger., Klampe; Ital., Rinforzo.
A cleat is a narrow strip of wood or of metal, fixed to something for the purpose of affording strength or of securing a piece of work in its proper position.

CLIP-DRUM. Fr., Tambour à tenailles; GER., Scherentrommel.
See Brake, pp. 586 and 619. Mechanical Movements.
CLOTH-WORKING MACHINERY. Fr., Machine à faire les draps; Ger., Tuchmaschino; Ital., Muchina da panni; Span., Muquinaria para fabricar paños.

See Calender.

CLUTCH. Fr., Clef d'arrêt ; Ger., Ausrück Kuppelung; Ital., Connessione a denti.
A clutch is a projecting tooth or piece of machinery for connecting shafts with each other or with wheels so as to be disengaged at pleasure. See Gearing.

COAL-CUTTING MACHINE. Fr., Machine à tailler des houilles; Ger., Kohlenbreck Maschine; Ital., Macchina da spezzare litantrace; Span., Máquina para cortar carbon de piedra.

The objects, says John Fernie, of Leeds, to be gained by the application of machinery to coal cutting are, firstly, the cheapening of the work; secondly, the saving of a large quantity of coal, which, in the ordinary process of holing or undergoing by hand labour with the pick, is broken up into slack and dust; thirdly, the removal of the danger attendant upon undergoing by hand labour; fourthly, the getting of a larger quantity of coal out of the pit with the same length of working faces opened; and fifthly, in the case of machines worked by compressed air, the collateral advautage of better ventilation and a cooler atmosphere in the mine, owing to the discharge of the compressed air after each stroke of the tool. The difficulties attending the application of machinery to work previously performed by hand are greatly increased in the case of coal-cutting machines, by their having to work at great depths below ground, and in the very confined passages of a mine.

Fernie, in the 'Institute of M. E.,' described two coal-cutting machines driven by compressed air, one having a pick worked by a bell-crank lever, with an action like that of the ordinary pick used in hand work, and the other working a straight-action tool somewhat in the manner of a horizontal traversing slotting machine. Both these machines had been successfully employed in regular work for a length of time at collieries in the neighbourhood of Leeds.

The coal-cutting machine of W. and S. Frith, shown in Figs. 1933 to 1935, is constructed for working a pick by means of a bell-crank lever, so as to give an action similar to that of the ordinary pick employed in hand work.

The pick $\mathbf{A}$ is fixed in a socket in one of the arms of the bell-crank lever $\mathbf{B}$, the other arm of which is worked direct by the piston-rod of the horizontal cylinder C. The slide-valve D, Fig. 1936, for the admission and discharge of the compressed air by which the machine is driven, is an ordinary slide, worked by a tappet-roller $\mathbf{E}$ upon the piston-rod; the machine is thus self-acting as regards the strokes of the pick, which is started to work as soon as the compressed air is turned on by the stop-cock $F$ in the supply-pipe $G$. The machine is mounted upon four wheels running upon the ordinary rails of the colliery, and is advanced the requisite distance between each blow of the pick by a hand-wheel H connected by gearing with the hind pair of carrying-wheels. The two pairs of wheels are coupled together, in order to render the full adhesion available for the forward motion of the machine; and by this means it is found that sufficient adhesion is obtained without the necessity of laying down a special rack-rail for the feed motion.

As the return of the pick after each blow is made by means of the self-acting tappet-motion working the slide-valve, it is necessary that the tool should go to the full extent of its stroke at each blow, before it can be withdrawn again. The amount of feed between each blow has therefore to be regulated by the attendant, according to the hardness of the seam of coal in which the machine is cutting, so that the pick shall complete an entire cut at each blow. In the event, however, of the pick being advanced too far at any blow, so as to put too much work upon it and stop it before the stroke is completed, it is only necessary to draw the machine back again by means of the hand-wheel H , until the pick is released from the cut; the unfinished stroke is then completed, and the pick goes on working again the same as before the stoppage. In order to allow of altering the height at which the pick performs the holing in the coal, the socket K carrying the pick is made to slide vertically upon the shaft of the bell-crank lever $B$, the height of the socket being adjusted by the forked arm $J$ controlled by the screwed rod and handle $L$.

One of these pick machines worked the whole of the undercutting in the West Yorkshire Coal and Iron Co.'s Colliery at Tingley, near Leeds, holing a seam of coal 3 ft .8 in . thick; and the compressed air for driving it was supplied by an air-compressing engine at the surface, with steamcylinder of 20 in . diameter and 3 ft . stroke, working an air-cylinder of 18 in . diameter and the same stroke, and compressing the air to about 50 lbs. the square in. pressure. The depth of the pit is 170 yds., and the air is conveyed down the shaft and along the mine in $2 \frac{1}{2}$-in. cast-iron pipes, with a $1 \frac{1}{4}-\mathrm{in}$. wrought-iron pipe laid up the bords to the working faces, and then a $1 \frac{1}{4}$-in. flexible tube to the coal-cutting machine. Small air-vessels are placed at intervals of 500 yds . along the air-main,


[^1]
for the purpose of maintaining the pressure of the air at the machine when working at a considerable distance in the mine: the machine is worked at a distance of as much as a mile from the shaft.

In a trial of this machine, it was found that a pick of 75 lbs . weight, cutting a groove to a depth of 24 in . in from the face, gave about 74 blows a minute. At the colliery the coal was got by the long-wall system of working, as shown in Fig. 1937, in which the machine is indicated at M working along the straight face of 50 yds . at one of the banks. The time occupied by the machine in
1937.

undercutting a length of 56 ft . was twenty-five minutes, including all stoppages for clearing the rubbish out of the hole and for backing the machine when the pick occasionally made an incomplete stroke. The machine was then run back to the starting-point, and set to work again with a longer pick of 90 lbs . weight, completing the previous cut to the final depth of 3 ft .9 in . from the face. With this pick the blows were about sixty a minute, and the half-length of 28 ft . was undercut in seventeen minutes, including all stoppages. The time occupied in running the machine back and changing the pick was sixteen minutes. The machine in this case was working at a distance of about a mile from the bottom of the shaft.

From this trial it appears that in undercutting to the depth of 24 in . in a single course, the work done by the machine was at the rate of about 30 square yds. an hour; and in undercutting in two courses to the total depth of 3 ft .9 in ., the work was done at the mean rate of about 15 square yds. an hour, including the time required for running the machine back and changing the pick.

The width or height of the groove cut out by the pick is 2 in . at the inner extremity, widening out slightly towards the face of the coal. It is necessary to stop the machine at intervals, in order to clear out the rubbish left in the hole; and the rails in front of the machine have also to be cleared of the material thrown out by each return stroke of the pick. Two men are required to attend to the machine, one working the hand-wheel for the advance of the machine, and the other clearing a way the stuff.

A good criterion of the actual rate of working that may be safely reckoned upon with this machine in regular practice is afforded by its performance upon an occasion when it was kept continuously at work for twenty-four hours consecutively, on 21 st and 22 nd of May, 1868. During this time the machine was employed upon five different banks of coal successively, requiring accordingly to be shifted four times for the purpose. The average depth of holing was 3 ft .6 in ., and the totall longth of work completed to that depth during the twenty-four hours amounted to 257 yds . This gives the practical rate of holing by the machine at rather more than 12 square $y d s$. an hour, including all stoppages for clearing the pick in working and for shifting the machine on the completion of each separate length of bank.

The other coal-cutting machine, before alluded to, shown in Figs. 1937 to 1939, is the inven tion of G. E. Donisthorpe, and may be described as a horizontal traversing slotting machine, the

1939.

work remaining stationary, while the machine traverses along the working face of the coal and cuts out a horizontal slot or groove along the bottom of the seam of coal or along a parting in the thickness of the seam itself.

The cutter-bar A carrying the cutting tools is fixed upon the upper side or the working cylinder C, which moves horizontally forwards and backwards at each stroke, the piston and piston-rod remaining stationary. This arrangement has the advantage of economizing space, and giving greater stiffness and a better attachment of the cutter-bar; and the whole is guided steadily between the guides B B by means of four sets of steel rollers DD attached to the cylinder and steadying it both vertically and laterally, as shown in the sectional plan of the cylinder, Fig. 1940. The piston-rod E is made hollow, as shown in Figs. 1940 and 1942, having two passages

communicating with the opposite sides of the piston ; and the alternate admission and exhaustion of the compressed air is regulated by a cylindrical slide-valve F, Figs. 1937 and 1943, worled by the hand-lever G. The ports in the piston are circular, and at each end of the cylinder a projecting plug $\mathbf{J}$ is fixed on the inside of the cover, opposite to the port, as shown in Figs. 1940, 1941; this plug entering the port at the end of the stroke prevents the complete escape of the whole of the air in exhausting; and the air thus retained in the cylinder forms an air-cushion at the end of the stroke, whereby the piston is prevented from striking the cylinder-cover at either end. A subsidiary port I, Fig. 1841, alongside the main port, and provided with a valve opening outwards, affords a passage for the admission of the compressed air at the commencement of the next stroke, until the main port is unstopped by the withdrawal of the plug $J$.

The working cylinder C is 6 in . diameter with 12 in . stroke; and the frame B in which it is carried is itself slung upon horizontal trunnions H, Fig. 1938, in another frame K sliding vertically upon the pillars L L, which are fixed on the base-plate $M$ of the machine; and by means of a screw N and worm-wheel, Fig. 1937, the working cylinder can thus be raised or lowered bodily to any height at which the cutter is required to work. The trunnions and the curved slot $\mathbf{H}$, Fig. 1938, also allow of the cutter being adjusted by means of the hand-wheel and screw X to work at an inclination to the horizontal, between the limits shown by the two dotted positions in Fig. 1938.

The cutter-bar A, Figs. 1938, 1939, is of cast steel, having six sockets for carrying the cutting tools. The tools are made of flat bar steel, with one-quarter twist in the shank, by which means the cuttings are thrown away from the face of the cut at each stroke of the bar A. The cutters are placed about 7 in . apart longitudinally along the length of the bar A, the average depth of the cut being about 3 ft .4 in . in from the face of the coal; and the length of stroke being 12 in. , the tool does no work during the first 5 inches of the stroke, but acquires a rapid motion whereby it strikes upon the coal with the percussive action of a blow, instead of cutting by a uniform steady pressure like an ordinary slotting tool. Previous to the adoption of this principle of working, the present pressure of air of 60 to 70 lbs . a square inch was found insufficient to work the machine; but with the percussive action now obtained this pressure is found completely effective, and the diameter of the working cylinder being 6 in., the blow is given with a total force of 1700 lbs . or $\frac{3}{4}$ ton upon the tool. The cutters are stepped $2 \frac{1}{2} \mathrm{in}$. successively in advance of one another, as shown in the plan, Fig. 1939 ; and the bar is fixed upon the working cylinder by a centre pin $O$ and set-screws P P, allowing it to be set obliquely in plan at an inclination to the line of the stroke, as shown by the dotted lines in Fig. 1939. By this arrangement the inclination of the cutters to the work can be slightly increased or diminished, according as may be required by the quality of the coal, so as to obtain under all circumstances the most effective action of the cutters. The width of their cutting edges ranges from $1 \frac{3}{4}$ in. in the cutter at the outer extremity of the bar to 3 in . in the cutter nearest the working cylinder; and the slot cut out by the machine is therefore of the slightly tapered form shown in Fig. 1938.

The machine is mounted on four double-flanged wheels, Fig. 1939, running on rails laid for the purpose. The rail on one side is a plain wrought-iron bar $Q$ placed on edge; and the other rail $\mathbf{R}$ is made of two bars placed 2 in . apart, with a series of cross-pins at $1 \frac{1}{2} \mathrm{in}$. pitch, forming an open rack-rail, into which gears a pinion $S$ worked by a hand-wheel $T$, whereby the machine is advanced for each cut, the ordinary amount of feed being $1 \frac{1}{4} \mathrm{in}$. a cut. As the advance is given by hand by the man working the machine, any number of strokes can be given by the tool before the machine is moved forwards, in case the coal is too hard for the cutters to accomplish the full depth of cut at a single blow. The traversing pinion S gearing into the rack-rail is mounted in a slide on the base-plate $\mathbf{M}$ of the machine, whereby it can be raised out of gear with the rack when the machine has to be drawn away from one part of the mine to another.

The pressure of the air working the machine is also employed for steadying it against the
blows of the tool, so as to ensure the full effect of the blow being expended upon the coal at each stroke; otherwise the tool would be liable to be bent or broken, and the work would be irregular. This is accomplished by means of two vertical cylinders U U, each $8 \frac{1}{2}$ in. diameter, the piston-rods of which stand up towards the roof of the mine, and are connected together at the top by a horizontal frame V carrying a grooved wheel at each end. A loose iron rail on a wood batten W is placed above the grooved wheels; and the compressed air being admitted on the under-side of the pistons in the cylinders $U$, the rail is pressed up against the roof of the mine, as shown in Figs. 1937, 1938. The total pressure exerted by the pair of cylinders $\mathbf{U}$ U amounts to 6800 lbs., or upwards of 3 tons, which is found amply sufficient to resist the blow of the tool. When the machine in its forward traverse has arrived at the end of the roof-rail W, the pressure is shut off from the cylinders U , and the air is let out from them through cocks opened by hand, thus lowering the frame V as shown by the dotted lines in Fig. 1937; the roof-rail is then shifted forwards to the extent of its length, and the pressure being readmitted to the cylinders $\mathbf{U}$, the machine is again ready for continuing its forward traverse as before. The pressure exerted against the roof being elastic allows the top rail to accommodate itself to any irregularities in the level of the roof.

In working this coal-cutting machine at the West Riding Colliery of Pope and Pearson, at Normanton, it was observed that the number of strokes made by the tool was from 75 to 80 a minute, the pressure of air at the machine being from 65 to 70 lbs . a square inch at the time. The average depth of the cut during the time of observation was about 2 ft .9 in , as there was a clearance space of more than 1 ft . left between the side of the machine and the face of the coal, the rails not having been laid close enough to the face of the coal. The width of the cut was about 2 in . at the inner end, widening to about 3 in . at the face of the coal; and as the cut was in this case being made in a parting of dirt of about the same thickness, no waste of coal was produced by cutting the groove. One man is required to work the machine, working the air-valve $\mathbf{F}$ by the hand-lever $\mathcal{Q}$, Fig. 1937, and also giving the feed of the tool by the hand-wheel $T$ advancing the machine; and from three to five strokes were given by the tool between each advance of the machine, for completing the cut to the full depth of the groove; about three turns were then given to the traversing hand-wheel, advancing the machine about $1 \frac{1}{4} \mathrm{in}$., after which the same number of strokes were again given with the tool. The cuttings are cleared out of the groove by a man following the machine with a narrow curved rake; and the height of the cutting tool in the machine is regulated occasionally, according to the slight variations in the level of the parting in which the groove is made, so as to keep the tool always working in the parting.

The width of the groove cut by this machine is shown in Fig. 1938, the width at the face of the coal being not more than about $3 \frac{1}{4} \mathrm{in}$. in holing to the full depth to which the machine can work. In undergoing by hand to the same depth, the size of the cut is as shown in Fig. 1938, having a width of at least 13 in . at the face of the coal, making the whole size of the cut about three times that of the narrow groove excavated by the machine, which is shown by the dotted line in Fig. 1938. Supposing the machine to be holing in the solid coal, the waste produced by the undergoing in a seam of 5 ft . thickness, such as is being worked at the West Riding Colliery, would amount to only about 4 per cent., as compared with about 12 per cent. waste with hand work. An estimate of the absolute quantity of waste made by hand work in good seams of coal amounts to as much as 12 per cent., the whole of which is rendered nearly worthless; and taking the whole quantity of coal raised per annum in this country at $100,000,000$ tons, the waste produced in undergoing by hand labour amounts consequently to as much as $14,000,000$ tons a year.

The compressed air for working the machine is supplied by an air-compressing pump of 18 in . diameter and 2 ft .10 in . stroke, worked by an engine that is employed for winding coals up an incline at the bottom of the pit. The compressed air is delivered by the pump into a large airvessel, 3 ft . diameter and 30 ft . long, from which it is conveyed along the mine by a wrought-iron pipe $2 \frac{1}{2} \mathrm{in}$. diameter and $\frac{1}{4} \mathrm{in}$. thick, put together with ordinary screwed sockets, and terminating in a portion of $1 \frac{1}{4}-\mathrm{in}$. pipe and a long length of flexible tubing of the same size, which is coiled on the floor and drawn out by the machine as it advances along the face of the coal. Great difficulty was originally experienced in maintaining the pressure of the air at the machine when working at considerable distances from the compressing pump; but this has been completely obviated by the introduction of a second smaller air-vessel at the working face where the machine is in operation. This smaller air-vessel is 2 ft . diameter and 6 ft . long, and is mounted on wheels for the convenience of being quickly removed to any part of the mine. The air-compressing pump is capable of supplying air enough for working three of the coal-cutting machines.

By the adoption of compressed air as the medium for transmitting the power to work these coal-cutting machines, the important collateral advantage is obtained of improved ventilation at the working faces. The discharge of the exhaust air from the machines delivers a supply of fresh' air, free from noxious gas, during the whole time that the machine is working; and the coldness of the discharged air, consequent upon its expansion at the moment of liberation, has a highly beneficial effect in reducing the temperature at the working faces. During the trial made by Fernie at the Tingley Colliery, a shot was fired at a short distance from the machine, for breaking down a mass of coal; and although the smoke at first completely obscured the working face and the machine, the whole of it was cleared away in so short a time as two minutes, having been carried off by the current of exhaust air discharged from the machine. In the case of any danger arising from accumulation of gas in the goaf or elsewhere, the means of safety is at hand, as it is only necessary to detach the india-rubber supply-pipe from the machine, and discharge a continuous jet of fresh air in the required direction, whereby the gas is speedily dispersed, and the dangerous place rendered safe. Also, as the undercutting of the coal is done so much quicker by the machines than by hand labour, the employment of the machines requires a smaller extent of working faces to be kept open for getting the same qr ntity of coal in the same time; the air has therefore a shorter distance to travel and becomes le loaded with gas, and is consequentiy safer and more wholesome for the colliers.

In reference to the greater quantity of coal that is got by the use of the machine from a given length of working face in a given time, the quantity got by one man in a day, including undercutting, breaking down, and filling the coal, is 3 tons when undercutting by hand labour, assuming a yield of 1 ton a cubic yard; and taking a working face of 48 yds. length, the four men working on that length of face would therefore get 96 tons in eight days. The machine, however, will undercut 96 yds. in one day, and this quantity would then be broken down and filled in by four men in two days, making only three days for getting the 96 tons with the machine, in comparison with the eight days required for hand labour. Hence, with the machine, as much coal can be got from 18 yds . length of face as from 48 yds . by hand in the same time.

In regard to the danger to which the collier is exposed in holing by hand labour, from the risk of the coal falling down and crushing him while lying with his body inscrted partly within the groove which he is excavating, it is evident that this danger is entirely removed by the employment of a machine for performing the holing, as the men are altogether clear of the working face, and the tool alone enters the groove in process of excavation.

Hydrutlic Coal-cutting Machine, employed at Kippax Colliery, near Leeds.-This machine, Figs. 1944 to 1948, is intended to take the place of manual labour in nicking, or kirving, or, as it is called in Yorkshire, baring the coal. The seam of coal worked at Kippax Colliery is the wellknown Haigh Moor seam.

Its depth from the surface is 120 yds., and its section is as follows:-

> Ft. In.

| Coal |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Band of hard shale, with |  |  |  |  |  |  |
| Coal |  |  |  |  |  |  |
| Corites | .. | .. | .. | .. | 3 | 8 |

The roof is soft shale, containing thin beds of coal. In the Kippax and surrounding districts this coal is worked in its entire thickness, the band of shale having thinned out to 2 in., as above mentioned; but at Horbury, three miles west of Wakefield, the Haigh Moor is worked in two distinct beds, the upper and lower divisions being at that place 12 or 13 yds. apart.
'T. W. Embleton, speaking in 1865, observed that "the band at Kippax was the only part of the seam that was removed by this self-acting coal-cutting machine. The direction of the workings was towards the rise of the coal, or north cnd, as it is locally called."

While the machine proceeds with the baring, completing the work at once going over it, square pieces of wood and wedges are inserted loosely into the baring, at intervals of 4 or 5 ft ., to keep the coal in position till the colliers come to remove it. This slight support, however, does not prevent the coal so bared from detaching itself from the unbared part of the bed. The line of fracture was, in the case referred to, a few inches beyond the extremity or back of the baring, and in one even straight line.

The quantity of coal obtained for cvery yard of face was 2 tons, and the yield of small coal produced by breaking up the detached coal and the bottom coal was about 8 per cent.

The lower division of the coal was blasted with gunpowder in the usual way.
Water was the medium employed to actuate this coal-cutting machine, and water being for all practical purposes incompressible, its full power, diminished only by the friction of its passage through the pipes, can, therefore, be transmitted and applied at any distance from its source.

This self-acting machine consists of a hydraulic reciprocating engine, having a cylinder of $4 \frac{1}{2} \mathrm{in}$. diameter and 18 in . stroke, working horizontally, or at any angle to suit the inclination of the coal, or at any required height above the floor. The piston-rod is a hollow trunk or ram, into which is fitted a cutter-bar easily removed, carrying three or more cutting tools. The cutting tools can be adjusted so as to enter the coal at any angle with the line of the face. The position of the cutting tools will be most readily understood by reference to Fig. 1944. Although the length of the stroke of each of the cutting tools is 18 in., the practical cutting length of the stroke into the coal is about 16 in ., and, consequently, the three cuttcrs jointly give a total depth of 4 ft . at each stroke. The cutting cylinder has a valve-motion, which is self-acting; and the length of stroke of the tools can be varied, or any number of strokes can be given at any part of the entire length of the stroke. The cutting action of the tools being a steady push or thrust without any percussion, it is necessary that the machine should be firmly held upon the rails during the cutting stroke, and be released so as to traverse forward at the end of the return or back stroke, and this rigid fixing of the machine upon the rails is effected by means of a vertical self-acting holder-on, which is a prolongation of the piston-rod of another cylinder, mounted upon and becoming part of the machine itself, Fig. 1948.

The piston of this cylinder is actuated by means of the same self-acting valve-motion as that of the cutting cylinder, and the holder-on is retained in its dead-fast position by means of a small keep-valve, which retains the water during the cutting stroke. At the return or back stroke the valve-motion opens the keep-valve and relcases the water, thus enabling the holder-on $\mathbf{F}$ to descend and to slacken its pressure against the roof, and thus the machine is free to traverse upon the rails the requisite distance for the next cut. This traversing or progressive motion is also self-acting. For this purpose a chain is made fast ahead, close upon the floor, and passes over a grooved pulley on the machine, which gets the necessary bite upon the chain. This pulley makes part of a revolution, to suit any length of traverse, at the moment when the cutter-bar is completing its back-stroke.

The amount of feed or distance of cut is easily adjusted, and the self-acting traversing power is sufficient to move the machine upon skids, where they are found to be more convenient than wheels.

Unless the cutting tools complete a full stroke, the traverse-motion does not come into operation, but the tool will continue to cut in the same place until the full stroke is completed, and theu

only will the traverse-motion come into action; thus the back of the baring is always parallel with the rail upon which the machine travels, thereby causing the coal to break off in one uniform line.

The cutting tools, Fig. 1946, are of a form easy of construction, and are very strong, and capable of penetrating hard material with little risk of breaking. The cutting edge is nearly a $\frac{1}{4} \mathrm{in}$. thick, and any or all of the cutters can be removed and replaced in a few moments.

The machine can be adapted to any gauge in use for the roads of the mine, and is easily moved from place to place on its four wheels. The supplementary wheels at each end are used only when in operation, in order to secure a greater base.

The water pressure by which the machine is actuated is produced, in this instance, by an engine placed at the bottom of the shaft

The cylinder of this engine is 14 in . diameter; stroke, 2 ft .3 in . Attached to the engine are wo double-action pumps of $4 \frac{1}{2} \mathrm{in}$. diameter, and 12 in . stroke. These are capable of maintaining a pressure of 360 lbs . a square inch.


When the machine is not working, the engine regulates its own speed accordingly, and is further used for forcing water out of the mine to the surface. When convenient, the water pressure may be obtained from any pumping engine on the mine, by connecting the supply-pipes with the ordinary pumps, thus obviating any special outlay of capital for the moving power. The machine when working makes 25 strokes per minute, and uses 40 gallons of water.

The water is conveyed from the engine to the machine chiefly by 2 -in. wrought-iron pipes, the remainder being $1 \frac{1}{2}$-in. bore. The total length of these pipes is about 600 yds .

To allow the machine to traverse on the rails, it is connected to the $1 \frac{1}{2}$-in. pipes by means of an india-rubber tube of the same diameter. This tube will bear a pressure of 500 lbs. to the square inch. The wrought-iron pipes are the ordinary butt-welded steam-tubes, tested to a pressure of 500 lbs. to the square inch. They are used in preference to cast-iron pipes, because they occupy little space, are easily screwed together and bent, and will accommodate themselves readily to the varying floor of the mine, with no risk of breakage or leakage.

The exhausted or waste water is conveyed away from the machine by $2 \frac{1}{2}$-in. india-rubber hose, and by 2 -in. ordinary gas-tubes, to the place whence it was forced, and thus a very small quantity of water is required to work the machine, namely, as much as is necessary to fill the circuit of the pipes. The water may be used over and over again as in the ordinary hydraulic press.

The pressure at the engine is 80 lbs . in excess of that at the machine when working at 20 strokes per minute. Whenever the cutting tools meet with any extra resistance, the retarded speed allows this pressure to equalize itself, and thus overcome any obstacle.

The amount of pressure used for working the machine varies from 150 lbs . to 300 lbs . an inch, according to the hardness of the material to be cut.

Fig. 1944 is a ground plan when in working position, angle ano nelgnt adjustable. F holderon or feeder, $l$ traverse chain. D, self-acting valve-motion. X Z H, self-acting traverse movement. N, guide-bar and roller. A A A, total depth cut at each stroke; stroke of each cutting tool 1 ft .6 in . ; total stroke 4 ft . B, stem of cutters.

Fig. 1946, details of cutters.
D C, Fig. 1945, section through valve-chest.
D K, Fig. 1948, sectional end view taken through W W, Fig. 1944. Y, elevating screw.
Fig. 1947, end view in working condition, cutting at an angle.
COAL MINING. Fr., Exploitation de houillère; Ger., Bergbau; Ital., Arte del cavatore di litantrace; Span., Extraccion de carbon de piedra.

Parkin Jeffcock, in a paper published in the 'Proccedings of the Inst. of M. E.,' 1862, ably examined this subject, and applied his reasoning to the general features of the South Yorkshire district with reference to the circumstances affecting mining engineering.

The accompanying general plan, Fig. 1919, given by Jeffcock, represents that portion of the


Yorkshire coal-field which is more particularly called the South Yorkshire district; extending from Sheffield on the south to Wakefield on the north about 25 miles, and from west to east about 20 miles altogether, on either side of Barnsley. The plan shows the general extent of the coal-field, indicated by the shaded portion; the outcrops of two of the principal seams of coal, the Silkstone and the Parkgate seams; the positions of the principal faults; the localities of the more important collieries and iron-works; and the lines of railway and water conveyance.

The horizontal section, Fig. 1950, which is reduced from Thorpe's published section, is taken through Barnsley along the dotted line W E upon the plan, Fig. 1949, extending from the millstone grit on the borders of Derbyshire on the west to the eastern boundary of the coal-field at E .

The vertical section, Fig. 1951, represents the position and thickness of the principal beds of coal and mines of ironstone, as they were proved by borings at Wath Wood, near Lundhill Colliery, on the plan, Fig. 1949. Five beds of coal, between the Woodmoor seam and the Kent's Thin seam, do not occur at this place; a second vertical section, Fig. 1952, is therefore placed alongside, showing these beds in their corresponding position as they were proved in sinking at the Oaks Colliery, near Barnsley, Fig. 1949.

The South Yorkshire coal-field is a continuation northwards of the Derbyshire coai-field. On the east it is bounded by the overlying and unconformable magnesian limestone and Permian strata, and the extent of the coal-measures in this direction is yet unproved. On the west the millstone grit rocks crop out, forming the bleak moors of North Derbyshire; and the coal-measures extend northwards and constitute the North Yorkshire coal-field. The general dip of the coal strata is from west to east at an average angle of 1 in 9; this, however, is much modified in many localities by main faults, the principal of which are shown on the plan, Fig. 1949, by the strong black lines. The total number of coal seams is very great, as shown in the vertical section, Fig. 1951, and many of them have been worked in various localities.

The following are the principal seams of coal in their geological order, with their average thick-
 ness:-


The most important seam of the series is the Barnsley Thick coal, which, under the name of the Main, or Top Hard coal, has been very extensively worked in Derbyshire. In the South Yorkshire district its average thickness is about 8 ft 。 6 in ., but the thickness varies exceedingly
 at different places. It is most fully developed in the neighbourhood of Barnsley, but extends through the greater part of the district, and has been principally worked at Woolley, Gawber, The Oaks, Edmund's Main, Wombwell Main, Darley Main, Elsecar, Warren Vale, Rawmarsh, Hoyland, Lundhill, Mount Osborne, Thryburgh, Darfield, and Car House. The hard coal from this seam is in great repute for steam purposes, and stood high at the irials made at Woolwich in 1851 relative to the value of steam coals. North of Woolley the Barnsley seam is subdivided into two or three others, which are worked in the neighbourhood of Normanton under different names. In Derbyshire it appears to the best advantage at the large works of Mr. Barrow, at Staveley, where it is known as the Staveley Hard coal, which has been extensively used for steam purposes and in the manufacture of iron.

The Swallow Wood seam occurs about 60 yds. below the Barnsley Thick coal, its thickness varying, from 3 ft .4 in . to 6 ft . It has been worked only to a very limited extent, principally at Swallow Wood, and is known in Derbyshire as the Dunsil or Oldgreaves coal, lying there about 30 yds. below the Top Hard seam.

The Parkgate or Thorncliffe Thick seam occurs at an average depth of 219 yds. below the Swallow Wood, and has been chiefiy worked at Parkgate, Thorncliffe, and Pilley. Its average thickness is 5 ft .6 in ., but the thickness varies considerably, from 4 ft .10 in . to about 6 ft . It is known as the Bottom Soft coal in Derbyshire, where it has been very extensively worked.

The Thorncliffe Thin seam, called the Bottom Hard in Derbyshire, is found 24 yds. below the preceding; its thickness is from 2 ft .6 in . to 3 ft ., and it has been principally worked at Thorncliffe, Pilley, \&c.

The Silkstone or Sheffield seam lies about 61 yds. below the Thorncliffe Thin, and has an average thickness of about 5 ft . It is a very well defined seam, and may be taken as a sort of datum line in identifying the position of the other beds. It has been principally worked in the neighbourhood of Sheffield, and at Chapeltown, Thorncliffe, Pilley, Mortomly, and Silkstone, and is identical with the Black Shale or Clod coal of Derbyshire. The coal is of great value for house fire purposes, competing with the celebrated Hetton Wallsend.

By far the most important and valuable of the seams of coal are the Barnsley Thick and Silkstone seams. At the Woolwich trials, made by the Admiralty in 1851, relative to the strength
and value for steam purposes of the Barnsley Thick coal from Darley Main, West Hartley coal from Newcastle, and Welsh coal from Merthyr Tydvil, the total weight of water evaporated in each case was $24,960 \mathrm{lbs}$., and the evaporation a lb. of coal was $8 \cdot 10 \mathrm{lbs}$. by the Barnsley Thick and West Hartley coals, and 8.25 lbs . by the Merthyr coal. Trials were also made of the Barnsley Thick coal in 1858 at Doncaster, on the Great Northern Railway, when the evaporation obtained was $7 \cdot 64 \mathrm{lbs}$. of water a lb. of coal, the total weight of water evaporated being $448,281 \mathrm{lbs}$., and the coal used being a mixture of steam coal and house fire coal, consumed under Cornish boilers, working at a pressure of 45 lbs. The Barnsley Thick coal lights easily, burns freely, and raises steam rapidly. It produces only a very small quantity of white ashes and cinders, giving little trouble to the stokers, and the less it is disturbed the better; it does not clog or adhere to the bars, and makes no slag, maintaining a good clear fire with little sulphur. It is a most economical coal for marine engines, and in using it a light thin fire is particularly recommended.

The mines of ironstone occur between the Barnsley Thick coal and the Silkstone coal, as shown in the vertical section, Fig. 1951.

The first mine of importance is the Swallow Wood, about 60 yds . below the Barnsley Thick coal, which has been principally worked at Milton for the supply of the furnaces there. It consists of three measures of ironstone ; and an analysis of a sample of the ore by Spiller, of the Geological Museum, gave $26 \cdot 79$ as the percentage of metallic iron.

The Lidgate mine, next below the Swallow Wood, has been extensively worked at Milton, Tankersley, and Thorncliffe.

The Tankersley mine is usually found about 50 yds. below the Lidgate, and is called also the Musselband ironstone, from the number of fossil shells it contains. It has been worked chiefly at Tankersley, and yields about 1500 tons of ironstone an acre.

The Thorncliffe Black mine lies about 70 yds. below the Tankersley; it is worked principally at Parkgate, and used in the furnaces at Milton and Elsecar; and an analysis by Spiller gave $31 \cdot 16$ per cent. of metallic iron.

The Thorncliffe White mine lies immediately below the Parkgate seam of coal, and consists of three measures, containing about 32 per cent. of metallic iron, and yielding about 1500 tons of ore to the acre. It has been worked principally at Parkgate and Thorncliffe, and was formerly worked extensively at the Holmes.

The lowest mine is the Clay Wood or Black mine, consisting of three measures, containing about 32 per cent. of iron and yielding about 1600 tons of ore the acre. It has been got to a great extent at Thorncliffe, and is identical with the Black Shale or Stripe Rake of Derbyshire, which is so much prized by the ironmasters of that county.

The principal iron-works of the South Yorkshire district are at Parkgate, Holmes, Milton, Elsecar, and Thorncliffe, in blast; and at Chapeltown and Worsborough, out of blast.

The modes of working the coal in the South Yorkshire district may be considered as modifications of the long-wall system, so extensively and successfully practised in the Midland counties. The pillar-and-stall mode of working adopted in the North of England has not been much used in South Yorkshire; and the long-wall system being principally confined to the Midland counties, the South Yorkshire system of working may be regarded as a combination of the two. Where the circumstances are favourable, the long- wall system is being extended in the Yorkshire coal-field; and wherever it can be adopted, it is to be recommended, on account of the simplicity of arrangement both for working and ventilation, and also as being the most economical method of getting the coal.

The principal modes of working the coal adopted in Yorkshire are the Narrow Work, Long Work, Bords and Long Work, Wide Work, and Bank Work. These are shown in the ideal diagrams, Figs. 1949 to 1968. They can be represented only by ideal plans, becanse none of them are carried out in their integrity at any collieries in the South Yorkshire district; and in some instances one mode is adopted in one part of the workings, and another elsewhere in the same colliery. These different systems of working, some of which, however, are falling into disuse, have been rendered necessary by the variable nature of the roofs and floors of the coal seams in the South Yorkshire district. The same reference letters are used throughout all the diagrams.

Fig. 1953 is a plan of the mode of working by Narrow Work, on the end of the coal. P is the downcast pit, and B the main bord (road cut transversely to the grain of the coal, against the face of the coal), from which pairs of headings or endings E E (roads cut against the end of the coal, lengthways of the grain) are driven at intervals of about 30 yds . When these endings have been carried to the requisite distance on either side of the main bord $B$, a communication is made between their extremities, and the coal is worked by short faces homewards, as shown at W W. The whole of the coal being thus got out, the roof is allowed to come down in the goaf as the working progresses, being temporarily kept up immediately behind the working faces by props or puncheons, which are afterwards withdrawn successively and shifted forwards. U is the upcast shaft, and $\mathbf{F}$ the ventilating furnace. The main current of fresh air from the downcast pit $P$ is carrijd up the main bord $B$ and along the farthest pairs of endings $E$, as shown by the arrows, and is then passed through the face of the workings W . The course of the air is determined by stoppings $S$ built to block up the various crossgates between the bords and endings; and by doors D , through which the coal is brought down to the shaft from the workings W , and from the endings $E$ that are in process of being driven. At C is an air-crossing, where the current of foul air proceeding from the workings to the upcast shaft $U$ crosses over the current of fresh air entering the mine from the downcast pit P. At R R are regulators to control the quantity of air passing through each portion of the mine; when these are closed, the whole of the fresh air has to pass through the workings before reaching the upcast shaft; but when they are opened, sportion of the air finds a shorter course through the regulators direct to the upcast shaft, and a smaller quantity of air therefore passes through the workings. This mode of working is falling into disuse in Yorkshire, and is seldom adopted except under special circumstances, where the coal is of a soft
or friable nature and where the rcof is not strong, the coal being therefore got in very short lengths, as shown at W W, with only a very narrow face in process of working at a time, whence the name of this mode of working.
1953.


There are two modes of Long Work, the first of which is shown in Fig. 1954. This and all the subsequent modes of working are on the face of the coal, the workings W being carried forwards transversely to the grain of the coal, against the face of the coal, instead of against the end of the coal as in the previous narrow work. In Fig. 1954 it will be seen that there is a long
1954.

face of work in progress at once in each portion of the mine: the workings are started from the main headings or endings E , and the coal from the working faces is brought down through the goaf by means of packed roads G, shown by the strong black lines, the walls of which are built up of rock and shale; the packed roads are carried forwards continuously as the working faces advance. The fresh air from the downcast shaft passes along the endings $E$ and the packed roads G up to the working faces W, and thence by the bords B to the upcast shaft U, as shown by the arrows, the regulators R R controlling the ventilation in each portion of the workings. At H H are doors or stoppings with apertures to allow of passing some of the air through the packed roads $G$ in the goaf, according as may be required to keep them clear of gas.

In the second mode of Long Work, shown in Fig. 1955, the workings are subdivided into separate lengths of face by the pillars L being left between them at first, about 30 yds. thick; but when the workings have been carried forwards as far as intended, the intervening pillars are then also worked, beginning from the farther end and working backwards, as seen at A, whereby
the current of air is always kept up against the pillar face A until the whole pillar is removed. The packed roads $G$ are required for bringing out the coal through the goaf in this plan of

working, the same as in the first mode of long work; the strong dotted lines through the goaf in the neighbourhood of the pillar working A show packed roads that are no longer required to be maintained, and have been abandoned. The course of the air is shown by the arrows.

The mode of working by Bords and Long Work is shown in Fig. 1956. Here pairs of bords B B are driven from the main heading or ending E, at intervals of about 20 yards; and when they have reached the extreme distance intended, the whole of the intervening coal is worked homewards, downhill, and is brought out from the working face W through the bords B. In bords and long work, therefore, the bords form a marked feature in the system, being driven to the extreme extent in the first instance, as shown in the right-hand half of the plan, Fig. 1956, before the working of

the whole coal is commenced; and when this has been begun, as shown in the left-hand half of the plan, no packed roads are required in the goaf for bringing out the coal from the working face, but the coal is brought down through the bords themselves, which are thus not obliterated till all the coal is won, but remain of service to the last. In the previous modes of long work, on the contrary, shown in Figs. 1954, 1955, the progress of the work is in the opposite direction, uphill, and the face of work is opened without driving bords; and accordingly packed roads are required to be maintained through the goaf for bringing down the coal from the working face. The course of the air is shown by the arrows in Fig. 1956, and the air regulator is placed at $R$; but in bords and long work
there is no need of any arrangement for coursing part of the air through the goaf, as is required in long work.

In the Wide Work method, shown in Fig. 1957, the coal is got in banks W about 60 yds. long, each subdivided into bords 7 or 8 yds . wide, separated by pillars of an average thickness of 1 yd. , as shown by the thick black lines in the goaf. Crossgates K are made to the main roads B at suitable intervals, according to the state of the atmosphere in the mine and the ventilation. For
1957.

ventilating the workings the current of air is passed up the farthest bord B, across the face of the work in the first bank W, and out at the other end of the bank; it is then carried forwards up the intervening pillar bord B to the next bank, and across the working face in the same manner, as shown by the arrows. This method of working is now being abandoned where possible for the long-wall system.

In the Bank Work, shown in Fig. 1958, the coal is got in banks W about 60 yds. long, as in the last mode, but each bank is worked all in one length without any intermediate pillars being left in each bank. The method of ventilation is the same as in wide work, as shown by the arrows. The mode of working by single bords B, as in both bank work and wide work, instead of by pairs of

bords, is, however, to be condemned on account of the difficulty and expense of maintaining packed roads through the goaf for the winning of the pillars B B at the last; or if they have not been maintained, of making new packed roads for ventilation : and again, these pillars being liable to a heavy pressure, the coal in the pillar working is rendered of little value.

The plan of the Long Wall system of working, Fig. 1959, shows the difference of this system from any of the ordinary Yorkshire methods of working described above. This is not an ideal plan, but a plan of the actual long-wall workings of the Parkgate seam at the Wharncliffe Silkstone Colliery near Barnsley, Fig. 1949. There is here no loss in getting out pillars, as all the coal is
excavated at one operation. The ventilation of the mine is at the same time considerably simplified, the current of air having altogether a shorter and less tortuous course to follow from the downeast shafts $\mathbf{P}$ to the upcast U , as shown by the arrows. The thick dotted line $\mathbf{M} \mathbf{M}$ shows the position of a fault in one portion of the mine, and the workings are therefore laid out at that part conform-

ably with the course of the fault. By the long-wall system a working face of 430 yds . is here obtained in a single length without interruption, as shown at $W$; and in the lower portion of the workings along the fault MM another face has been opened of the same total length, but divided into two shorter faces by a pillar bord, for safety and convenience of working in the neighbourhoed of the fault, the intervening pillar being removed before that portion of the mine is abandoned.

Various supports for the roof are used in the Yorkshire seams: wooden props or puncheons are adopted in some cases; in others, piles of wooden blocks called chocks or clogs; and in others, packs of rock and shale. Cast-iron puncheons also are now being extensively introduced, one of which is shown in Figs. 1965, 1966.


Two of the grearest difficulties that have to be contended with in mining are Water and Gas. With regard to Water, the mines in the South Yorkshire district are not in general heavily watered in comparison with other mining districts; the workings nearer the outcrops or bassets of the seams are generally more watered than the rest. Except in some special instances there are few collieries where large pumping engines are required: lift pumps are used exclusively, and even tubbing has
scarcely ever been resorted to. A remarkable inundation occurred in 1861 at the Woolley Collicry at Darton, near Barnsley, Fig. 1949, which is working the Barnsley Thick coal : the coal is drawn up a long inclined plane extending from the outcrop of the Barnsley Thick seam and following the dip of the seam; and the water is raised by means of flat pumps. On the 13 th April, 1861, a sudden irruption of water into the workings took place, to such an extent that they were almost entirely filled. The water entered through a fissure in the overlying rock, which is of considerable thickness and is full of cracks and fissures towards its outcrop. It is probable that a large amount of head or drainage water had accumulated in these fissures while they remained closed, and that they afterwards became opened by subsidence of the strata in consequence of the working of the coal: the water was found to be drawn away from a well in the rock at the surface, 170 yds. above the coal. The accumulation of water must have been very great, as it continued rising in the day drift a fortnight after the inundation had occurred, at the rate of more than 1 ft . an hour, although a double 10 -in. pump had been kept continuously at work; but its rise was subsequently stopped by additional pumping power.

In the amount of Gas generated by the different seams of coal there are great variations. The most terrible explosions have taken place in the Barnsley Thick coal, especially at the Darley Main Colliery, the Oaks, Warren Vale, and Lundhill; the Barnsley Thick and Silkstone seams being specially liable to sudden and powerful emissions of gas. The ventilation is produced by a furnace, shown in Figs. 1960 to 1963, situated at F in the diagrams, Figs. 1954 to 1958, at the bottom of the upeast shaft U, by which a fresh current of air is kept continuously flowing through the mine, so that any gas issuing from the coal is speedily diluted and rendered harmless. For distributing the air through the workings, the stoppings S , doors D , and regulators R are arranged in proper places. The division of the air into separate splits, each of which ventilates a distinct portion of the workings by means of the crossings or overcasts C, and the "scale doors" or regulators R , may be considered, if properly carried out, one of the best preventives of explosions in these very fiery South Yorkshire mines. All the return air should be conducted into the upeast shaft by a dumb drift N, Figs. 1960 to 1963 , so as not to pass through the fire of the furnace; and the underground furnaces, whether closed or otherwise, should be fed with nothing but fresh air direct from the downcast shaft.

At some of the mines in the district, belonging to Fitzwilliam, large fans driven by steam power have been substituted for the furnace generally used elsewhere; they are a simple and efficient means of mechanical ventilation, well worth the consideration of all interested in mining, and have now been continuously working with complete success for several years. In the early periods of mining the only ventilation was the natural ventilation, the current of air through the workings being produced simply by the colder and denser air from the downcast shaft displacing the hotter and rarer atmosphere of the mine. Sometimes rarefaction was increased by putting a pan of coals in the upeast shaft; but the consequence of such imperfect ventilation was that the workings were sometimes stopped for many days tngether. Natural ventilation could, of course, be adopted only when the shafts were of moderate depth and the workings on a limited scale.

In the South Yorkshire district, safety-lamps were first used exclusively at the Oaks Colliery, in the workings of the Barnsley Thick coal, where Stephenson lamps are used in preference to Dary's; and the use of safety-lamps has since extended to many other collieries. At the Wharncliffe Silkstone Colliery, near Barnsley, working the Silkstone seam, Stephenson and Davy lamps are used exclusively; and as the coal-field is very much cut up with faults, the gas cannot be bled away, but as each fault is cut through the greatest caution is required in dealing with the gas in the solid coal beyond, in bye. In addition to the use of safety-lamps, an abundance of air should be taken into the working places of fiery mines. Since the explosion at Lundhill in 1857, safetylamps have been exclusively adopted there. The importance of their use in fiery workings was strongly shown at the Oaks Colliery in 1857, when an outburst of gas took place in the workings down the engine plane, so violent that it was compared to the roar of a draught in the furnace. All the Stephenson lamps were put out, and the Davy lamps were ignited internally, the gauze becoming red hot. As the outburst of gas occurred within a hundred yards of the main intake to the upcast shaft, and a large quantity of air was passing this part at the time, the gas was soon diluted and carried away; and in less than an hour the only traces that remained were found at one or two places where the floor had been upheaved. Thus no doubt a terrible explosion had been averted by the use of safety-lamps; but if any one of the lamps had been out of order, or the gauze smeared with oil or coal dust, or if any naked light had been in this part of the workings, an explosion would inevitably have occurred.

COAL-WASHING MACHINE. Fr., Machine à laver les houilles; GEr,, Kohlenwaschmaschine; Ital., Macchina da lavar litantrace; Span., Máquina para limpiar carbon de piedra.

We give, Figs. 1969 to 1973, a set of coal-washing apparatus and coke ovens at one of the establishments of the Wigan Coal and Iron Company. The washing machinery, which is contained in a building 48 ft . long by 24 ft . wide, consists of four iron chambers, each of which is in free communication with a cylinder placed at one end of it, as shown in the plan, Figs. 1970 to 1972. These cylinders are each 3 ft . in diameter, and the pistons working in them have a stroke of $3 \frac{1}{2} \mathrm{in}$., and are driven at 150 strokes a minute, so as to keep the water within the chambers in a state of constant agitation.

The coal to be washed is discharged from the railmav magons into a large hopper outside the building, and it is lifted from this hopper, Figs. 1969, 1970, by the elevator shown in the section, Fig. 1970, this elevator delivering it to the crushing rollers, by which it is reduced to a uniform size. After passing the crushing rollers, it is taken up by a second set of elevators, and delivered to the washing chambers, or bashes, down a trough laid at an easy gradient, this trough having four compartments, down which the coal is washed by a small stream of water. The coal, during the process of washing, is placed on perforated copper screens or sieves slightly submerged in the water contained in the bashes, and the pulsation of the water, occasioned by the action of the pistons, causes the good coal, which is the lightest, to be washed over into the shoots with which
the chambers are provided. The pyrites, on the other hand, being heavier, fall down and lie on the perforated sieves; and at regular intervals the sluices are opened, and the pyrites allowed to fall into the chamber below. At the bottom of each chamber there is another sluice, which is opened occasionally to allow the refuse to be discharged into wagons placed to receive it. The good coal from the shoots of the bashes is also delivered directly into wagons.

The washing machinery, elevations, and so on, are driven by a beam engine, with 14-in. cylinder and 2 ft .6 in. stroke, arranged as shown in Figs. 1969, 1970. The loss by washing is about 10 per

cent., by weight, of the gross quantity of coal passed through the machine. Several machines of the kind we have described are in use at the Wigan Coal and Iron Company's works; and they turn out from 300 to 350 tons of small coal a day.

The general arrangement of the coal-washing machinery and coke ovens will be understood from the general plan and section, Figs. 1971 to 1973. From this it may be observed that the ovens, which are each 11 ft . in diameter and 8 ft .6 in . high, are arranged in two parallel double rows, a chimney being provided for every eight ovens. The charging is performed by the aid of a tramway laid over the tops of the ovens, Fig. 1973, the coal dust being brought from the washing apparatus in 25 -cwt. hopper wagons along this tramway, and dropped through holes left in the crown of the ovens for this purpose. The charge of each oven is 8 tons, and this charge is allowed to burn five days or 120 hours. This, when burnt, is cooled by the aid of a water-pipe inserted in the oven, and is afterwards drawn out by rakes.

One of the most successful of recent coal-washing machines is that constructed by M. Evrard for
the mines of La Chazotte, in France, Figs. 1974 to 1976. Figs. 1974 and 1976 are vertical sections through the machine in two directions at right angles to each other, and Fig. 1975 is a plan of the whole. The special object in view in this construction is the washing of very large quantities of coal of comparatively unequal size, and the separation of these different sizes by the aid of the washing process. This large washing machine is thereby enabled to combine the advantages of machine labour with the superior ruality of work performed by hand-washing. The coals are

thrown into a hopper A, Fig. 1974, with a revolving base upon a larger revolving screen, where the largest pieces are freed from stones by hand, two workmen being placed upon the screen B, and carried round by the latter in its revolution. There are two screens with narrow openings below this first, and they deliver the smaller coal by means of two elevators C , to the top of the washing machine. Each kind of coal is brought up separately, and thrown upon a circular table D, which revolves slowly round a large fixed cylinder E , containing the floating piston $\mathbf{F}$. The two elevators
bring the coal up to the circular table, and it is levelled there by two scrapers fixed in different heights ( $m n$ in plan, Fig. 1975), so that a stratum of small coal is spread over a layer of larger coal below. The central part of the machine is filled with water, and the floating piston $\mathbf{F}$ is moved up and down in the water by means of a lever L, Fig. 1975, worked by a cam. The lever

is counterbalanced by a float G, immersed in mater, and its movement causes an alternating up and down current of water through the screens in the revolving table D, by which the different materials mixed together in the unwashed coal slack separate themselves in parallel horizontal layers, according to their difference of specific gravity, and also according to their size of grain. There is, however, one thing of great importance required for a successful operation, namely, the agitation of the water must commence with great power, and must successively diminish in force as the washing proceeds, since the first action effects the settlement of the very largest pieces of stone at the bottom, and of the largesized coal in a stratum immediately above that. This being done, it requires a more gentle agitation, so as to leave the lowest layers undisturbed, while operating through these upon the superposed strata of small-grained materials, so as to give the small pieces of stone and other heavy matters an opportunity to pass through the coarse-grained coal down to the lowest stratum of stones. With a uniform action of the machine, M. Evrard obtains this important result, by inclining the table against the water lever, as shown in Fig. 1976. He thereby obtains a greater immersion of the table, and, consequently, a more powerful rush of water through its sieves at the one part of each revolution, and a less powerful action at the opposite side where the coal is partly raised
 above the water-level.

The body of this machine is constructed in masonry, in the shape of an inverted cone, with a diameter of 33 ft .6 in ., and a height of 32 ft .10 in . Its bottom opens into a drain-pipe for removing the mud collected there. The floating piston is $17 \mathrm{ft} .4 \frac{1}{2} \mathrm{in}$. in diameter. The table D is 32 ft .

10 in . in diameter, and $6 \mathrm{ft} .6 \frac{3}{4} \mathrm{in}$. wide. It is perforated with holes of $\frac{9}{25}$ in. in diameter. The inclination of this table amounts to one foot for the whole length of the diameter of the table, the depth of layer of coal is 6 in ., and the water-level 8 in . over the lowest point of the revolving table. The coal is removed from the table after having passed through one entire revolution by means of four scrapers placed at different heights, so as to remove each another of the superposed strata of coal ranged according to the size, the smallest coal being raked off from the top, while the lower layers give coarser-sized coal in proportion to the depth. The lowest stratum is formed by the stones and other heavy impurities of the coal, and this is removed whenever it has reached a certain height. The washing process, as carried on with this machine at Chazotte, reduces the contents of ashes of the coal slack from 15 to 17 per cent. on the unwashed slack down to 8 or 9 per cent. The production of one of these machines is 138 bushels for each revolution, and, taking the speed at the average adopted at Chazotte, namely, one revolution in five minutes, with 100 strokes of the piston during that time, the total production is 1650 bushels an hour. The total power required for the complete machine, with its revolving screen and elevator, is 10 horse-power.

A very simple mode of washing the impurities out of small coal has been at work at South Tyne Colliery, Haltwhistle. The machinery employed consists of an arrangement of boxes or troughs, into the upper series of which, A 1, A 2, A 3, Fig. 1977, the water for washing is conducted by a pipe P. At the end of A3 is a small screen, formed of $\frac{3}{8}-\mathrm{in}$. round iron rods, placed about $\frac{1}{8}$ of an inch apart; immediately below this screen another series of troughs, B 1, B 2, B 3, is similarly arranged, terminating with a perforated zinc plate placed in a sloping position like the screen in the upper series. The upper troughs are 2 ft .4 in . wide and 13 in . deep; the lower ones are 2 ft . 5 in . wide and 6 in . deep.

At $c, d$, and $e$, Fig. 1977, grooves are placed, into which slips of wood are inserted, one above another, at different stages of the washing; these partially check the current of water, and assist in collecting the stones, pyrites, and so on. W, Fig. 1978, is the wagon-way, by which the coal is brought to the washer.

It will be observed that the pipe $\mathbf{P}$ is bent so that the water pours against the end of the trough; this has been found to act better than when the water is poured in the contrary direction. The mode of working is apparent; an operator shovels the coal into the trough, and another with a rake spreads it and keeps it under the action of the current. The water that passes through the screen carries much of the smaller coal with it; to obviate this the lower series of troughs were erected, along which the water is conducted, and finally passed over the perforated zinc plate. The velocity of the current is regulated, when required, by raising or lowering the troughs, their arrangement being such that this is easily effected. With this apparatus two operators can wash three wagons of coal an hour.

See Artestan Well. Boring and Blasting. Cage. Fuel. Haulage. Iron. Lamps, Safety. Ventilation.

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 papers by various authors in the 'Transactions of the North of England Inst. of Mining Engineers,' 'Inst. of Mechanical Engineers,' and the 'South Wales Inst. of Mining Engineers.'

COAST DEFENCE. Fr., Fortification Maritime; Ger., Küstenbefestigung; Ital., Difesa delle coste ; Span., Defensa de la costa.

Coast Defences.-The defences of coasts and harbours, especially those of an island like England, where if attacked the assault must be delivered more or less immediately from the sea, forms an important branch of the general technical art of fortification; with this considerable difference, however, that while the works of an inland fortress or strong place are generally constructed with a strict regard to certain rules laid down for the different systems of celebrated engineers, the acknowledged heads of their profession, the defences of a coast must in a great degree depend on local circumstances, and, if secure from an attack on the land side, do not require to be furnished with the appliances for a protracted siege. The struggle with an enemy afloat and unsupported with land forces would then be short, sharp, and decisive, the victory remaining to the hardest
hitter, the biggest gun, or, perhaps, to the craftiest-laid torpedo. In like manner, the proper placing and construction of coast defences emancipates, as it were, the officer of engineers from the more mechanical operations of permanent fortification, and, if he possesses genius, enables him to take advantage of every natural feature, both above and below the water-level, either for defence or offence, and make the most of them.

Their History.-Although in the very dawn of Jewish, Greek, and Roman history we read of the walls and other fortifications of towns, we do not find any mention made of impediments to prevent invaders landing on their coasts, except occasionally a free fight on the beach, such as the Greeks appear to have experienced when they reached the shores of Troy. When Julius Cesar landed on the Kentish strand, he had, it is true, to break througl a bulwark of half-naked blue-stained Britons, which his henvy-armed legionaries soon proved too strong for. William of Normandy met with considerable opposition when he disembarked as an invader; but he eventually conquered. In most cases we find the invaders securing themselves a safe haven or tête-de-pont for return in case the fortune of war should declare against them. Thus' the Greeks fortified their ships on the shores of Troy, and the Romans created the germs of the future fortifications and defences of Dover and Portsmouth, and perliaps the Thames. But in the first few centuries after the Conquest the constant wars between England and France exposed both nations to the visits of hostile fleets, and subjected the unfortunate inhabitants of our sea-board to much misery, especially on the south and east coasts. We can yet see in the Isle of Wight the existing outline of a town (Newtown) burned by the French fleet in the time of the second Edward, and which remains, with its houseless streets and market-places, a desolation to the present day. Even localities so remote as the Welsh coasts were not safe, as we read that in 1405 the French landed at Miilford Haven and plundered Carmarthen and other towns, safely regaining their ships; whiile as late as 1758 we retaliated by sacking the now impregnable Cherbourg, and levying a tribute on its Monts du Piété and other public institutions.

As the invention of artillery, however, greev in importance, the attention of our rulers seemed turned to the necessity, on certain points of our coasts at least, of securing the cities and towns situated near them from being burnt or plundered with impunity; and the sequestrations of the rich monasteries in Hampshire and the Isle of Wight furnished the eighth Henry with funds sufficient to erect the castles of Southsea, Hurst, Sandown, Yarmouth, and Calshot, as well as to fortify the narrow entrances into the harbour of Portsmouth, which was then, as now, considered the first naval arsenal of England. Besides these, we owe to Heury VIII. the defences of Falmoutl, Plymouth, Pembroke, Deal, and other important points on the coast. In Queen Elizabeth's time the defences of Milford Haven, the Thames, and Plymouth, as well as minor works on the Kentish and eastern shores, were not neglected by her shrewd and far-seeing ministers, and the many existing traces of the reign of the Stuarts in the old fortifications of Portsmouth, Devonport, Sheerness, Gravesend, and other places along the coasts, although erected more probably from dread of a Dutch than a French invasion, show that neither Charles nor James neglected the defence of England on its then weakest and most accessible points. During the reign of Anne and the first two Georges the constant dread of descents from the partisinis of the Pretender, or his French and Spanish allies, caused further extensions of our coast defences; and when the French fleet rode triumphant in the Mediterranean, after the fall of Minorea and execution of Byng, we deemed it necessary to strengthen the land defences of Portsmouth by the first fortifying of the old Hilsea lines. It is a curious thing that one of our ancient northern coast defences, namely, the Old Fort on the Bass Rock, was the last spot in Great Britain that held out for the Stuarts. The long wars of the French Revolution, although our fleets finally swept the sens of our enemies, still demanded constant attention to various exposed and unprotected points of our coast; but it was not till France again had a navy, and the defences of Cherbourg. Cronstadt, and Sebastopol created, as it were, a new era, and the due progress of things produced the Crimean and American wars, that the subject was really taken up in a comprehensive and extended manner; our eyes seemed suddenly opened to the utter worthlessness of many of our existing works, and an honest effort made to effect all the improved modes of fortification now absolutely necessary by the monstrous strides that modern science has made in the weapons both of offence and defence. The Report of the Commissioners appointed to consider the defences of the United Kingdom, dated the 7th February, 1860, entered fully into the details of their existing state, and gave their recommendations for their augmentation or renewal; and until the present time immense works, costing an aggregate sum of at least nine millions both in coast and inland defences to our harbours and arsenals, have been carried on, subject however to the disadvantage of the ever-changing increase in the weapons of attack and a corresponding increase ever fcreed on them in those of defence, causing works which might have been deemed perfect a few years ago to be now nearly useless or obsolete.

Reconnaissance of a Sea Frontier.-In addition to the general deseriptions necessary in makiug a reconnaissance, that of a sea-board line should clearly describe the nature of the coast; whether it is bordered by dunes or sand hills, or edged with flat rocks in shallow water which would render access to it more or less dangerous, or by cliffs which would render it altogether inaccessible ; the points on which descents could be easily made, the rentrant angles affording sheltered creeks and anchorages; the high lands and capes affording eligible sites for the erection of forts and batteries should be clearly pointed out, with the height of each above the ordinary high-water level, also all islets and rocks capable of being made available for advanced works, such as towers, and so on; the shoals, creeks, roads, and ports, with their soundings, the nature of the winds necessary for entering and sailing out of these ports, of which it will be requisite to note the local peculiarities; the state, garrison. and armament of the different batteries established for the defence of the anchoring grounds; the field-works, if any, formed on poiuts where descents might be practicable; the camps, fortified towns, and other strong places which cover or protect the principal maritime establishments or military arsenals; finally, to analyse
carefully the existing system of defence, and to consider the best mode of forcing it. If rivers have their mouths on these coasts it is necessary to consider very carefully the influence of the tides on their entrances. It is not less essential to indicate the hours of high water at all the principal points and ports, and the period of tide more or less favourable to the approach to the different points of disembarkation. To these general remarks, taken from the French 'Aide Mémoire,' we may add that in modern warfare it will be very necessary to take into consideration the various lines of railway by which forces from the interior may be suddenly concentrated so as to crush any inferior or isolated body of men who may seek a hostile landing on a coast, and also the facilities which may exist for the placing of obstructions either floating or hidden.

General Principles of Coast Defence.-Before entering into this subject it must be premised that, while we write, some of the most important points of coast defence are involved in much doubt, and must be considered as subject to important changes, according as the experiments on our guns, our ammunition, our armour, and our obstructions, fioating or otherwise, present from day to day new phases. We can therefore only offer a few useful general principles on the subject from the best modern authorities, simply reminding our readers that the great question of ships versus forts and iron versus earth yet remains to be solved, when the giants of the earth again do battle.

Von Wurmbs, in his 'Lehrbuch der Kriegs Baukunst,' remarks that the defence of a seasurrounded country, or an extended line of coast, requires in the first instance movable bodies of troops cantoned near the most important points, and acting in co-operation with divisions of the fleet, flotillas of gunboats, and war steamers. Well-arranged lines of railway and telegraph will be here of great importance. Fortresses are, however, necessary at the principal arsenals, dockyards, and other sources of supply, as well as at those points where the landing of a large body of an enemy's troops might take place, and so enable them to attack us on the land side.

The fortification of the principal important points of a coast line, such as our war havens of Portsmouth, Sheerness, Plymouth, Milford Haven, and Cork Harbour, also demand an enceinte on the land side capable of withstanding a sudden assault, and outworks which are strong enough to defy a regular attack, to resist the operations of a hostile fleet, and finally to keep the enemy at such a distance as to prevent a bombardment of the place and its establishments. This has been the principle adopted in the modern land defences of our naval arsenals, which are now surrounded with substantial forts as well towards the land as the sea, so as to hold an enemy at bay until our forces could be hurried up from all sides. On other points, coast batteries or forts of a minor magnitude become necessary to defend roads, havens, or anchorages, to protect the interests of commerce, or to hinder the landing of an enemy; due precautions should in all cases be taken to secure the rear of these batteries from surprise by proper gorge walls, and so ou.

As, nevertheless, fortresses cannot defend every point, or be placed close enough to hinder the passage of a powerful fleet, we must therefore in such cases take advantage of floating and hidden obstacles of every kind, such as torpedoes, floating batteries, booms, chains, rope nets, and so on.

Coast fortifications, it must always be considered, are opposed against the power of a most formidable enemy. In the present day we must consider the enormous and still increasing armaments, both in number and weight, of modern ships, and their armour creeping up from 4 in . to 15 in . in thickness. Even frigates which a fev years ago were seldom equipped with anything hearier than an 18 -pounder, now carry 12 -ton guns, discharging a $300-1 \mathrm{~b}$. bolt. Also, in a proper depth of water, ships can change their position easily and quickly, and by the help of steam can tack in the face of contrary winds and lay their broadsides full on against the opposing forts, unless these are situated high enough above the water-level.

But, on the other hand, many circumstances favour the defence of fortresses against ships when unsupported by land forces. The most formidable position of ships in attacking a coast defence is no doubt with broadside on, and, theoretically at least, as close as possible; but in taking up this position, if the ship, not of course a turret, allow itself to be raked by the fire of the fort, it is in the first place quite powerless to return a gun, and its sudden destruction may in many cases take place. If the wind suddenly changes, the keel touches a shoal, or a shot knocks the machinery out of gear so as to hinder its withdrawal from an enemy's fire, then it may be destroyed by the guns of the fortress, if they are heavy enough to penetrate her iron plating; for in modern warfare we must consider all fighting ships as thus protected. This fate very nearly met our fine 90 -gun line-of-battle ship the 'Albion' and 50 -gun frigate 'Arethusa,' by the fire of two or three guns only of the little Wasp and Telegraph batteries, on the heights of Sebastopol Harbour, on the 17th October, 1854. In modern times, many brilliant examples of this have been given. One large shet striking a line-of-battle ship near the water-line may sink her, or a shell bursting through her decks explode her magazines.

Fig. 1979 shows the defences of the entrance to Sebastopol Harbour, and position of the cliff batteries and attacking ships. The figures show the height in feet above high-water mark.

We may here remark the circumstance that coast batteries have generally a very large object to aim at,-a line-of-battle ship, for instance, from 200 to 300 ft . in length, and, with her sails and rigging, 60 to 80 ft . in height; while the ship may have no target to fire at but an open coast battery, covered with a strong earthen shield, and with the muzzles of its guns not rising more than $1 \frac{1}{2} \mathrm{ft}$. above the crest.

It is desirable to have the guns of a small battery as much as possible of the same calibre, if not of the same nature. It must be also considered that a ship changing its position at the rate of 8 knots an hour moves 600 yds . in $2 \frac{1}{2}$ minutes; therefore to meet this, not a single gun, but a number on each battery, will be found necessary and most efficient to injure a passing fleet.

Previous to 1861 not less than twenty-three varieties of guns were mounted on our coast batteries, after which period the number was restricted to five varieties, namely, the $10-\mathrm{in}$. gun of 86 cwt., 68 -pounder of 95 cwt ., 8 -in. of 65 cwt ., 32 -pounder of 56 cwt , and 13 -in. mortars ; but with
the late imagined improvements in artillery our batteries will be soon generally armed with the 12 -ton 300 -pounder, and on important points with 25 -ton 600 -pounders.

Batteries for mortars, flanked, however, by a few heavy guns, are requisite on certain important points. That of Puckpool, commanding one of the eastern entrances to Spithead, is a formidable battery of this artillery arm, mounting thirty 13 -in. mortars.

Each description of gun should have a separate expense magazine for its ammunition.

Low barbette batteries should never be used when the work is at less elevation than 100 ft . above the sea-level, and then only they are safe from enfilade.

Barbette batteries have the advantage of a more extended range of fire than those with embrasures.

If a battery be less elevated, the parapet must be heightened, and the gun then fires through an embrasure, or it must be raised, when great range is required, to fire over a parapet of 7 ft . protected by bonnettes.

In earthen batteries the distanco between the guns must not be less than 35 ft . when mounted on traversing platforms, and traverses should be provided for every two guns. In casemated and iron-plated batteries the guns are generally spaced 24 ft . from centre to centre, $=$ to a 12 -ft. pier and a 12 -ft. shield.

Casemated batteries are necessary when it is required to place a work close to deep water, and more than one tier of guns is necessary to defend the
 position.

All coast batteries, whether casemated or not, should be retired as much as possible from deep water, unless by doing so the object of the battery might not be attained.

We in many cases utilize our casemates both in coast and land batteries by using them as barracks for the garrison, but all partitions and rear walls should be made movable and as slight as possible. Continental coast batteries, when casemated, are generally open in the rear; in double-tier casemated batteries the magazines and stores are usually placed in the basement story, which is faced with solid walls of great strength, and covered with bomb-proof arches.

It is almost useless to discuss here the question of embrasures. Modern science, however, plainly points to the adoption of wrought-iron shields both for casemated and earthen batteries when the Moncrieff system of gun-carriage is not used. The thickness of the shields may vary according to exposure from 9 to 15 in ., with the aperture for the gun's mouth as small as practicable.

Red-hot shot or shells filled with molten iron requiring a supply of furnaces and fuel to heat same, were until lately part of the equipment of every coast battery of importance. The universal adoption of iron-clad vessels has however diminished very much the importance of this mode of defence, and our best modern coast forts have no appliances of any kind for heating shot or melting iron except in very peculiar circumstances; an enemy, however, attempting a landing with a flotilla of wooden boats, might suffer severely from the use of heated shot.

The following additional observations are principally founded on Sir J. Burgoyne's 'Notes on the Coast Defences of Great Britain.'

In all arched vaults, such as casemates liable to be battered, the covering arches should be brought endways to the front, so as to form a revetment en décharge.

Batteries or brigades of guns to be brought up with troops to parts of a coast threatened, will be worthy of being organized in time of war in central secure stations.

The most effective fire against ships is doubtless from guns placed but little above the level of the water, but so placed they would be subjected to an overwhelming fire from ships in return.

In proportion as batteries are elevated they lose in the best effect of their own fire, but become far less exposed to suffer from that of ships.

There is also from elevated batteries, if close to the water, a certain space near them which they cannot command at all, and where vessels and boats would be consequently safe from them.

Sir J. Burgoyne considers an elevation of not less than 50 ft . as a good medium elevation : at that height the shot may be made to strike the water at a distance of 200 yds. , and will ricochet well, and at the same time the guns would be but little exposed to direct fire from shipping.

Rockets may be of great service in close quarters with ships. With reference to the exposure of guns and gunners, he remarks that in military operations the first aim is to obtain success;
to save life is a secondary consideration, excepting so far as may not impede the first, when of course it becomes an anxious and imperative duty.

He recommends the construction of packets and mercantile steamboats to carry guns in case of need, and the organization of the population of the coast, the coastguard and dockyard men to man our batteries.

The experiences taught in the last American war have thrown quite a new light on what we, a few years ago, learned in the Baltic and the Black Seas. Von Scheliha, in his work on 'Coast Defences,' lays down some general rules, which we here give, being worthy of consideration, and supporting his propositions by examples of different conflicts on the coasts and rivers of America, premising that he considers the Confederates in the first place committed a great mistake in attempting to fortify so many points as they did both on their coasts and rivers.
$a$. That railway communication with different points is preferable to fortifying places of secondary importance.
$b$. That exposed masonry or brickwork is not capable of withstanding the effect of modern artillery.
c. That earth, especially sand works properly constructed constitute a better protection against modern artillery than permanent fortifications built on the old plan.
d. That guns mounted en barbette, even when protected by properly-built traverses, may be silenced by a concentrated fire from ships.
$e$. No fort now built can keep out a large fleet unless the channel be obstructed.
$f$. A mere partial obstruction of a channel not sufficient to keep out a large fleet.
g. No fleet can force a passage if kept under the fire of heavy batteries by properly-placed obstructions.

These views of Von Scheliha are on the whole satisfactory to us with a few exceptions, when we consider the vast outlay we have made within the last few years on our coast defences. With torpedoes and other hidden obstructions, backed by the fire of our modern forts, no fleet could possibly pass narrow channels like that between Hurst Castle and Sconce Point, or the winding sinuosities of deep water between the Spithead Forts, the Thames, Medway, entrance to Plymouth Sound, and other similar localities leading to our principal arsenals and dockyards, as well as the river entrances to our great commercial emporiums, as Liverpool, Hull, Bristol, \&c.

The rast alterations in the armaments even of our older defences have rendered a great number of additions to our magazine and store accommodation necessary; increased provision must be made for shells, apparatus for large guns, and so on. In the larger earthen coast batteries the magazines are generally in the rear of the work (which should have its gorge closed by a defensible loop-holed wall), and constructed with bomb-proof arched brickwork, with a covering of earth varying from 4 to 6 ft .; smaller expense magazines are formed in the various traverses, and other secure places in the earth-works. There is considerable difficulty in this climate in preserving these magazines free from damp, principally arising from condensation of saturated air on the walls. If deeply seated in or covered with earth, the ventilation is often defective, and where wooden floors are used considerable ravages have been made by dry rot, the warm and damp air within the magazines appearing to favour the growth of fungi in the timber. Floors of asphalte laid pure without grit are now used in magazines with adrantage, and the application of steam in drying the internal air of magazines is a subject worthy of consideration.

The viems of Rear-Admiral Porter, U.S.N., on coast and river defences, as enumerated in his report, 1865, are these, namely :-
a. Too much importance cannot be attached to the employment of torpedves and other hidden obstacles.
b. Considers that no forts can withstand the united fire of monitors and iron-clad vessels.
c. Recommends the combination of military with naval attacks. These formed an important feature too much overlooked, perhaps, in the attack and defence of coast forts during the American war, as at Charlestown Harbour, Vicksburg, \&c.
d. Little difficulty occurs in running past the strongest forts if no obstructions natural or artificial exist, but on forts only no safe reliance can be placed.
$e$. He knows no instance in which troops and ships properly combined have attacked a fort but they have taken it.
$f$. One gun on shore being considered equal to ten afloat is a principle not at all to be depended on.
g. Even if gums in a land battery are protected by traverses they are not safe from vertical fire.
$h$. It must, however, be allowed that there are certain points on every coast where forts can be built unassailable by ships, and that Federal Point, Wilmington River, where Fort Fisher stands, is one of them; also if the engineer who built the latter had placed it one mile inside of where it now stands it would have been inaccessible both by sea and land.
i. Guns placed in a land battery should not be less than from 60 to 100 ft . apart.
$j$. Our frowning stone walls with their guns standing en barbette, or looking through small ports in the casemates, look strong, while both arrangements are serious defects.
k. Latterly the rebels revetted the stone walls of their forts, external as well as internal, with sand bags.
l. Fort Fisher was the key to all the immense system of defences to Wilmington Harbour, and will furnish food for study to our military engineers for years to come; and in many cases the system inaugurated by the rebels will be adopted by us.
$m$. Never erect a fort without consulting the hydrography of its approaches. In Wilmington River a chain of the strongest works ever erected was lost because Fort Fisher, the key to them all, was placed within reach of ships.
$n$. The shells of navy guns most effective against land batteries.
o. He recommends that all our works shall be of earthwork, or that those now built should be covered with earth; the guns to be mounted on monitor turrets; iron to any thickness and guns of any measure may be thus used, and so far have an advantage against a floating enemy, in which the use must be limited.
$p$. The value of land fortifications is, he considers, not in the least diminished by the late results; their importance is greater than ever, but they must be properly built.

Fig. 1980 shows the position of Fort Fisher at entrance to Wilmington River.

Fig. 1981, plan of the defences of Fort Fisher.

Classification of Coast Defences.-The sea defences of our naval arsenals, commercial harbours, and most accessible landing points on our coasts, may be classified as follows :-

1. Existing old castles erected originally in the times of the Tudors and Stuarts, many of them still forming the nuclei of our present works.
2. Coast towers, martello towers, \&c., of more modern construction, dating principally from the period of the earlier wars of the French Revolution.
3. Simple earthen batteries firing through embrasures or en barbette, in many cases open or undefended at gorge.
4. Earthen batteries of improved formation, with closed defensible gorge, provided with iron shields to embrasures, and permanent accommodation for gunners, magazines, \&c.
5. Double or single tier casemated composite batteries, with granite facings, iron shields, and defensible keeps and other works at rear.
6. Iron-plated sea forts, in many cases isolated, but covered with iron wherever exposed to shot from direct fire of floating armaments.


In this classification we have not included numerous coast forts similar to class No. 5, constructed on the old principles of unprotected embrasures, with walls of brick or similar materials, in some cases faced insufficiently with stone. Such is Fort.Victoria and Cliff End Fort, on the right-hand side of the Solent passage, the batteries at the entrance to Portsmouth Harbour, and nunerous other old though costly forts of the same construction, which must be considered to all intents and purposes obsolete, and should be reconstructed sooner or later, in one of the classes we have given above. This applies to defences constructed by Russia, France, America, Spain,

Holland, and other Powers, as well as by our own Government, during the last couple of centuries, when the principles of Vauban and Coehorn were perhaps more considered in theory than the advantages of site, local marine difficulties, \&c., which to-day afford room for the display of the peculiar talents in coast defence by our modern officers of engineers.

Class 1.-Previous to Henry VIII.'s time we had, on many points of our coast, castles in the feudal style, such as at Dover, Scarborough, Bamborough in Northumberland, and many others, erected, however, before the invention of artillery. Many of them were allowed to fall to decay when that important element began to come into operation, and the earliest improved form of coast defence appears in the many castles erected by Henry VIII. in dread of an invasion by the Catholic Powers of Europe after his quarrel with the Pope. They consisted generally of a central keep, with curved casemated batteries at a lower elevation, and were constructed of strong and substantial masonry or brickwork, available for store and barrack purposes to the present day. The castles of Southsea, Hurst, Yarmouth, and those of Falmouth and Dartmouth Harbours, Deal, and Walmer, are of this period.

Fig. 1982 shows Southsea Castle as originally built.


Scale, $12 \frac{12}{}{ }^{\prime \prime}$ to a mile.


A, Keep. B, B, Batteries. D, D, Ditch.

Fig. 1983 shows Hurst Castle, once the prison of the first Charles. It is now a place of much strength, large casemated granite batteries, same as class No. 5 , mounting sixty-one heavy guns, being erected in front and on each flank of the original castle, and showing that the engineers of Henry VIII. had a good eye for a site for defence. Besides these heavy guns, it will probably be eventually equipped with an armament of three turrets on the upper platform, mounting six additional guns, certainly not less than 600 -pounders each.

Fig. 1984 shows the modern extensions to the old Hurst Castle.


Scale, 48 ft . to an in.
Fig. 1985 shows the old castle of Yarmouth, still mounting half a dozen light guns, and affording accommodation for a few stores and gunners. This is a curious relic of mediæval times, although as a work of defence it is now obsolete and rendered useless by the more modern works in its immediate neighbourhood at Fort Victoria and Cliff End.

The old works at Dover and Sheerness of this period are quite absorbed by the modern works of defence surrounding them. The still-existing important coast work at Landguard Point, mounting twenty heavy guns, has been a good deal modernized, but dates from the time of

James I. The old defences of the sea entrance to Portsmouth and Plymouth Harbours are principally of the Stuart period.

Fig. 1986 shows a bird's-eye view of the defences of Plymouth, land and sea, anno 1643.


Class 2.-In the early part of the revolutionary war the damage inflicted on one of our ships of the line and a frigate by a single-gun tower in Martello Bay, in Corsica, caused rather an exaggerated idea to be formed of its power of defence, and numbers of them were erected on our southern and eastern coasts, as well as on the weakest points of the Channel Islands, on the coast of Ireland, in the vicinity of Dublin, Cork, and Lough Swilly, \&c. Rear-Admiral Porter, in 'Von Scheliha,' reports a similar case where a one-gun, 24 -pounder tower, cut up and repulsed a 20 -gun frigate that he served in.

Sir J. Burgoyne considers towers may be usefully applied in detached positions, and no doubt, if properly covered by an earthen envelope, would still be found of advantage in defending our coasts in minor operations, or in groups of four to six opposing the landing of an enemy during a period of war, on many points of the 300 miles of shore accessible for landing troops of the 800 miles extending from the Humber to Penzance.

Martello towers were nearly all built on the same model, in England of brick, and in Ireland and the Channel Islands of granite. They generally mount a $24-$ pounder gun on top, are loopholed more or less, and accommodate a garrison of from six to twelve gunners. In some cases they are surrounded by a ditch with small bridge, and covered by a glacis; in other cases are merely accessible by a ladder, movable and defended by a machicoulis. Few of those towers are now equipped, and they are generally occupied by a few coastguard men.

Figs. 1987 to 1989, from 'Von Wurmbs,' show the plans and section of a martello tower, the stores and ammunition being kept in the basement. The outer walls are generally from 6 to 8 ft . in thickness.

Figs. 1990 to 1992 show one of the series of towers of the same nature approved by the first Napoleon for coast defence purposes. They are generally square instead of round, and accommodate a larger garrison and mount more guns. They were intended for the same purposes as our smaller towers. On many land frontiers where a small garrison has to hold out against numerous assailants unprovided with heavy guns, such as those of New Zealand and the Cape, these towers would be found very useful, and on the frontiers of the latter colony several of them have been erected. They should, however, be carefully furnished in this case with tanks or wells.

Class 3.-Earthen Batteries undefended at Rear or open at Gorge.-On many points of the coast elevated from 100 to 300 ft , above high-water mark these batteries will be found economical in construction, and effective. Our experience in the Crimea, which we have already alluded to, where, on the 17th October, 1854, the Allied fleets attacked the outer defences of Sebastopol


Plan of upper floor.


Plan or vaseement.
1989.

10
20 $\qquad$ 3015

Se tion on line $\Lambda B$.


Harbour, and the warm reception our heavy-armed ships met from the guns of the little Wasp and Telegraph cliff batteries, which inflicted more damage on them than all the guns of the granite casemated forts put together, will render even iron-clads cautious how they approach those elevated batteries, capable as they are now of being armed with guns of the heaviest calibre. Batteries of this nature in time of war would be supported and supplied by magazines and depôts situated inland, and easily accessible by tram-roads or otherwise.

Earthen batteries may, at an emergency, be easily thrown up, and by the use of strong timberframing and earth even mount a second tier of guns. Many of the Confederate batteries during the late American war were of this construction; but the materials they are composed of should be always at hand, and therefore they are inapplicable on rocky cliffs, shingly beaches, and similar sites.

Fig. 1993 shows a cheap earthen battery open at rear for seven guns of any calibre up to 300 lbs ., the cost of which with bomb-proof magazines, traverses, \&c., will not exceed S00l. a gan. It is similar to a very excellent battery lately constructed on Hatherwood Cliff, near the Needles, in the Isle of Wight.


Figs. 1994, 1995, show sections of an earthen battery covering a strong wood framing, forming casemate below, similar to many erected by the Confederates during the American war.


Longitudinal Section. Scale, 24 ft , to an in.
Figs. 1996, 1997, plan and section of Fort Powell, in Mobile Harbour, a combination of earthwork and timber.

Class 4.-Earthen Batteries, with Accommodation in Rear for Troops and Stores, \&c., with closed Loopholed Gorge and enfladed Ditches in Front and on Flanks.-Mounted with heavy guns, batteries of this nature are a most useful element of defence for our commercial harbours, and the rivers leading to our principal maritime emporiums, \&c. They are constructed generally on the most commanding positions and have ample acco mmodation, in low flat-roofed buildings, for their garrison
in the rear, and well-protected bomb-proofs for magazines and stores, wells and water-tanks, so as to be enabled to resist a coup-de-main, if the enemy should land a force in their neighbourhood. The guns, which may be of the heaviest calibre, fire either en barbette or through earthen embrasures protected with iron shields. The ditch in front and on flanks is often further defended by a loop-holed Carnot-wall and flanking caponnière chambers. Of this class are batteries lately erected to protect the channel of the Humber, the entrance to the Orwell and Stour rivers near Harwich, the new batteries on each flank of Southsea Castle, and various other points on the coast. Moncrieff gun-pits will be equally applicable to batteries of this construction as the last; ample shelter for gunners is provided in the interior of traverses, passages to magazines, \&c. These batteries in connection with floating or hidden obstructions would make the approach to our principal maritime towns, situated a fer miles inland, nearly impossible; the gunners also enjoying good quarters, not subject to the inconvenience or dangers of
 casemated batteries.
1997.


Section on line AB.
In the construction of batteries of this description the immensely increased penetrating power of modern artillery on earth-works must be taken into consideration. The penetration of a $600-\mathrm{lb}$. shot fired from a $13-\mathrm{in}$. gun at 200 yards is estimated at 50 ft ., and a $150-1 \mathrm{~b} .9-\mathrm{in}$. shot at the same distance, 30 ft .

Fig. 1998 shows Southsea Castle, with auxiliary earthen batteries and defensible gorge-wall in rear.

Figs. 1999 to 2001 show a 2-gun battery similar to those on the Humber and near Harwich, with Carnot-wall in ditch, soldiers' quarters, and stores in rear. The unit for such batteries may be considered the distance from centre to centre of each gun mounted, or generally from 35 to 40 ft . American engineers consider this distance ought to be enlarged.

Earthen batteries are sometimes armed with mortars which will make, where they are mounted, a different arrangement necessary.

Figs. 2002, 2003, show a unit of a mortar battery containing platforms of from four to six


Scale, $6^{\prime \prime}$ to a mile.
A, Old Central Work. B, New West Battery.
C, New East Battery mortars in a group. These may be placed in the most eligible positions for vertical fire, and the remainder of the battery armed with heavy guns.

Class 5.-Single or Double Tier Casemated Batteries, faced with Granite and provided with Iron Shields to Embrasures.-When the Defence Commission of 1860 made their report, granite-faced casemated batteries, from the experience the then unarmoured ships had received in the Baltic and the Crimea, were in high favour; it was then a question of granite versus oak, not as now granite versus iron; and in the representations in the pictorial papers of the day, the granite forts


Plan. Scale, 300 ft . to an in.
References:- $a, a$, Glacis. $b, b$, Slope of ditch. $c, c$, Ditch. $d, d$, Carnot-wall. e, e, Banquette. $f$, Caponnière. $g, g$, Traverses. $h, h$, Expense magazine. $i, i$, Embrasures. $k$, Gun en barbette. $i$, Terreplein. $m$, Parade. $n$, Magazine. $\quad o$, Quarters in rear. $\quad p$, Entrance. $q, q$, Defensible gorge-wall.


Section on line AB.
2001.


Unit showing two guns and traverses. Scale, 100 ft . to an in.

2003.


Section on line $A^{\prime} B^{\prime}$. Figures show height above high water.
of Cronstadt and Sebastopol presented a most threatening and formidable appearance. Those of Cronstadt had defied our attacks, because, as military critics say, they were never made; and in the little tussle between the Allied fleets and the Sebastopol batteries it must be confessed that we stood in considerable awe of them, and kept, with one or two exceptions, at a most respectful distance, and when we did approach nearer got the worst of it. The destruction of the Bomarsund defences would naturally be attributed to their unfinished state, as well as to the co-operation of a land attack; therefore the Defence Commission were rather inclined to look with favour on granite walls and embrasures, even when unprovided with iron shields or platings; and we find our most important and expensive works were proposed to be executed in that material, and,
with the (in the course of time) necessary alterations and additions for iron shields to embrasures, have been so constructed. We may instance-the Garrison Point and Isle of Grain defences, near Sheerness; the Thames and Medway forts; the important position of Gillkicker and Hurst Castle in the Spithead and Solent defences of Portsmouth Harbour; many works also in the Plymouth and Milford Haven defences. But more recent experience has shown us, in batteries at the waterlevel, or only moderately elevated above the same, even when furnished with iron shields to embrasures, that solid stone piers offer but a poor resistance to the heary guns of modern warfare. Unfortunately, many of our forts were already thus built before the experience of the late American war was acquired; and to plate them with iron now would cost nearly as much as their original construction; and, indeed, in many cases when they are retired from the water's edge would not be necessary. It is, however, probable that for the future granite will enter much less into our coast defences as an element of resistance than it has hitherto done. An inspection of the diagram we give, of a granite-faced iron-embrasured coast battery, Figs. 2014, 2015, shows a rather defective construction, for if the intermediate piers be battered down, the abutments of the groined arches necessary for the movement of the guns within will be destroyed, and the whole of the bomb-proof masonry and guns on terre-plein above will become a heap of ruins. The attention of our engineers is now directed to a simpler mode of construction, in which wrought-iron girders and iron-buckled plates covered with concrete and earth will probably supersede brick arches altogether, unless in cases where the end of the arch itself goes through the external walls, and direct fire will not affect its stability. Batteries of this nature are composed of units, the unit being a casemate for one gun; a battery of any number of guns therefore consists of as many units arranged in as many tiers as may seem desirable. The store and magazine accommodation is generally in the basement, where the external walls are solid, and of great thickness, and it will be necessary to provide for at least 200 rounds of shot and shell for each gun, besides all necessary tramways, lifts, \&c., for the easy conveyance of the heavy missiles we now use, and the fall of one of which might endanger the safety of the whole battery. These magazines are lighted by oil lamps, with thick plate-glass fronts and reflectors, and fixed through apertures in the external walls. In many cases during the late American war it was found necessary to revet stone-faced batteries with sand-bags.

Figs. 2004 to 2006 are a plan and sections of Fort Sumter, which after a long resistance was finally destroyed by the Federal American fleet. It is not a favourable example of a casemated battery, as the casemates are small and confined, and the thickness of the walls appears insufficient.


Figs. 2007, 2008, show plan and sections of the unit of a single-tier casemated battery. Turret guns or heavy guns en barbette may be mounted on top. In this case the magazines are detached and not in basement.

Figs. 2009 to 2011 are a coast battery, with a small keep in rear, mounting thirteen guns in casemates and eight heavy rifled guns on terre-plein, from Von Wurmbs' 'Lehrbuch der Kriegs Baukunst.'

Figs. 2012, 2013, show two casemates of the old Blockhouse Battery at the entrance to Portsmouth Harbour, and section through same. The limited dimensions of the casemates are a great contrast to those of the present day, alınough with the old guns this battery was considered a most efficient one.

Figs. 2014, 2015, show an example of one of the most approved single or double tier casemated battery, with magazine and stores beneath, and casemates fitted to accommodate troops, with verandah behind. Before the adoption of iron external plating, these batteries were probably the strongest construction used. Their cost, including the shields, buildings, and wall in rear, would be about $3500 l$. a gun. This construction is applicable to any portion of a curved surface, or to the entire circle.

Many of the batteries of the Russian arsenals, and those of France and the United States, are formed on this principle, which we are beginning reluctantly to consider obsolete. Russia and other countries have already commencel plating their granite batteries with iron. We fear,

2011.

Section on line AB.


Scale, 48 ft . to an in.
however, that total reconstruction will be in many cases found necessary, as it was with our men-of-war a few years ago.

Class 6.-Iron-clad isolated Forts plated on Stone Walls, or independent Iron Constructions.-There are in progress now the stone bases of four marine forts, 200 ft . diameter, at Spithead, and one of an elliptical shape, but nearly of the same mean dimensions, on the Shovel Rock, behind the Plymouth Breakwater. Recent experiments at Shoeburyness would seem to point out that the resisting face or iron construction of these forts should be quite unconnected with, and independent of, the internal arrangements of masonry or brickwork, forming, as it were, an external shell round same. The plating of granite walls with iron plates, connected by heavy bolts and screws, we have always considered to be a mistake, as the elasticity and continuity of material which might permit it on the wooden sides of ships are much wanting either on walls of granite or brick. The Russians are attempting a sort of plating on their Cronstadt forts; but practical men doubt of its ever being effective, and the expense of plating walls with railway iron and such small material in the American war led to no satisfactory results when opposed to heavy guns. The internal arrangements of forts of this nature will differ little from those of the casemated granite forts shown in Figs. 2014, 2015.


Plan. Scale, 80 ft . to an in.
References :- $a, a$, Powder lifts. $b, b$, Shell lifts. $c, c$, Light boxes. $d, d$, Iron shields. $e, e$, Ventilating and light apertures. M, Stone piers. P, M, Powder magazines. S, S, Shot and shell store. L, Laboratory. N, Terre-plein.
Sir J. F. Burgoyne, in his letters to the Secretary of State with reference to the defence works, dated 20th February, 1867, observes:-"The circumstances under which it is nost requisite to apply iron defences on the fullest scale are where batteries are necessarily placed in very exposed and isolated situations, and where the area of the site on which guns can be placed is unaroidably restricted; such as in the sea or in a very few analogous positions on shore, where foundations are costly and where the works are subject to a concentrated fire from a large number of powerful ironplated men-of-war.
"On the shore where the space is not limitcd and where guns are more or less dispersed, or in channels where there is no probability of ships being able to


Section on line A B. Scale, 32 ft . to an in. engage batteries for a lengthened period, it is only requisite to apply iron in the form of shields for strengthening that part of the work about the embrasures, which in batteries of an earthen or granite construction would otherwise be very weak. In cases where batteries are at a considerable height above the sea, no embrasures are necessary, and iron shields are not required."

General Notes on the Practical Construction of Coast Batteries.-When stone is used in the facing of batteries it should be of the hardest and most compact nature, in as large blocks as possible, and set on its natural bed; even in granites this should be regarded, although they are often erroneously supposed to have none. If intended to be afterwards iron-plated, all the necessary holes for bolts, chases, sinkings, \&c., should be previously made: Granite or other stone facing may be backed with brickwork, and the same may be used in all revetments, walls of magazines, \&c., not exposed to direct fire.

The greatest care should be taken to ensure ventilation to all floors and other woodwork in earth-covered buildings.

In throwing up earthen batteries of any size, the engineer should study the geological peculiarities of the site, nature of the formations, \&c. Many serious failures in our earth-works have occurred by the neglect of this precaution, especially near the junction of the chalk and tertiary formations, and in the various drift deposits on our southern coasts.

When earth-works are to be thrown up, the proper drainage of the natural surface beneath them should be considered, so as to prevent the accumulation of water in the earth formed as a rampart. This should be a preliminary step, and is too often overlooked, causing constant slips in the slopes formed.

Earth-works should be formed in layers not more than 15 in . in thickness, and well rammed. Occasional thin beds of burnt ballast or shingle should be used, and agricultural drain-pipes so laid as to conduct off all superfluous water as quickly as possible.

Slopes steeper than $45^{\circ}$ can hardly be formed without the employment of built sodwork, which experience proves is very liable to decay. Flat slopes may be either covered with flat sods or soiled with fine mould and sown with selected grass seeds, with roots calculated to form a fibrous network.

Conduct all rainfall off your works as rapidly as possible, by means of surface, pipe, or other drainage.

Give your guns plenty of room, and let their platforms or racer-blocks be firmly bedded.
If iron plating be used, let its form be simple and with as few bolts as possible, and let all ironwork of embrasures be painted the same tint as the surrounding material, whether grasswork or granite, so as not to present a distinguishing mark at a distance.

Earth-works should be formed as free from shingle or loose stone on the external covering as possible, although in some cases a core of such material may be found convenient to use; one of chalk forms an excellent nucleus for a battery.

The comparative cost of earth, concrete, granite, and iron, in relation to their comparative strength, may be estimated, taking earth as the unit, as $1,3,10,50$. Concrete costs 10 times as much as earth, but offers 3 times the resistance; granite 100 times as much, but 10 times the resistance ; and iron 40 times as much, but from 2000 to 3000 times the resistance.

Every battery, no matter how small, should be provided with a well or tank for the supply of water.

The position of the flag-staff of a coast battery should be such as to draw off the fire of an enemy to a point where it will do as little harm as possible, as it is often the only target for an enemy to fire at.

Let all artificial lights in magazines, \&e., be so placed that no concussion can displace them.
No ventilating shaft or passage should be made which could admit of the direct entrance of any missile into a magazine or passage of same.

And finally, gunners should be made to feel perfect confidence in the stability of the batteries they are appointed to defend, and in the sufficiencr of shelter and means of supplying ammunition to the guns, and cover for the wounded.

Table I.-Showing probable Maximum Penetration of Projectiles in various Materials. (Appendix to Report of Committee on Fortifications, 1868.)

| Nature of Gun. | Maximum Penetration in |  |  |  |  |  | Remarks. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Earth. | Concrete. | Brickwork. | Rubble Masonry. | Granite. (a) | $\begin{array}{\|c} \text { Iron } \\ \text { Plating.(b) } \end{array}$ |  |
| 12-in. .. | ${ }_{50}^{\text {ft. }}$ | ${ }_{15}^{\text {ft. }}$ | ${ }_{15}{ }^{\text {ft. }}$ | ft 10 | ft. | 15 | (a) The destructive |
| 10 „, .. .. | 45 | 14 | 14 | 9 |  | 14 | effect of a shot in frac- |
| 9 ", ... | 40 | 12 | 12 | 8 | 2 | 11 | turing granite, extends |
| 8 ", .. .. | 35 | 11 | 11 | 7 | .. | 10 | far beyond the limits of |
| 7 ", .. .. | 30 | 9 | 9 | 6 | .. | 9 | actual penetration. |
| 110-pounder .. | 28 | 8 | 8 | 5 | .. | 6 | (b) The thickness |
| 70 " | 24 | 7 | 7 | 4 | .. | 5 | given are those of single |
| 40 " | 16 | 5 | 5 | 3 | .. | $4 \frac{1}{2}$ | plates, which are just |
| 20 " ${ }^{20}$ | 12 | 4 | 4 | 2 | $\cdots$ | 4 | capable of keeping out |
| 12 " .. | 5 | 2 | 2 | 1 | . | 3 | a shot. |

Table II.-Showing probable Maximum Penetration of Spherical Mortar-Shells impinging with the greatest Velocity which they can acquire in Air.

| Nature of Shell. | Maximum Penetration in |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Earth. | Concrete. | Brickwork. | Rubble Masonry. |
| 13-in. 10 8 | $\begin{array}{ll}\text { ft. } & \text { in. } \\ 6 & 0 \\ 4 & 0 \\ 3 & 0\end{array}$ | $\begin{array}{cr}\text { ft. } & \text { in. } \\ 1 & 6 \\ 1 & 0 \\ 0 & 8\end{array}$ | $\begin{array}{cc}\text { ft. } & \text { in. } \\ 1 & 6 \\ 1 & 0 \\ 0 & 8\end{array}$ | in. 8 7 6 |

Table III.-Showing Covering for Bomb-proof Constructions, Semiclrcular Arches, and Arches of $120^{\circ}$. (Report, 1869.)

| Span. | Covernig for Bomb-proof Casemate. |  |  |  |  |  | Remarks, |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Semicircular Arches. |  |  | Arches of $120^{\circ}$. |  |  |  |
|  | Biock Stone. | Brick. | Rubble Stone. | Block Stone. | Brick. | Rubble Stone. |  |
| ft. | ft. in. | No. | ft. in. | ft. in. | No. | ft. in. |  |
| 5 | $0 \quad 9$ |  | 011 | 010 |  | 10 | The thicknesses given in |
| 6 | 010 | 1 | 10 | 011 | $1 \frac{1}{2}$ | 12 | this Table for arches are |
| 8 | 10 | $1 \frac{1}{2}$ | 12 | $1{ }^{1} 2$ | $1 \frac{1}{2}$ | 14 | not in themselves adequate |
| 10 | 12 | 11 | 14 | 13 | 11 $\frac{1}{2}$ | 16 | to resist the concussion |
| 12 | 13 | 112 | 16 | $14 \frac{1}{2}$ | 2 | 18 | of the fall or explosion of |
| 14 | 14 | $1 \frac{1}{2}$ | 18 | 16 | 2 | 110 | the charge of $13-\mathrm{in}$. shells. |
| 16 | 15 | 2 | 19 | 17 | $2 \frac{1}{2}$ | 21 | They are calculated in all |
| 18 | 16 | 2 | 110 | 18 | $2 \frac{1}{2}$ | 22 | cases to be protected by a |
| 20 | 17 | $2 \frac{1}{2}$ | 111 | 19 | 3 | 24 | further covering of from 2 |
| 22 |  | $3 \frac{1}{2}$ | 22 | 110 | $3 \frac{1}{2}$ | 26 | to 3 ft . of concrete, or from |
| 24 | 19 | $3 \frac{1}{2}$ |  |  | 4 | 30 | 4 to 6 ft . of earth. |

Table IV.-Showing Weight and Energy of Guns and Projectiles.

| Description of Gun. | Weight of |  |  | $\begin{gathered} \text { Initial } \\ \text { Velocity with } \\ \text { Highest } \\ \text { CCarge. } \end{gathered}$Unarge. | Energy or Punching Power per Inch of Shot's circumference |  | Total Energy of Projectile at 1000 yds . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Gun. | Projectile. | Highest Charge. |  | $\stackrel{\text { At }}{\text { Muzz.le. }}$ | $\begin{gathered} \text { At } \\ 1000 \mathrm{yds} . \end{gathered}$ |  |
| Cast-iron Smooth Bore. | tons. cwt. | lbs. | 1 lbs. | ft . | foot-tons. | foot-tons. | foot-tons. |
| 32-pounder .. .. .. | 218 | 32 | 10 | 1690 | for-tos. | .. | .. |
| 8 -in. shell .. .. .. | $3 \quad 5$ | $49 \frac{1}{2}$ | 10 | 1488 |  |  |  |
| 68 -pounder .. .. .. | $4 \quad 15$ | 68 | 16 | 1579 | 46 | 18 | 452 |
| Wrought-iron Muzzle-loading Rifles. |  |  |  |  |  |  |  |
| 7-in. .. .. .. | $6 \quad 10$ | 115 | 22 | 1430 | 75 | 52 | 1143 |
| 8 , .. .. .. .. | 90 | 180 | 30 | 1330 | 88 | 66 | 1659 |
| 9 ", .. .. .. .. | 120 | 250 | 43 | 1340 | 111 | 85 | 2403 |
| 10 " .. .. .. .. | 180 | 400 | 60 | 1290 | 148 | 123 | 3863 |
| 11 " .. .. .. .. | 250 | 600 | 70 | 1212 | 163 | 137 | 5165 |
| 12 " .. .. .. .. | $30 \quad 19$ | 600 | 100 | .. | .. | .. | .. |

The following statements will show the financial position of our defence works from the period of the first report of the committee until 1869 .

Table A.-Original Estimate of the Defence Comimssion of 1860.


When the report of the commission was considered by Government, it was determined to raise the money for works and lands by loan, and to provide for the armaments and floating defences in the annual estimates.

Omitting, therefore, these last two sums ( $1,500,000 l$.), the commissioners' estimate may be divided thus:-

$$
\begin{aligned}
& \text { For completion of works authorized previous to } 1860 \text {.. .. ... } 1,610,000 \\
& \text { For new works recommended by the commission, including }\} \\
& \text { 8,740,000 } \\
& \text { Total .. .. .. .. .. .. .. £10,350,000 }
\end{aligned}
$$

This amount was, however, in the first instance reduced by Government, by the omission of several important items, namely:-


In proposing, however, the first vote to Parliament, further works were omitted, namely:-

| Amount proposed by Government |  | £. | $\stackrel{£ .}{8,720,000}$ |
| :---: | :---: | :---: | :---: |
| Portsmouth .. | Connecting lines | 20,000 |  |
| Plymouth | $\left\{\begin{array}{c}\text { Part of N.E. defences and the whole } \\ \text { of the Saltash .. }\end{array}\right\}$ | 1,800,000 |  |
| Chatham.. | W, defences | 700,000 |  |
| Pembroke | $\left\{\begin{array}{cccccc}\text { Further reduction } & \text { in } & \text { number } \\ \text { land-works } & \text {.. } & \text {.. } & \text {.. } & \text {.. } & \text {.. }\end{array}\right\}$ | 250,000 |  |
|  |  | 2,150,000 |  |
| And a further reduction for works provided for in A Estimates, 1860-61 |  | 390,000 |  |
|  |  |  | 2,540,000 |
|  | Remaining to be provided | by loan | £6,180,000 |

Table B.-Estimated Expenditure for Completion.


$$
\text { Carried forward .. .. .. .. } £ 5,155,000
$$

Table B.-Estimated Expenditcre for Completion-continued.


And if turrets and turn-tables be provided, as recommended by the Fortification Committee, probably $270,000 l$. in addition.

A furfher summary of the cost of these works given in the 'Report of the Committee appointed to inquire into the Construction, Cost, dcc., of the Fortifications,' 1869 , gives a total of
$£ 7,951,437$
Table C shows the Number of Heavy Rifled Guns required for the Arvament of Sea Batteries erected, or in course of erection, under the Loan for Defences.


Table C—continued.

| Name of Work. |  |  |  | Description of Gun. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $12^{\prime \prime}$ Gun, 25 Tons. | $\begin{aligned} & 10^{\prime \prime} \text { of } \\ & 18 \text { Tons. } \end{aligned}$ | $\begin{gathered} 9^{\prime \prime} \text { of } \\ 12 \text { Tons. } \end{gathered}$ | $\begin{gathered} 7^{\prime \prime} \text { of } \\ 6 \frac{1}{2} \text { Tons. } \end{gathered}$ |
| Medway .. |  |  |  | . | 11 | 410 | .. |
|  |  |  |  | .. | 11 |  |  |
|  |  |  |  | . | .. | 36 | ... |
|  |  |  |  | . | . |  |  |
|  |  |  |  | .. | .. | 11 | . |
|  |  |  |  | $\cdots$ | $\cdots$ | 2 | $\cdots$ |
| Cork | $\left\{\begin{array}{l}\text { Spike Island .. } \\ \text { Carlisle } \\ \text { Camden Fort.. }\end{array}\right.$ | $\begin{array}{ll}. . & . \\ . . & . \\ . . & .\end{array}$ |  |  | 44 | 123 | 12 |
|  |  |  |  | .. |  |  |  |
|  |  |  |  |  |  | 8 | 6 |
| General Total .. .. 896 |  |  |  |  |  |  |  |

* Two guns in turrets.

Fig. 2016 shows a general plan of the defences-coast and inland-of Portsmouth, Spithead, and Isle of Wight.


REFERENCES.

| Land Defonces. | 11. Fort Gomer. <br> 12. Hilsea Lines. | 9. Puckpool Battery. <br> 10. Bembridge Fort. | 21. Fort Monckton. <br> 22. Blockhouse Point. |
| :---: | :---: | :---: | :---: |
| 1. Fort Purbrook. |  | 11. Yaverland Battery. | 23. Point Battery. |
| 2. " Widley. | Coast Defences. | 12. Redcliff Battery. | 24. Southsea Castl |
| 3. ", Southwick. | 1. Needles Battery. | 13. Sandown Fort. | 25. Lump Fort. |
| 4. " Nelson. | 2. Hatherwood Battery. | 14. Atherfield Battery. | 26. Eastney Fort. |
| 5. "Wallington. | 3. Warden Point. | 15. Brook Battery. | 27. Fort Cumberland. |
| 6. "Fareham. | 4. Cliff End. | 16. Hurst Castle. | 28. Spit Fort. |
| 7. " Elson. | 5. Fort Victoria. | 17. Calshot Castle. | 29. Horse Sand. |
| 8. " Brockhurst. | 6. Goldenhill. | 18. Browndown Battery. | 30. No Man's For |
| 9. ", Rowner. | 7. Freshwater Battery. | 19. Stoke Bay Lines. | 31. St. Helen's Fort. |
| 10. „ Grange. | 8. Yarmouth Battery. | 20. Gillkicker Battery. |  |

Fig. 2017, plan showing defences of Plymouth as recommended by commission of 1860 .


Fig. 2019, plan of Portland Harbour and its defences.

Scale, $\frac{1}{2}$ in. to a mile.
References:-A, Verne Battery. B, Nothe Fort. C, Breakwater Fort. D, Blacknor Fort.
2019.

Fig. 2018 shows the defences at the entrance of the Thames and Medway. The channels will be further protected by obstructions, floating and hidden.
 REFERENCES.

1. Isle of Grain Fort and Battery.
2. Grain Tower.

| 4. Sheerness Lines. | 7. Chatham Lines. |
| :--- | :--- |
| 5. Hoo Fort. | 8. Tilbury Fort. |
| 6. Darnet. | 9. Coalhouse Point. |

10. Cliffe Creek.
11. Coalhouse Point.
12. Shornemeade Battery.
13. Slough Battery.
14. Garrison Point.

See Armour. Artillery. Battery. Fortification. Gunpowder. Materials of Construction. Ordnance.

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COFFEE MILL. Fr., Moulin à café, Ger., Kaffeemühle; Ital., Macinello, Span., Molino de café.

See Mills.
COFFEE-HULLING MACHINE.
Fr., Machine à égrèner les grains de café; Ger., Maschine zum schälen der Kaffeebohnen; Ital., Mucchona da sgusciare il caffè ; Span., Maquina para descascarar
 cafe:

Sections of A. Angell's Coffee-hulling Machine are shown, Figs. 2020, 2021. The coffee is faxl
in through a hopper I , and passes between the roughened surfaces of a hulling cylinder B , and the serrated spring strippers D. During this operation the hull or husk is effectually separated from the kernel.

Refcrring to Fig. 2021, it will be seen that the strippers D are arranged side by side, one in advance of the other, in a series, within a hollow lid $\mathbf{C}$, to which they are secured. The lever $b$ serves to regulate the pressure on hulling cylinder B , and the machine is operated by the cogwheel G gearing into a pinion H. See Gin.

COFFER-DAM. Fr., Bâtardeau; Ger., Fangdamm; Ital., Diga; Tura; Span., Ataguia.
See Dams.
COG-WhEELS. Fr., Rozes dentées; Ger., Zahnräder; Ital., Route dentate; Span., Rueda dentada.

## See Gearing.

COHESION. Fr., Cohésion; Ger., Cohäsion; Ital., Coesione; Span., Cohesion.
See Materials of Construction, Strength of.
COIL. Fr., Glène; Ger., Aufgeschossenes Tau; Ital., Rocchetto d'induzione ; Span., Bobind.
Electro-magnetic Induction.-An induction coil has an influence by which an electric or galvanic current produces magnetic polarity in certain bodies near or round which it passes.

Magneto-electric Induction is the influence by which a magnet excites electric currents in closed circuits.

The Induction Coil of J. Kidder, Fig. 2022, consists in arranging the fine insulated wire which composes the helix or helices. U, O, used upon a magnet S, G, for obtaining an induced current or currents, either wholly or in greater quantity at or near the centre of the length of the maguet, or what is termed its neutral portion, in combination with the arrangement of the inner or primary coil S , and its core of soft iron $G$. The said helix or helices are made adjustable upon the magnet between the poles of S, D, and the so-called neutral portion thereof, for the purpose of
 varying the power of the induced current. S, G, D, is securely fixed to the base T. See Boring and Blasting, p. 573. Telegraphy.

COKING OVEN. Fr., Four à coke; Ger., Coaksofen; Ital., Fornace da arso; Span., Horno para hacer coke.

See Ovens.
COLLAR. Fr., Crapaudine ; Ger., Spur-ring oder Spurscheibe ; Ital., Collare; Span., Anillo.
A collar is a ring-like part of a machine, used commonly for restraining irregularity of motion or for holding something to its place; as the collar of a pump or steam-cylinder, which is a plate of metal screwed down upon the stuffing-box, with an aperture through which the piston-rod passes; the collar of a shaft used to prevent the shaft from shifting its place endwise, and the like.

A collar-beam, or collar, is a horizontal piece of timber connecting and bracing two opposite rafters.

COMBING MACHINE. Fr., Peigneuse; Ger., Kamm Maschine; Ital., Macchina da pettinare. See Flax Machinery. Wool-working Machinery.
COMPASS. Fr., Boussole ; Ger., Compass ; Ital., Bussola; Span., Brújula.
A compass is an instrument consisting essentially of a magnetized needle turning freely on a point, used to determine horizontal directions, in reference to the north and other cardinal points.

An Azimuth compass is one constructed like the Mariner's, except that the card is accurately divided into $360^{\circ}$, and the instrument is furnished with two sights, and has a motion in azimuth. It is chiefly used to note the actual magnetic azimuth, from which is determined the variation or declination of the magnetic needle.

The Mariner's compass is one which has its needle permanently attached to a card so that both move together. The card is divided into thirty-two parts or points, called also rhumbs, and the glass-covered box containing it is suspended in gimbals, in order to preserve its horizontal position.

A Surveyor's compass is one having the needle su pended by itself, and witl a graduated circle of $360^{\circ}$, on which the needle indicates the angle between a given direction and the magnetic north. It is also furnished with two sights.

Variation compass, one of delicate construction employed in observations on the variations of the needle. See Compasses.

COMPASSES. Fr., Compas ; Ger., Zirkel ; Ital., Compassi; Span., Compas.
A compasses is a simple instrument for describing circles, measuring figures, and so on. It consists of two, or rarely more, pointed branches or legs, usually joined at the top by a rivet, on which they move.

Compasses used for drawing purposes admit of many refinements, and vary in construction in many ways. With draughtsmen the first consideration is a perfect joint, of the making of which there are two distinct methods, one adopted pretty generally on the Continent and sometimes in England, called the long joint, and the other the sector joint. In the long joints, the head extends some distance down the body, as shown in Fig. 2023; consequently, with the closing of the compasses, a larger amount of surface comes continually into action, producing much greater friction and stiffness when the instrument is nearly closed than when it is wide open. This is the fault of the joint ; its only merit is that it requires little skill in making, as it admits of fitting up if the work has been commenced improperly. It is universally used for common instruments, for which it answers very well if properly made. The other form of joint in use, technically called the sector joint, Fig. 2024, is now made to all compasses with any pretence to good quality of workman-
ship. In this joint the working surfaces are of circular form, equally distributed around the centre ; consequently, the compasses move with equal pressure whether nearly closed or wide open. It is not necessary to screw the centres of sector-jointed compasses tightly, as the surfaces are not required to come in perfect contact-for this reason, that after the sector joint is made, the workman lubricates between the joint with hot beeswax, which aids in producing that peculiar deadness in movement so much esteemed in the sector joint. If the joint be made true the wax will never leave it. The writer has parted sector joints which have had twenty years of constant wear; the wax appeared the same in quantity as when first introduced, although it was blackened; the amount of wear on the plates not having been sufficient to take out the fine file-marks which were left in making the joint. This is a peculiarity of the sector joint which no other joint possesses.

The sector joint, to attain perfect movement, should be tightened and loosened as little as possible, thus allowing it to form its own surface. Care should be taken that no oil gets upon the joint, as it speedily dissolves the wax and spoils the joint.

The method of trying if a joint is perfect is to open the compasses until they are in line, and then to close them again very slowly, noticing if equal pressure is required at all openings; this will test the evenness of the joint. Another important consideration is, that the centre should fit perfectly; to examine this it will be necessary to take the compasses about half open, and close and open them alternately and quickly for as small a distance as possible, as it were to feel the joint. In doing this, if the centre should not fit, a slight jerk will be felt immediately after commencing to open or close them. Improperly-made sector joints are worse to work with than improperly-made long joints.


Long Jomit.

With regard to compasses with changeable points, there are two ways of fixing the points to the compasses: in one manner by an angular pin which fits into a socket and is secured by a screw, and the other very much better plan of a cylindrical socket opened on one side, so that it forms a spring. In the first plan the screw is always slipping its thread or getting lost. The opening legs of Drawing compasses are occasionally made of tubes; these are termed Tubular compasses. These have inner tubes, or, what is better, solid bars, fitted telescopically, which extend the legs to take higher radii if required. With tubular compasses, the points turn on an axis, so that there are no loose pieces; these kind of compasses are preferred by many mechanical draughtsmen. A smaller kind of compasses, termed bows, have a handle above the head joint. In the common English construction of this instrument the opening joint is placed very near to the centre of the instrument. This is obviously wrong; the opening joint should be placed as near to the top of the instrument as is consistent with allowing just sufficient length to form a handle. The hollows are better placed sloping inwards than outwards. In compasses, Figs. 2025, 2026, of a superior class, the points of which are made to carry needles, Stanley, of Great Turnstile, Holborn, has made improvements which are said to obviate the general shakiness of these points. Instead of splitting the point down in the English fashion, or boring a cylindrical hole up the point, in the Continental fashion, to contain the needle, in the new plan a conical hole is bored from the back of the point, so that when the needle is pressed into the hole it is wedged sufficiently to hold it firm. The back of the needle is secured by a cross-bolt, which holds the needle more firmly than it is held in any other pian in use. Stanley also makes another point, especially for tracings. In this point the needle is pressed forward by a soft spring, so that it merely bites the surface of the paper sufficient to make a circle without entering it in a perceptible degree. This is a very useful point for tracing from old drawings with large centre-holes, or in a general way for a draughtsman with a heavy hand.


Beam Compasses, Fig. 2027.-These are distinct from those described above, in having no joint; the distance between the points being obtained by a pair of heads sliding on a lath or beam. The points are made plain, or for ink and pencil, as in the ordinary drawing compasses. The general method of fine adjustment is by a screw at the end of the beam which is made to carry one of
the heads with its motion. A. Strange has introduced a new adjustment in these beam compasses, now being used in plotting the survey of India. This is shown in Fig. 2027. It consists of a point with a socket joint fitted into the end of the beam compasses. The point, after being properly ground to its socket and fitted with a milled head above, is bent below the socket, so that the point is made an eccentric. In taking a distance off with this beam, after clamping the movable head as nearly as possible to the distance required, the milled head is slightly turned either way, and the point brought exactly to the position with great precision and ease.

Reconnoitring Compass, for the use of military men, travellers, geologists, and so on.-Every traveller, when exploring a new country, should be provided with a portable compass, to find his way when the sun or the stars are not visible. Such an instrument, to be complete, should be capable of being used in the hand, as well as of being placed on a stand, for the purpose of observing horizontal angles. We ought also to be able to place it upon a board M, Figs. 2028, 2029, for the purpose of setting it, by means of notches $d$ in the centres of the squares of the plane.

The compass which we give appears to satisfy all these conditions. As usual for this kind of instrument, it is in the form of a watch; it has a small pendulum $\mathbf{P}$ for observing vertical angles. The following are the additions which have been made to it:-

1. A small flat piece $b b$ has been fixed to one side; the use of this part is to observe horizontal or vertical angles. For these latter angles it offers special advantages when the observer wishes to take inclines which do not reach the eye, as in the case of the profile of a distant mountain.
2. The bottom of the compass has been provided with a female screw C, Fig. 2028, for the purpose of fixing it to a table by means of the screw $d$, or to a stand by means of the movable rest $\mathrm{T}^{\prime}$, Fig. 2030, similar to the one described for Burel's level. They are affixed to the compass in the same way as a key to a watch.
3. A small reflector A, quicksilvered half upon one face, half upon the other, and turning upon a hinge F, Figs. 2028, 2029, is fixed to the ring of the compass. This reflector serves as a limb when the instrument is placed upon a stand, or even when held in the hand. When on a stand, dipping or descending angles are observed on the upper side of the reflector, and ascending angles on the lower side.

When held in the hand, the eye, being situated above the limb to observe descending angles, may at the same time read the degree marked by the needle; but in the case of ascending angles, the eye, being situated below the compass to see the opposite side of the reflector A, cannot see the degrees. To remedy this, Captain Hossard quicksilvered a portion $\mathrm{A}^{\prime}$ of the glass of the compass. By this means we may see the image of the eye in the reflector ' ${ }^{\prime}$ ', and make it coincide, in the same vertical plane, with the image of the object observed reflected by the reflector A. When the planes perpendicular to $A$ and $A^{\prime}$ coincide, they form a plane perpendicular to the hinge F, and consequently a new reflecting limb, which is such that, the point observed being situated above the eye, the eye may at the same time be situated above the compass, and thus read the degrees marked by the needle. A small flat piece of mica $g$, adapted to the extremity of the com-pass-needle, enables the observer, by inclining it a little, to apply friction to the limb, and so arrest its oscillations.

The reflector A, being quicksilvered on both sides like those of Burel's level, the compass may, in a case of need, be transformed into a level. For this purpose it is sufficient to apply a screw to the reflector to regulate its verticality when the instrument is suspended by the cord H H, Figs. 2028, 2029. This cord, passing through in $H$ to the opposite side of the compass, allows it to be suspended for the purpose of taking directions in mines. This compass costs 55 francs.

Field-glass with Support and a Micrometer for determining distances.-Having procured a level and a compass, a third instrument not less useful is requisite: this is the field-glass, not merely for distinguishing distant objects, but for determining distances by the addition of a micrometer at its focus. Those are most portable and clear which reverse the objects; this is a small inconvenience for those who are accustomed to such glasses. A small micrometer, engraved upon gelatine, in which the millimètre is divided into 8 or 10 , is sufficient to determine distances with tolerable accuracy by means of the formula $X=\frac{H}{h} \phi \times 100$, in which $X$ is the distance from the object to the object-glass ; $H$ the size of the object; $h$ the number of divisions which it intercepts upon the micrometer ; $\phi$ a constant coefficient, determined by experiment when X, H, and $h$ are known. It is the focal distance of the object-glass, measured in parts of the micrometer. The formula $\mathrm{X}=\frac{\mathrm{H}}{h} \phi \times 100$ is engraved upon the cover of the field-glass with the value of $\phi$, which varies from 11 to 12.

We find also upon the cover another formula $\mathbf{X}=\frac{a c \cdot h^{\prime}}{h^{\prime}-h}$ relative to the case in which, H being unknown, two successive observations, $h$ and $h^{\prime}$, are made at a distance $a c$ from the object. The divisions of the micrometer are marked in fives by a small bar, for ease in reading; Fig. 2031 represents the appearance of this micrometer. There is a horizontal and a vertical one, enabling the observer to draw upon paper divided into squares the portion of the landscape comprised in the field of vision.

In order to make a careful observation with the field-glass, we must be able to fix it. This is easily effected by means of a gimlet fixed to the rim by a hinge movement like that of compassheads, Fig. 2032.

Burel's Reflecting Level.-The advantages possessed by this instrument are :-

1. The simplicity of its construction and rectification.
2. The length of its line of sight.
3. The readiness with which it may be used, even in the hand without a stand to rest it upon.
4. The smallness of its size.


Description of the Instrument.-The principle upon which this level is constructed is this, namely, that the line A A, Fig. 2033, which joins the centre of the eye and the centre of its reflection, is horizontal when the reflector is vertical.

The instrument consists of a reflector B fixed in a small pendulum C, which naturally retains the vertical position by turning about the horizontal axis D , by means of a single piece of ribbon. A lid E, Fig. 2034, fitted to the tube F, shuts in the instrument and protects it from the wind. An aperture K K in the tube F allows the reflector to be seen; this aperture is closed by turning a second tube G. A stopper $H$ closes the tube $F$ at the bottom.

To use the instrument, withdraw the stopper H, open the aperture $K K$, and looking into the reflector, bring the image of the eye-ball upon its edge, so as to make it coincide with the object towards which the operator is looking.

When great accuracy is required the instrument must be placed on a stand. The results thus obtained have been equal in point of accuracy to those given by the spirit-level. [A result established from experiments made at Lyons by order of Lieutenant-General Rohault de Fleury, who was the first to conceive the idea of applying the principle of reflection to levels.]

When an approximative value is sufficient, the tube may be held in the hand.
These levels were first constructed with a double suspension; it has since been discovered that one is sufficient. They also bore in the middle of the reflector a horizontal line; this line was found to be embarrassing, especially when the instrument was used in the hand; it necessitated the holding of the image of the eye upon it, which was always a difficult operation; many forgot it, looked only upon this horizontal, and thus obtained an inclined line of sight.

Rectification of the Instrument.--The reflector B, Figs. 2033, 2035, is quicksilvered, half upon one side, half upon the other. It follows from this arrangement that the instrument is reversible, and that, if it is properly set, we ought to obtain the same result by using either face of the reflector. This arrangement renders the rectification of the instrument very easy, and enables us at any moment to verify its adjustment. When its adjustment is not correct, it may be rectified by means of the screw $L$ which presses upon the reflector and changes its inclination.

This mode of rectification supposes the two faces of the reflector parallel.
Measuring and Tracing Inclines.-To render the instrument capable of measuring or tracing inclines, a rod J furnished with a heavy head I is introduced into the hole M in the pendulum C, Fig. 2033. The reflector thus assumes an inclined position, and the trigonometrical tangent of the angle of inclination sought, is proportional to the distance from the centre of gravity of the weighted rod to the reflector.
,The division of the tube J is founded upon the following demonstration, due to Goulier.
A heavy rod O P, Fig. 2036, is suspended at the point O. Its centre of gravity is at A. Under the action of weight, this rod is vertically in equilibrium.

Suppose at the point A, at right angles with OP, another rod AB fixed to it in any manner ; suppose the centre of gravity of this second rod at the point $B$, what will be the position of equilibrium of the system?

Let $p p^{\prime}$ be the weights of the two rods applied at the points A and B ; their resultant $p+p^{\prime}$ passes to the point G of the line AB determined by the relation $\frac{\mathrm{GB}}{\mathrm{AG}}=\frac{p}{p^{\prime}}$

The system, under the action of the force $p+p^{\prime}$, will turn about the point O , until G has arrived at $\mathrm{G}^{\prime}$ upon the vertical passing through the point O .

Let us now determine the angle $\phi$.
We deduce from the relation $\frac{\mathrm{GB}+\mathrm{A}}{\mathrm{A} \mathrm{G}}=\frac{p+p^{\prime}}{p^{\prime}}$, or $\frac{\mathrm{A} \mathrm{B}}{\mathrm{AG}}=\frac{p+p^{\prime}}{p^{\prime}}$.
Now" A G is the trigonometrical tangent of the angle $\phi$, therefore $A G=R \tan . \phi$, whence $\tan . \phi=\frac{\mathrm{AB}}{\mathrm{R}} \times \frac{p^{\prime}}{p+p^{\prime}}$

But $\frac{p}{\boldsymbol{R}\left(p+p^{\circ}\right)}$ is a constant quantity K , therefore $\tan \phi=\mathbf{A B} \times \mathbf{K}$; tan. $\phi$ is, therefore, proportional to A B.

If the rod A B be divided into equal parts, we shall have the measure of tan. $\phi$.
To effect the division of the rod, the tangent of the angle $\phi$ must be determined directly, and this value introduced into the equation; from this we may deduce the oumber of divisions corresponding to this particular value of the angle. We shall then have merely to continue the division upon the rod.

If now we wish to take the angle of a plane with the horizon, the rod must be thrust in to a greater or less degree; this will cause the mirror to incline, and when the eye sees its image coincide with the object looked at upon the line of inclination, the reflector is perpendicular to the incline. The angle of the reflector with the vertical (the angle $\phi$ ) equals, therefore, and serves to measure the angle of the inclined plane with the horizon.

As the reflecting pendulum is inclined during the observation, the tube $\mathbf{F}$ should also be held inclined. If the tube be mounted on a stand, the stand should be provided with a movable rest T, of wood, Fig. 2037, fitted to the tube F ; or of copper, 'T', Fig. 2030, fitted to the stopper H.

When we have to measure an incline the tangent of which exceeds the length of the rod $J$, the solid cylinder I must be drawn out, and the second numbering of the tube $J$ used. The division upon the end of the tube $\mathbf{N}^{\prime}$ may be adapted at pleasure to the pendulum. Care must be taken to screw it on the side of the screw L , and to introduce the tube J into the tube $\mathrm{M}^{\prime}$ on the same side. Ascending inclines are always observed on the face of the reflector corresponding to the screw $L$, and descending slopes on the opposite face. If, therefore, after having observed an ascending incline, it be required to observe a descending one, $\mathrm{M}^{\prime}$ must be unscrewed, the opposite face of the reflector brought forward, and $\mathrm{M}^{\prime}$ re-screwed on the side of the screw L .

It is obvious that the axis D must be horizontal to enable the ribbon to which the reflector is suspended, and, consequently, the whole system, to turn freely under the action of the rod. It has been found, however, that on holding the instrument in the hand, its weight is sufficient to move the apparatus in a vertical plane. Thus we have the required condition of exactness.

Level reduced to the Reflecting Pendulum and its Axis, Fig. 2035.-The instrument has been reduced to its simplest expression by retaining only the pendulum C , with its reflector B , and its axis. In this condition it is still very convenient to use in the hand, even in taking inclines; but it is more exposed to the wind, and it can be placed upon a stand only by means of a screw-ring fixed on one side of the stand and passing, on the other, through a little cylinder placed for this purpose on the upper portion of the level. This little cylinder is not represented in the Fig. 2035.

Level serving as a Goniometer.-The base of the tube F, Figs. 2033, 2034, may be easily divided at every five degrees, and a corresponding portion of the rest T, Fig. 2037, inlaid with copper, serves as a vernier.

Figs. 2038 to 2040 represent front, back, and side elevations respectively of a mariner's or ship's compass invented by A. Albini; Fig. 2041 is a plan of the same; Fig. 2042 is a plan of the under-side of the compass card, and Figs. 2043, 2044, are sections of the said card; Figs. 2045 to 2047 are front elevations of the mechanism by which the sliding bar hereafter referred to is raised and lowered; Figs. 2048, 2049, are a side elevation and a plan respectively of the said mechanism; Figs. 2500, 2051, are detached parts of the same. The same letters of reference indicate the same parts in each figure.

$\mathrm{A}^{1}, \mathrm{~A}^{2}$, are the front and back plates carrying the several parts of the compass; O is the compass card, the position of which is seen in the several figures. The said card $O$ is furnished with a magnetized bar $o^{1}$ carrying sliding weights $o^{3}, o^{3}$, by means of which the balance of the card can be adjusted as required. $o^{2}$ is a metallic ring affixed to the under-side of the compass card 0 , the ring being furnished with raised letters or printing types indicating the points of the compass, the letters or types of the ring $o^{2}$ being situated under and corresponding to those on the compass card 0 . The said compass card fitted with its printing ring is suspended by and
turns on a central pointed support in the usual manner, as represented in the figures. The construction of the compass card and its metallic printing ring will be readily understood by an examination of Figs. 2042 to 2044, Fig. 2042 representing a plan of the under-side of the compass card and ring, and Figs. 2043, 2044, cross-sections of the same taken at right angles to one another. Underneath the printing ring $o^{2}$ of the compass card 0 is a bar $\mathbf{R}$ provided with set screws $r, r$, which screws $r, r$, by being adjusted as required under the ring $o^{2}$, prevent the excessive vibration of the said ring and card. $d^{2}$ is a drum on the axis of the fusee hereafter described. Over this drum and the pulleys $d^{4}, d^{4}$, an inking ribbon $d^{3}$ passes, which ribbon is kept in a state of tension by the said pulleys $d^{4}, d^{4}$. One of the axes $d^{5}, d^{5}$, upon which the pulleys $d$, $d$, are mounted is fixed to one of the bars $d^{6}$, and the other of the said axes is fixed in an arm $d^{7}$ which is jointed to the other bar, the said arm $d^{7}$ being pressed outwards by a spring $d^{5}$, Figs. 2038, 2039. The required tension in the inking ribbon $d^{3}$ is thus maintained. The

drum $d^{2}$ is hollow, and constitutes a reservoir for the ink, which ink passes through perforations in the periphery of the drum to a strip of cloth with which it is covered. The inking ribbon $d^{3}$ is by this means kept properly charged with ink. $\mathbf{H}^{1}, \mathrm{H}^{2}$, are drums situated within the inking ribbon $d^{3}$, Fig. 2038, around which drums a slip of paper $h^{3}$ is coiled. These drums turn on the axes $h^{1}, h^{2}$, and are steadied by springs $h^{4}, h^{4}$. I, is a horizontal cushion situated a little below the printing ring $o^{2}$ of the compass card $O$. On this cushion the strip of paper $h^{3}$ is printed. By an examination of Figs. 2038, 2039, the manner in which the strip of paper $h^{3}$ passes over the cushion I and between it and the inking ribbon $d^{3}$ will be readily seen. $J$ is a sliding bar having guide-slots $j^{1}, j^{1}$, working on fixed guide-pins $j^{2}, j^{2}$, Fig. 2039. The bar J has a rising and falling motion communicated to it by means of mechanism actuated by the clockwork combined with the compass. To the top of the sliding bar $J$ a small presser bar $j^{7}$ is affixed. The said presser bar $j^{7}$ is situated over the edge of the type or printing ring $o^{2}$ of the compass card, and at each descent of the said bar J presses down the said ring $o^{2}$ upon the inking ribbon $d^{3}$ and prints upon the strip of paper $l^{3}$ the course followed by the ship. Parallel
with the presser bar $j^{7}$ is a rod $j^{8}$, which on each descent of the bar $\mathbf{J}$ presses upon the inking ribbon $d^{3}$, and thereby leaves on the paper $h^{3}$ a dot indicating the exact direction of the ship's head, the pressure exerted by the said rod $j^{8}$ being moderated by a coiled spring $j^{10}$. The rising and falling motion of the sliding bar J and the feeding forward of the inking ribbon $d^{3}$ and strip of paper $h^{3}$ are effected at the proper times by mechanism constructed and actuated by clockwork in the following manner:-B is a clock dial for indicating the time, the said dial being fixed to the plate $\mathrm{A}^{2} ; \mathrm{C}$ is the spring or going barrel of the clock; $\mathbf{D}$ is the fusee, which is wound up by a handle $d^{1}$ on its axis. On the axis of the fusee is the drum $d^{2}$ over which the inking ribbon $d^{3}$ is passed. By the motion of the said fusee the ribbon $d^{3}$ is moved slowly over the drum, and a fresh portion of it is successively brought over a blank portion of the strip of paper $h^{3}$ for the next impression of the type or printing ring and rod. Motion is communieated from the fusee D by means of toothed wheels $d^{9}$, $\mathbf{E}^{1}, \mathbf{E}^{2}, \mathbf{F}^{\mathbf{1}}, \mathbf{F}^{2}$, and $\mathbf{G}$ to an axis $g^{1}$, on which are fixed a counter-balance $g^{2}$ and snail-shaped cam $g^{3}$. The speed of the axis $g^{1}$ is retarded by a fly-wheel $S$ on a shaft $s^{1}$, on which is fixed a pinion $s^{2}$ which is driven by a toothed wheel $g^{10}$. To the axis $g^{1}$ a lever $g^{8}$, Fig. 2050, is fixed, the end $g^{4}$ of which lever constitutes a detent. To the free end of the said lever $g^{8}$ is a segmental bar $g^{5}$, which is furnished with detents $g^{6}, g^{7}$. The detents $g^{4}, g^{6}$, and $g^{7}$, operate in conjunction with the cam $L$, as hereafter described. The said cam L is shown separately in Fig. 2051. Above the lever $g^{8}$ is another lever K turning on an axis $k^{1}$ and pressed down by the spring represented. The said lever $K$ is supported by an arm $k^{2}$ and cam $g^{3}$. The lower end of the arm $k^{2}$ of the lever K is turned at right angles, and upon this bent part the cam $g^{3}$ acts. The lower end of the sliding bar J, by which the descent of the printing ring $o^{2}$ upon the strip of paper $h^{3}$ is effected, is jointed to the free end of the lever K.
 The sliding bar $J$ is raised, supported, and allowed to descend at the proper times by the operation of the cam $g^{3}$ upon the arm $\kappa^{2}$ of the lever K, to which the said sliding bar J is jointed combined with the operation of the cam L upon the lever $g^{8}$. The cam L is double-acting, the portions marked $l^{1}, l^{2}$, having flat faces,

Fig. 2051. The said cam $L$ is fixed on an axis $l^{3}$, which carries the seconds' hand of the clock, and gives motion by means of a toothed wheel $l^{4}$ to a pinion M, which is connected to a chronometer escapement N, by which the motion of the apparatus is regulated. The toothed wheel $l^{4}$, connected with the said escapement $N$, carries a movable pin $l^{8}$, which once a minute acts on a

pin $l^{3}$ on the periphery of a wind-up barrel $l^{10}$, so that in case the "going" or spring barrel C is run down, the motion of the works is stopped. The action of the parts for giving the rising and falling motion to the bar $J$ is as follows:- When the parts are in the respective positions represented in Figs. 2038 and 2045, the sliding bar $J$ is supported in its raised position by the arm $k^{2}$ resting on the cam $g^{3}$. At every half minute the detent $g^{4}$ of the lever $g^{8}$ escapes from the part $l^{2}$ of the cam L by the rotation of the axis $l^{3}$ of the seconds' hand, and the said lever $g^{8}$ falls from the position represented in Figs. 2038 and 2045 to that represented in Fig. 2046 by the partial rotation of the axis $g^{1}$, upon which the said lever $g^{8}$ is fixed, the cam $g^{3}$ partaking of the same motion. On the completion of the minute the detent $g^{6}$ of the lever $g^{8}$ escapes by the further rotation of the cam L from the part $l^{1}$, and the cam $g^{3}$ releases the arm $k^{2}$ and allows the lever K and bar $J$ to fall, the said parts assuming the positions represented in Figs. 2047 to 2049. By the descent of
 the bar J the printing of the ship's course upon the strip or paper $l^{3}$ is effected, the ring $o^{2}$ of the compass O being depressed by the presser bar $j^{7}$, and the rod $j^{8}$ at the same time being caused to make a dot on the paper. The detent $g^{6}$ of the lever $g^{8}$ having thus escaped from the cam $L$ and caused the descent of the bar $J$ and the printing of the ship's course, the detent $g^{7}$ of the said lever,
which is of slightly greater length than the detent $g^{6}$, rests on the said cam L for two seconds, in order to defer for that space of time the revolution of the cam $g^{3}$, and thus prevent the undue

vibration of the compass card, which would result from the too sudden rise of the presser bar $j^{7}$ from the ring $o^{2}$ of the said compass card. On the expiration of the two seconds the rotation of the
cam L permits the escape of the detent $g^{7}$ therefrom and allows of the further rotation of the cam $g^{3}$, whereby the said cam is brought round to the position represented in Fugs. 2038 to 2040. and 2045, and by acting on the arm $k^{2}$ of the lever K raises the bar $J$ and hifts the pressure bar $j^{7}$ and rod $j^{s}$,


The lever $g^{8}$ having been brought round with the cam $g^{3}$ to the position Figs. 2038 and 2045, the said lever $g^{s}$ is again ready to be released at the proper times by the cam L and cause by its action the descent of the cam $g^{3}$ and the consequent fall of the bar $J$ and the printing of the ship's course as hereinbefore described. In
 Fig. 2041 the printing of the ship's course and the centre of the ship's course at intervals of a minute upon the strip of paper $h^{3}$ is indicated in dotted lines. The strip of paper $h^{3}$ is shifted on the raising of the bar J so as to bring a blank part in position for the next impression in the following manner:-On the face of one of the drums $\mathrm{H}^{2}$, upon which the strip of paper $h^{3}$ is conled, is a series of pins $l^{5}, h^{5}$, one of which pins is acted upon at each ascent of the bar $J$ by the pawl,$^{3}$ jointed to the lower end of the said bar $J$; an intermittent rotatory motion is thereby given to the drum $\mathrm{H}^{2}$, the return motion of the said drum being prevented by the pawl $j^{4}$, which turns ou one of the guide-pins $j^{2}$ of the bar J, Fig. 2038. Attached to the pawl $j^{3}$ is an elbow lever $j^{5}$, by means of which the said pawl $j^{3}$ can be thrown into or out of gear with the drum $\mathrm{H}^{2}$, as may be required. The said pawl $j^{3}$ is furnished with a catch $j^{6}$ which, when the said pawl is out of gear, as indicated by dotted lines in Fig. 2038, rests on the bearing $l^{5}$ in which the axis $l^{3}$ turns and thus prevents the descent of the sliding bar J. By means of a milled button $T$ fixed on an axis $t$ connected by toothed wheels to the motion work of the hands of the clock, the said hands may be set as required. The parts of the mechanism of the clock which I have not described are of the ordinary kind. The whole of the compass is suspended on gimbals P P, to allow for the motion of the ship. Instead of causing the descent of the sliding rod $J$ and the printing of the ship's course at intervals of a minute, the clockwork may be so arranged as to cause the descent of the said rod $J$ at other short intervals. In England, this compass is constructed ouly by Elliott Brothers, London.

Hedley's Mining-Compass, or Dial, Figs. 2052, 2053, invented by the late John Hedley, and manufactured extensively by John Davis of Derby. This dial possesses many advantages over the

ordinary dial, inasmuch as it gives an operator all the advantages of the ordinary one, and in addition, by its construction, allows longer sights to be taken in pits of any declivity, by means of
a ring moving on centres. This ring, Fig. 2053, carries the sights, and can be moved to any angle, the centres remaining in all cases horizontal. To the ring is attached, when required for use, a graduated arc, which gives the vertical angle and difference for hypothenuse and base. A vernier is fixed to the horizontal circle for dialling with the fast needle. See Geodesp. Hand-Tools. Surveying. CONCREIE. Fr., Béton; Ger., Grundmörtel; Ital., Calcestruzzo; Span., Hormigon.
See Cement. Construction. Lime and Mortar.
CONDENSER. Fr., Condensateur ; Ger., Condensator ; Ital., Condensatore ; Span., Condensador. Sec Details of Engines.
CONSTRUCTION. Fr., Construction ; Ger., Konstruction; Ital., Costruzione ; Span., Construccion.

This article, placed under what we consider an appropriate term, is designed more for artisans engaged in ordinary building operations, than for civil and military engineers.

Foundations.-The foundation of a building is the horizontal platform, either natural or artificial, prepared for carrying the walls and superstructure. It must not be confounded with footings, which are the bases of walls made broader to distribute the weight more equally over the foundation; nor with piers, although it is not always easy to define where a foundation ends and where a pier begins: in general, all those parts of a structure which arc sunk in the natural soil, the conditions of which are therefore different from those parts above ground, are foundations.

There are three important points which should be considered in all foundations :-
1st. That the weight to a unit of area imposed upon it should not be more than it and the subsoil below it can bear.

2nd. That it should be as nearly as possible homogeneous and equally strong throughout.
3 rd. That the upper surface should be horizontal: if not in one, then in several plaues.

## Foundations particularized.

1. Rock, gravel, and such unalterable grounds.
2. Clay, sand, and such alterable grounds.
3. Firm ground underlying soft ground.
4. Firm ground overlying soft ground.
5. Soft ground of indefinite thickness.
6. Concrete.
7. Fascines.
8. Piling.
9. Hollow cylinders.
10. Foundations in water.
11. Loose stones.
12. Coursed masonry and concrete.
13. Caissons.
14. Coffer-dams.

Rock.-It is generally supposed that rock is a dangerous substratum to make a foundation platform from; for it is rarely that rock is found so homogeneous as to provide a large horizontal surface without artificial filling in; and it is difficult to make the filling in as hard as the rock itself, which it should be, that the settlement, if any, may be uniform. Also in many cases of inclined strata there is the danger of one part of the strata slipping over the other from the additional pressure of the building.
licn cul, from experience, recommends that a foundation in rock should never be less then $0^{n \cdot 3}$ in depth, for security against slipping and detrusion.

Hughes gives an example of a pier of an aqueduct 50 ft . high, which being founded partly on rock and partly on gravel, was split. owing to unequal settlement.

Gravel.-Many consider a sound thick stratum of gravel to be the most secure foundation possible. In such cases it is only necessary to sink a little into the stratum, rather more than into rock, and to take care that the area of foundation is proportional to the weight a square unit the gravel is calculated to bear. When the gravel is not sound, besides the latter precaution, it is advisable to sink deeper and fill in with an artificial foundation of concrete or large stones or hard durable timber.

Sand.-When in thick strata, and not liable to be moved by water or other disturbing cause, sand forms a very yood foundation ; it is desirable to sink dceper into sand than into gravel, and to fill in with an artificial foundation to counteract any irregular settlement of the sand. When exposed to the action of water or any other moving action, however slight, sand is a dangerous foundation to trust to, on account of its great mobility.

Clay appears to be considered an uncertain and troublesome substratum for a foundation, on account of the irregularity of its strata, and its action on being disturbed; for there is a tide in the land as well as in the sea. In consequence of clay's plasticity and its retention of water, it is liable to yield unequally to the pressure of a building, and to move irregularly when exposed or cut into : consequently, care must bc taken both to spread the structure over a large area of foundation and to load the foundation uniformly in the course of the construction. Dobson gives examples of the expansion of clay on exposure to air by cutting and also on saturation with water. In the Box-hill Tunnel 6 in. was allowed for the expansion; and in the Metropolitan Railway the expansion of the clay on cutting had to be provided for in the centering and staging. A bed of clay can be sometimes made firmer by piling or by making holes in it and filling them with stones or gravel: the elasticity of clay is sometimes so great that Renaud states that piles are often forced up again by the action of driving the neighbouring piles.

It frequently happens, especially in the alluvial banks of rivers, that below the soft ground of the immediate surface lies a hard stratum, and when the thickness of the soft superstratum is not great ( 30 ft . may be considered a maximum for ordinary cases), a secure foundation may be obtained by carrying piles or piers down to the hard ground below and supporting a horizontal platform on their tops. These may be wooden or iron piles driven till they enter the hard bottom ; or piers formed by sinking well-holes through the soft ground and filling them up with masonry, loose stones, or, Renaud says, even sand, though this last should only be used when the superstratum is sufficiently firm to resist the lateral pressure of the sand. The tops of these, if piles, may be connected by beams and planks forming a horizontal platform; or if piers, by arches filled in at the spandrils to a horizontal surface. These piles or piers must be considered as columns fixed at the bottom and calculated accordingly, without trusting to the lateral support of the intermediate strata.

Firm Ground overlying Soft Ground.-In some cases of alluvial foundations, a stratum comparatively firm of gravel or clay is found at the surface or near it, the substrata below that being much softer. In such cases, if the weight of the structure is not very great, it is frequently desirable to leave the hard crust unbroken ; but then the area of foundation should be enlarged, beyond what would be used for the same stratum, if of considerable thickness; and special care should be taken to distribute the pressure equally. Also in these cases the hard crust should be cut into as little as possible for any purpose; if it is clay there is danger of it yielding by exposure to air and wet; if the substratum is sand there is danger of its being moved by the action caused by drainage or any operations of that kind, consequent on the building.

Soft Ground of Indefinite Thickness.-When the soft superstratum is of indefinite or very great thickness, and not hard enough to float the building upon it, by extending the area of the foundation, it must be supported upon piles or piers, carried sufficiently deep that the friction on their sides will be enough to carry the weight. In the case of piling, they should be closer together than in the former case, and the heads of the piles, besides being connected together with timber framework, should be surrounded with a mass of masonry or concrete, to distribute the weight and add to the resistance. If piers are employed they may be of masonry, sunk in the manner that wells are formed, and which are used as foundations by the natives in India, or they may be hollow cylinders of iron.

When the ground is exceedingly soft, there is considerable danger of the pressure on that part underneath the bulding causing that part surrounding it to rise above its original level; to counteract this, as far as possible, the pilng or piers should be extended beyond the area of the foundation, and the ground in the immediate neighbourhood should be consolidated or weighted with stones or concrete, and as few excarations as possible should be made in the natural soil. It is also necessary in these cases to equalize the pressure all over the area of the foundation, because there is sure to be a settlement, however small, and the smallest irregular settlement will cause a break in the structure. Equalization of the pressure on the foundation will not, however, prevent an absolute settlement, nor a rising in the neighbouring ground, which latter can only be counteracted by piling and counterbalancing the pressure by weighting the surrounding parts.

Concrete. -The nature of concrete that should be used for a foundation depends on the nature of the soil it is to be laid in: the object in all cases being to get as nearly as possible a homogeneous bed under the structure. If the soil is dry, a concrete of sand, gravel, and as much ordinary lime as is necessary to produce a coherence of it altogether is sufficient; as it is little more than a bed of coherent gravel; but then it must be spread over such an area that it might be sloped at an angle of $45^{\circ}$ from the outside of the footings of the walls, down to the bottom of the foundation; and of such a thickness that it will not be liable to crack under the pressure. For ordinary buildings probably from 2 to 3 ft . is sufficient. If the soil is wet, or the building is of great weight or special character, the concrete should be made of hydraulic lime and sand and broken stones, in about the same proportions as would be used in rubble masonry; that is to say, the lime should be about $\frac{1}{7}$, the sand about $\frac{2}{7}$, and the broken stones about $\frac{4}{7}$. These, however, must be considered only as a verage proportions for medium hydraulic lime and ordinary wet soils; the proportion of lime must be varied inversely as its quality is better or worse, or as the circumstances are more or less important. In such cases the concrete, if properly constituted and laid, may be considered as a solid coherent mass, capable of bearing without crushing the weight the square foot mentioned m recognized tables as the crushing resistance of different kinds of concrete, a proper coefficient of safety being used. The bed of concrete must also be thick enough not to break by transverse strain, but so as to settle in one mass if the subsoil yields. These two considerations will determine the area of the bed for the foundation.

With moderate hydraulic limes and common limes there will be an expansion of the mixed concrete, consequent on the slaking: mome cases the lime increases to double its original bulk; this may be almost entirely provided for by allowing time for the lime to be thoroughly slaked before laying the concrete; in some cases, however, tlie lime, or parts of it at least, will take so long to slake, that the process is completed after the concrete is laid, and it is therefore generally desirable to consider this expansion in preparing the site for the concrete.

As the principal object in laying a bed of concrete is to form a solid cohesive mass when it hardens, it has been sometimes recommended that it should be thrown in from a height to consolidate it; this practice, however, has the disadvantage of separating the fine from the coarse particles: it is better to lay the concrete from barrows or boxes on the level of the site, and to consolidate it afterwards by ramming; in ordinary foundations, to effect this properly and to allow the lime to set, the concrete should be laid in strata of not more than 1 ft . thick each; it is very desirable to bond these strata into each other in the process of laying, as the joint between two days' work is always a weak part in the mass. In large foundations, or with strong hydraulic lime, it is better to make the strata 2 or 3 ft . thick; on that account, for the same reason, the whole of one strata should be laid as quickly as possible.

In large foundations in water, a slight coffer-dam is generally made round the area required ; if there is rock underneath, the coffer-dam is made of framework filled in with sheeting; the soft ground is then dredged out from the inside of the coffer-dam, and the space is filled with concrete to the height at which the regular masonry begins, which would be generally at 3 or 4 ft . below the lowest water-line. In this method the concrete is generally lowered into the water within the coffer-dam, in a box with a movable bottom, which is made to open when the box is near the place required, and so allow the concrete to fall gently into its position; of course, a strong hydraulic lime must be used, and therefore the concrete should be laid as quickly as possible in a mass.

Fascines.-In soft marshy ground of great depth, a foundation of fascines is frequently employed in places where suitable brushwood is plentiful, in Holland for instance; and in such places it is highly approved of. Its recommendations appear to be that when carefully made it is elastic, durable, and uniform.

Authorities differ as to the best size of fascine for foundations; Pasley recommends 6 in . diameter; Lewis used them 12 in. diametcr successfully, as a foundation for a 3 -gun martello tower at Hollesley Bay on the coast of Suffolk, built in 1812. Hughes, U.S., says that on the Dutch railway from Amsterdam to Utrecht, they used two kinds, a saucisson of $0^{\text {n }} \cdot 4$ circumference and a fascine of $0^{\mathrm{m} \cdot 5} 5$ circumference. But all agree that they should be compactly made of carefully-selected wood.

When the sward of a site of the town was removed, preparatory to laying the foundation, speaking of the operations in Holland, the ground was found so soft, that a $10-\mathrm{ft}$. rod was easily thrust down the whole length of the rod with the hand.

Extract from Puper by Hughes, U.S.-Description of the Fuscine Foundation of the Embankment of the Amsterdam and Utrecht Railway.-The fascine foundation is to be formed by an under and upper framework of hurdles (saucissons) $0^{\mathrm{m} \cdot 4}$ thick, with a backing of fascincs between them. Including the second laycr of saucissons of the under frame, and the first layer of the upper frame, the mean depth of the backing ought to be $0^{\mathrm{m}} \cdot 5$. The saucissons to be of a uniform texture of straight sticks, strongly connected together by not less than eight twig bindings for every mètre in length. Those of the first layer of the under frame to be placed lengthways, in the direction of the road, at 1 metre central interval; those of the second layer at right angles to the first, also at 1 metre central interval. The longitudinal saucissons to break joint; the two layers to be tied together at every crossing with strong twig bindings. Upon this under frame, which must be well filled up with sand to the level of its upper surface, are to be laid the first course of fascines; then two transverse layers crossing the first at right angles; and on this a fourth layer, like the first, at right angles across the axis of the way, but having the larger ends of the fascines turned to the other side : they are to be well rammed together. The packing must have a depth in the centre of $0^{\mathrm{m} \cdot 6 \text {, and at the }}$ sidle $0^{n} \cdot 4$. The upper frame, of the same construction as the lower frame, to be placed over it, so that the upper layer of saucissons comes parallel to the axis of the road. Through each crossing, stakes are to be driven, passing through the hurdles of the under frame.

The fascines to be made of sound and clean willow wood of not less than three years' growth, or of oak brushwood of not less than five or six years' growth; the wood to be straight and cut green last season, to be firmly bound with two bindings and well tightened. The length to be $3 \frac{1}{2}$ to $4 \frac{1}{2}$ mètres.

Piling.-There are two modes in which piles may be used to form a foundation:
1st. When the soil is soft for a considerable depth; in which case a large area should be covered with piles connected together by framework at the top, and so forming one united body, which would resist settlement chiefly by the friction of the subsoil against the sides of the piles.

2nd. When there is a stratum of hard ground below the soft ; in which case the piles should be driven into the hard stratum and each pile would act as an independent column bearing a certain proportion of the whole weight, and resisting settlement both by friction and by its own transverse strength.

1st. Soft ground throughout.-M. Renaud says the piles should be from $0^{\text {m. }} 8$ to $1^{\mathrm{m} \cdot 2}$ central interval, which agrees with the general practice in England; they should be at that interval both ways, that is, the area should be studded with piles 1 yd. apart both ways; in England the size of piles is generally determined by the size of the logs or balks in which pine timber is sold in the market, and which are about 12 in . square on the average; a pile made of one such piece is commonly called a whole pile; when one such balk is cut into two piles they are called half pilcs. M. Renaud says the diameter should be $\frac{1}{24}$ of the length, and never less than $0^{\mathrm{m}} \cdot 18$. Mr. Dobson recommends the area to be enclosed with sheet piling before the main piles are driven, in order to consolidate the ground better; he says soft clay is too compressible to be well consolidated by piling; it is so elastic that sometimes a pile is forced out of the ground by the driving of a neighbouring pile.

When the tops of the piles are connected by timber, Dobson says the first pieces should be those across the breadth of foundation, and the longitudinal pieces running parallel to the length of foundation should be over them: on these latter, planking should be laid to carry the masonry ; it distributes the weight better to lay the planking diagonally. When the upper stratum of the ground is very soft, or when the timber framing at the top is fixed at considerable intervals, the upper part of the ground should be taken out and filled in with stones, or concrete or regular e masonry, to increase the resistance of the piles and to form a more uniform platform for the superstructure; when of concrete, this top filling-in should be about 3 ft . thick for large buildings not excessivcly heavy.

2nd. Hard ground below.-In this case, as each pile resists partly by its resistance to deflection, the diameter or cross-section should be greater in proportion to the length: when the depth is considerable, say above 25 ft ., and the weight of the structure is considerable, two or more piles should be driven close together and well connected together at top; the piles should come immediately under the weight to be borne, and their number should be determined from that weight, and the timber framework at top should be strong and well tied together. It is advantageous in this case to arrange the superstructure of the building so that it may be ultimately carried on piers at intervals, and consequently that piling will only be required underneath those piers. It is evidently of great consequence that the piles should be quite vertical.

Various kinds have been used when the piling was of uniform character in straight lines, and when labour was expensive. Nasmyth's steam pile-driver has been used in England; it is an ingenious adaptation of his steam-hammer ; the steam-cylinder rests on the head of the pile, so that the action of raising the ram tends to drive the pile down, as well as the fall of the ram itself. Another and more common mode of applying steam is by making the steam work an endless chain, which passes over a pulley at the top of the driving machine and round an axle at the bottom; the chain is made so that it lays hold of the catch of the ram by a self-acting apparatus. It is released at any height required in a similar manner to that of the crab-engine, and immediately on coming to rest on the pile is caught by the endless chain again.

The upright part of the pile-engine must be truly vertical when the pile is to be vertical, because the ram is guided in its fall by the two uprights; when the pile is to be driven in an inclined position, if the framing of the engine admits of it, the guiding pieces should be inclined accordingly. but if (as is usually the case) the framing does not admit of it, temporary guiding pieces must be laid against the uprights to the slope required, and the ram must work in them.

The statical weight which any given pile will support, without sinking, may be theoretically calculated from knowing the effect upon it of a known ram falling a known height. If the weight of the ram be $w$, and the weight of the pile $p$, and the fall of the ram be AB, Fig. 2054, and the distance through which it has driven the pile be BC; then the velocity of impact is $v=\sqrt{2 g} \mathrm{~A} \dot{\mathrm{~B}}$, and the velocity with which the pile and ram move on together is $v_{1}=\frac{w}{w+p} v$. Then the weights $w+p$ have passed through the space $\mathbf{B C}$ with a velocity commencing with $v_{1}$ and decreasing to $o$; therefore the accelerative velocity of the motion is $\left(\frac{(2 \mathrm{BC}) w}{w+p} v\right)^{2}$, and as forces are proportional to the velocities, they generate in a unit of time; then $g: w+p::\left(\frac{(2 \mathrm{BC}) w}{w+p} v\right)^{2}: \mathrm{P}:: \mathrm{P}$ being the pressure which would generate the accelerative velocity, and, consequently, the statical pressure which the pile will bear without moving farther.

This calculation is more applicable to piles which resist by the friction on their sides; it is applicable to those which are driven through very soft ground into a hard substratum, but in this case, in addition to this calculation, a further one should be made, considering the pile as a column fixed at its lower end, and liable to break by deflection.
M. Renaud has given an empirical rule which will serve as a guide. He says, first of all, that piles of a diameter of $\frac{1}{24}$ of their length will carry $0^{\mathrm{k}} \cdot 5$ a millimètre square; this must mean resistance to deflection, or crushing the wood. He also says, if a ram of 600 kilogrammes falling from a height of $3^{m} \cdot 6$ drives a pile
 blows; then in either case the pile will bear 25,000 kilogrammes permanent statical weight. By which it is apparently supposed that the statical resistance varies directly as the square of the fall and weight of ram, and inversely as the depth driven in by a blow. He does not, however, place much dependence on his formula, for he recommends a practical experiment to be made in each case.

Pasley points out that if a scarp revetment 30 ft . high is built on piles 3 ft . apart each way, each pile will carry about 10 cub. yds. of masonry and about 3 cub. yds. of earth; and taking the former at 160 lbs . and the earth at 100 lbs . the cubic foot, the total weight on one pile will be 51,300 lbs.

He also says that at Neuilly Bridge, the arches of which are 128 ft . span, the weight of masonry on each pier is $15,417,648 \mathrm{lbs}$., which is borne by 135 piles. Therefore the weight on each pile is $114,200 \mathrm{lbs}$. They are of oak, $12 \frac{3}{4} \mathrm{in}$. diameter and 13 ft . long.

Also, that Perronet considered that oak piles 9 in . diameter should not be loaded with more than $50,000 \mathrm{lbs}$.; and if 12 in . diameter, with $100,000 \mathrm{lbs}$.

Also, at the Bridge of Tours, one pier which gave way was on sixty-five piles of oak 8 ft . long, $9 \frac{3}{4}$ diameter, and carrying each $166,212 \mathrm{lbs}$., the toe of the pile being in firm soil.

The tops or heads of wooden piles are generally protected from the action of the ram by an iron hoop, the head being cut round for the purpose. The foot or toe of the pile is generally shod with an i.on-pointed shoe to save the wood. Cast-iron shoes are recommended in preference to wrought iron by some engineers, because it is more difficult to make the wrought iron to it the toe of the pile exactly, which it should do to save the wood fully. The fastenings of the cast-iron shoe should be of wrought iron.

Wrought-iron piles have been generally used in the form of hollow tubes, and the mode in which they have been driven in England has been by attaching a broad-bladed screw at the end, and weighting the pile when vertical, and turning it by an apparatus at top, and thus screwing it into the ground. This method is only applicable in sand or soft soil. The chief difficulty appears to be to keep the pile vertical. In the 'Minutes of Proceedings of the Institution of Civil Engineers' for 1848, is a description of Mitchell's wrought-iron screw-piles, as used at the Southend Pier.

Hollow Cylinders.-As before mentioned, in soft soil of indefinite thickness, or sometimes when it is desired to reach a hard substratum, the superstructure of the building may be supported on piers at intervals, the foundation of which consists of hollow cylinders of masonry or iron, extending to the depth required, and sustained chiefly by the friction on their sides. The mode of sinking these cylinders is by excavating the soil from the inside, and allowing them to sink by their own weight, in the manner in which the steining of masonry wells are formed. The practice appears to have been long in use in India, where it is the custom to sink many of these masonry wells over the area of the foundation required, and connect them at the top by arches or flat stones. Owing to the use of iron, the English engineers have been able to enlarge upon this method, and it has become a common method of obtaining foundations for piers of bridges and such structures
in water. When they are applicable, they are economical and expeditious, and they can be carried to great depths. They are generally made of cast iron, but the largest cylinders are of wrought iron. They are generally made of from 5 to 10 ft . diameter, and in lengths of from 6 to 10 ft .: probably 3 ft . diameter is the smallest size that could be used, and sometimes they are of much larger diameter. The soil inside is excavated by hand labour; therefore in passing through watery soil it is necessary to keep the water out, which is done by forcing into it a pressure of air sufficient for the purpose. There must in such cases be an arrangement of aur-tight partitions across the interior of the cylinders with trap-doors in them for the passage of men and materials, for the men are working, like in a diving-bell, under a heavy pressure of air. M. Renaud mentions a cylinder foundation at Bordeaux, on the Gironde, a gravelly bottom movable by floods; the cylinders were $3^{\mathrm{m} \cdot 6}$ diameter. There were $13^{\mathrm{m}} \cdot 35$ of water, and $7^{\mathrm{m} \cdot 5}$ of foundation below that, making a total pressure of $20^{\mathrm{m} \cdot 85}$. The thickness of the cast iron was $0^{\mathrm{m}} \cdot 0 t$. The joints were planed, and had a ring of india-rubber between them, laid in a groove in the joint. In this case the cylinders were forced down by hydraulic pressure. He also mentions a foundation in the
 ness of metal, the depth of water being 11 mètres and total depth of cylinder 17 mètres. It was filled with concrete.

The centre pier of Saltash Bridge, near Plymouth, rests on a wrought-iron cylinder, 37 ft . diameter and 70 ft . deep from the surface of the water. The cylinder rests on trap rock, and is filled with granite masonry.

It is the general practice to fill the interior of the cylinder with cement or masonry, so that when the iron is destroyed by rust the masonry column will resist by its own strength.

Foundations in Water.-The difficulties in making foundations under water, which are added to the ordinary difficulties on land, are, 1st. The presence of water, impeding all operations; 2nd. The stratum of mud or gravel generally found on all beds of streams; 3rd. The danger of the foundation being undermined by the current. There are five different methods used for making foundations in water, so as to meet these difficulties.

1st. By piling or cylinders.
2nd. By throwing in loose stones to form an artificial bank up to the surface of the water.
3rd. By using a diving apparatus, and building regular masonry under the water.
4 th. By sinking a box or caisson of the size of the foundation required, and building inside it.
5 th. By enclosing the area of the foundation in a coffer-dam, and pumping the water out of it.
Before the introduction of iron cylinders and of the use of concrete so extensively, the ordinary method of obtaining a foundation for the piers of a bridge was by piling under the area of the pier, and cutting off the piles as low as the water admitted, and planking over the tops of them to form the platform. To prevent the current, increased by the obstruction of the piers, from carrying a way the soil about the piles, a mass of loose stones was generally laid round the upper part of the piling; but this was not always efficacious in strong streams with soft bottoms. In such cases M. Renaud recommends that the area of the pier should be enclosed with sheet piling and filled in with concrete. He mentions an example of a difficult case of foundations in a stream with a lied of moving gravel, at Moulins, on the Allier. Three bridges had been carried away by the undermining of the foundations. A flat bed of masonry, $1^{\mathrm{m} \cdot 65}$ thick, and 1 metre below low water, was laid on the gravel from bank to bank for a length on the stream of 34 metres, and protected above and below stream with a wall. Several of the bridges over the Thames at London have given way in consequence of the undermining of their pile foundations by the increased current caused by the improvements in the river. Therefore piling is not used now generally, except for long sea-walls and in special cases.

Loose Stones.-By this method masses of loose stone, just as taken from the quarry, are thrown into the water from a stage built on piles over the area of the foundation required, and are left to find their own position of equilibrium. It is therefore a method chiefly applicable to breakwaters, moles, and such extensive sea-works. The principal point for consideration is the size of the stones to be used in the different parts of the work. As the action of waves is greatest near the surface, and, according to experiments made in the Gulf of Lyons, does not extend below 24 to 30 ft . even during hurricanes, the smallest stones should be placed at the bottom and the largest at the top. In forming the foundations for the walls of the Basin Napoleon at Marseilles, the French engineers laid down precise rules for these sizes. The stone was a limestone, and they divided all that came from the quarry into the following four classes:-

| 1st. Rubble, in pieces weighing from | 5 to 100 | kilogrammes each. |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 2nd. Blocks | $"$, | $"$, | 100 to 1300 | $"$ | $"$ |
| 3rd. Blocks | $"$ | $"$ | 1300 to 3900 | $"$ | $"$ |
| 4th. Blocks | $"$ | $"$ | 3900 |  | "pwards. |

And the following proportions of each class were used:-

| 1st. | . | .. | .. | .. | .. | .. | .. | .. | .. | $\frac{2}{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2nd. | .. | .. | .. | .. | .. | .. | .. | .. | .. | $\frac{3}{9}$ |
| 3rd. | .. | .. | .. | .. | .. | .. | .. | .. | .. | $\frac{2}{9}$ |
| 4th. | .. | .. | .. | .. | .. | .. | .. | .. | .. | $\frac{2}{9}$ |

Dobson quotes from Poirel's account of a mole constructed at Algiers in 1841, of loose stones, of which the minimum size was from 18 to 27 cub. ft., and the general size under 100 cub. ft . It was found that up to a level of from 13 to 16 ft . below low water the stones preserved a slope of 1 to 1 , or 1 height to $1 \frac{1}{2}$ base, and above that level they preserved a general slope of 1 to 6 and 1 to 10 , becoming flatter nearer the surface; but that nearer the surface the stones never rested long in one position, but were altered by every storm.

In building a superstructure of regular masonry on the top of a foundation of loose stones, it is
desirable that the junction of the regular masonry with the loose stones should be well covered on the exposed side to allow for the subsequent movement of the stones.

The breakwaters of the harbours of refuge at Portland and Hoıynead are excellent, and rewarkable examples of foundations of loose stones in water.

Regular Masonry.-In places where squared masonry is not expensive, or where, from circumstances, it is necessary to have a vertical face to the foundations and walls, the foundation is sometimes built up from the bottom in regular masonry, with the help of a diving-bell or other diving apparatus. It is advantageous, when practicable, that sea-walls should have vertical faces (or nearly vertical) to a depth of 20 or 30 ft . below low water, because the waves will not break generally in that depth, and the wall would in that case have to resist ouly the oscillating action of the waves, and not the progressive action. With the assistance of a diving apparatus, the process of construction is carried on in much the same manuer as on dry land. The bottom is levelled, and any superficial strata removed, and the stones are lowered and placed singly; of course a strong hydraulic lime must be used for the mortar.

C'aissons.-In this method a large case or box (called a caisson in France, which name, from the more frequent use in that country of this method, has also been adopted in England), is floated over the area of the foundation, and sunk by building the foundation inside it; as it sinks, the sides of the caisson are also carried up sufficiently high to keep out the water, and thus the great box containing the masoury gradually sinks to the bottom; the bottom must be levelled and prepared for it beforehand, with the help of a diving apparatus. The caisson has been generally made of wood, but wrought iron has been used. Renaud calls it a great flat-bottomed boat, and says the method is applicable to deep water. Both he and Dobson agree that the great difficulty is in obtaining a hard level surface for the bottom of the caisson to rest upon. Any soft strata should be dredged away; and if the surface is irregular, a bed of hydraulic concrete should be laid over it; if the natural bed is too soft to carry the caisson, piles must driven over the area, and a platform of concrete or planks formed on the top of them for the caisson to rest on.

Coffer-dams.-When circumstances require that a perpendicular wall should be built in not very deep water-say 20 ft .-and when there is good holding ground for piles, and the area of the foundation is not very large, then it is a very efficacious method of obtaining a foundation, to enclose the area with a temporary water-tight wall, and pump out the water from the interior, and proceed to form the foundation, as if it was on dry land. This temporary water-tight wall is commonly called a coffer-dum, and is generally made by driving two rows of piling, about 4 ft . apart, the piles in each row touching each other, and the space between being filled up with clay. In constructing the coffer-dam, a row of what are called gunde-pules are driven aloug the line of the coffer-dam; they are generally made of whole timbers, and are at, about, 10 ft . intervals; on the inner (or coffer-dam) side of these, two longitudinal timbers are fixed, called wale-pıeces, one as low down as the water will permit, and the other near the top, thus connecting the guide-piles together: inside of and close against the wale-pieces are driven what are called shect-ples, about 3 in. thick and of the breadth of whole timbers, driven as close together as possible. At about 4 ft . distance, auother row of guide-piles, wales, and sheet-piles is formed, the sheet-piles being on the inner side, so that the two rows of sheet-piles form a long narrow deep trough; from this trough the natural mud or sand must be excavated by dredging down to some impervious stratum, and must then be filled up to above the lighest water-line with an impervious clay, well worked and kneaded into its place, which working is generally done by men's feet: the clay so worked is commonly called puddle. This is a general description of a good-sized coffer-dam, but the dimensions of its part must vary with the size and circumstances of the whole; and it may be made sometimes without piling, and sometimes the rows of piles may require additional piles, and braces, and struts, to resist the pressure of the water. For when the water within the area of the foundation is removed, the piles of the coffer-dam will be in the condition of beams fixed at the lower end, and subject to a trausverse strain arising from the pressure of the water acting at its centre of pressure. The principal points to be considered in using coffer-dams are:-

In soft ground there is danger that before there is any weight put on the area of foundation, it may be forced up by the pressure round the outside of the coffer-dam.

It is very necessary that the piles should have good hold into a sound substratum. There is always great danger of leakage between the bottom of the puddle wall and the natural bed of the water; leakage through the puddle is also to be guarded against; sometimes gravel is mixed with the puddle to lessen its tendency to crack, and so leak. Springs of water arising from the ground inside the foundation area should be provided for by draining to a suitable spot and pumping. It is probable that after the water inside has been removed, a pump will have to be kept constantly at work to keep down leakage and springs. The clay for the puddle wall should be cut small and well rammed or punned, as it is called, in layers.

Coffer-dams have been much used in England for foundation work in water, because it enables the bottom to be well examined and carefully prepared for the superstructure. It is an expensive and laborious method, because a part of the piling has to be driven from barges, and because of the puddling.

Sheet-piles must be shod and capped similarly to other piles; their toes are generally cut with a bevel or incline, forming a point, which in driving is placed next the last driven pile, so that the pressure against the bevelled part in driving forces the toe close to the last pile.

Sometimes coffer-dams are only used to form shores or' revetments to the sides of the area of foundations, and the water is not removed; then the soft strata required to be removed is dredged out, generally by hand dredging; and the foundation is formed by putting in hydraulic concrete in mass between the rows of piling of the coffer-dam.

Walls.-A wall of a house has two duties to perform: First, to support the floors and roof; sccond, to screen the interior from the weather. These two duties should be always clearly distinguished in considering the construction of every wall. For the first duty, it will generally be
sufficient that the wall should consist of piers or columns at intervals, extending in an unbroken perpendicular line from roof to foundation. For the second duty, the spaces between these piers may be filled in a slighter manner, and even with different material, because the utmost pressure this part of the wall has to carry is that of its own weight, and part of that may be frequently thrown upon the piers. In ordinary dwellings these two parts are generally united into one common thickness of wall, in which apertures are left for the doors and windows; but it is not less necessary to keep them distinct in the mind, and to ensure that the solid blocks between the apertures, which are the real piers carrying the whole weight of the structure, extend in an unbroken line from roof to foundation, and that the spaces above and below the apertures have as little weight to carry as practicable.

Squared Masonry.-Stone walls are built sometimes of regularly squared masonry laid in uniform horizontal courses; sometimes of masonry only roughly squared and laid in courses, horizontal, but no ${ }^{2}$ regular, and which is technically called Random rubble ; sometimes of rough masonry without any courses, but roughly shaped to fit into each other, and which is technically called Uncoursed rubble; and sometimes of rubble faced with squared masonry, and which is technically called Ashlar. The choice of either of these modes of building must depend on the material available, and the object of the work. For walls of dwellings the first method is the strongest and most certain. With such masonry the mortar or cement should be of a fine description, and only sufficient of it used to make a layer between one stone and another, as the main strength of the wall depends on the bonding and mutual pressures of the stones, and the chief object of the mortar is to ensure the pressures being distributed over the surface.

The danger to be apprehended in stone walls of square blocks is that the blocks will not press evenly on the beds of their courses, and so cause fracture of a stone; therefore stones should not be too long, compared with their breadth. Of whatever the quality of stone, the wall should consist of as much stone and as little mortar as possible.

The courses of such masonry should be horizontal and of equal or nearly equal heights : the horizontal joints in a wall may extend through it, as the pressures are generally vertical, but the vertical joints should be broken both transversely and longitudinally, otherwise the wall approximates more or less to the condition of an assemblage of thin columns. Subject to this condition, it is immaterial, as far as construction is concerned, where the vertical joints occur. Bonding is the technical name for breaking joint so as to connect the parts of the wall into one mass as much as possible, and where it occurs regularly the stones are called headers and stretchers; occasionally a through bond stone should occur extending through or nearly through the wall. Renaud recommends that the length of stones in squared masonry should not be more than four or five times the height: and Hosking recommends that the through bond stones should not be very long, but extend only about $\frac{2}{3}$ through the wall, laid alternately from each face. Tredgold recommends that even through bond stones should not be more than three times their thickness in length. As a general rule, the stones should be cut with the natural quarry-bed horizontal.

Rubble masonry, if coursed, is subject to the same general conditions as squared masonry as respects bond and joints. More mortar is necessary, on account of the greater irregularity of the surfaces; for the stone is only roughly dressed with the hammer and chisel, and the stones being of irregular sizes, the horizontal joints are broken; therefore the bonding is not so regular nor so good, and more through bond stones are required. This description of walling is, therefore, more suited for thicker walls than the squared masonry; it is very effective when the stone is of a highly-stratified character and capable of being roughly squared without much difficulty. It was much used by the Gothic builders, who had a difficulty in obtaining large blocks of squared stone. Squared masonry is the exception in Gothic buildings, and is used for columns, groins, and such like principal parts, the main body of the walls and arches being filled in with rubble in random courses.

Uncoursed Rubble Masonry is the term applied to masonry of unsquared stones, roughly dressed to fit into each other, and therefore without regular horizontal or vertical joints. It requires higher skill to build such a wall well, than one of horizontal courses, to fit the stones properly into each other, and so make the wall as much as possible one mass of tightly-wedged stones; therefore, it shows a skilful builder to build a dry rubble wall without mortar; but uncoursed rubble requires more mortar than coursed and of a better quality, for as there is a greater liability in the wall to slip over its joints from the vertical pressures, a greater part of its strength depends on the resistance of the mortar to shearing. It is a method most suitable with unstratified stones having irregular planes of cleavage, and it should be the object in fitting the stones together to make use of those natural planes of cleavage.

This method was first used in what is called cyclopean or polygonal walling, which, in the Mediterranean, was of large blocks dressed to the form of the natural cleavage, sometimes with care, and so fitting closely without mortar, and forming a very strong wall, though expensive in labour. Many of the old Venetian walls in the Mediterranean and also Gothic walls in many places are built of small stones in this manner. The stones should be of uniform size, with bond stones to tie the two faces of the wall together. But the most usual and efficacious mode of bonding a wall of this masonry is by horizontal bands of a few courses of squared masonry or of brickwork extending through the thickness of the wall and all round the building. Such a method was commonly employed by the Romans; walls of rubble-work, sometimes very rough, sometimes no better than coarse concrete, and alternated with bands of the thin Roman brick, constitute a general type of Roman masonry. M. Renaud says the intervals of the courses of bricks were from 1 metre to $1^{\mathrm{m}} \cdot 4$.

But the most advantageous employment of this description of masonry is in revetments and such like walls of fortifications. Such walls are generally of considerable thickness, and have generally only one fair face-two favourable circumstances for its use. If well built with stones of nearly equal size and with good mortar well filled in, it offers a better resistance to artillery fire than
squared masonry, unless of very superior description. It is necessary in such eases to put a faeing to the wall of rather larger and more regular stones, and to make the exterior face perpendicular, in order to avoid the growth of vegetation on it. Renaud says truly that in all masonry of small pieces the bond is not so important as the mortar and the care of execution.

Brickwork.-If the stones of a wall of squared masonry were obliged to be all of one size, then, in order to ensure the effective bonding in the horizontal courses, the length of the stones should be double the width; then the stones in one horizontal course would be laid lengthways and in the next course crossways, or, as it is technically called, stretchers and headers. See Bond.

Mortar.-The mortar and the workmanship are of more consequence in brickwork than the bricks themselves. If the pressures on a wall were always exactly vertical, then all that is required in the building would be to distribute those pressures over the parts intended to carry them; but there is almost always a liability of inclined pressures on all walls, and a certain inclined pressure on many: hence the mortar is subject to a shearing strain, and much of the efficiency of the wall depends on its strength and adhesion to the bricks. This is particularly the case in walls of fortifications exposed to artillery fire. It is invariably found in cases of breaching brick walls by artillery that the brickwork is shaken for considerable distances beyond the actual rupture, and that the line of shake almost always follows the joints. The line of pressure in this case being nearly horizontal, the only resistance to motion besides the weight of the portion of the wall is the resistance to shearing of one or more joints of briekwork. Hence it is especially important in walls of brickwork and squared masonry so exposed to arrange them in large masses.

The proper thickness of the joint of mortar in briekwork must depend somewhat on the nature of the lime used. If the mortar is equal in strength to the bricks, the joints may be as large as the bricks, as was the case in the old Roman brickwork, the joints of which were from $0^{\mathrm{m} \cdot 02}$ to $0^{\mathrm{m} \cdot 03}$. This was partly on account of the thinness of the Roman brick compared with its area. When the mortar is inferior in strength to the bricks, as is generally the case, the less there is of it the better. Hosking recommends that with bricks $2 \frac{1}{2}$ in high no four courses should reach $11 \frac{1}{2}$ in., which allows less than $\frac{4}{10}$ in. for a joint. He also points out that thick joints of mortar (unless of superior description) are more liable to injury by wet and frost, and to cause a greater amount of settlement in the wall. The minimum size for a joint is simply that no two bricks should touch each other.

The necessity of using clean and moist bricks will be pointed out in our article on Limes and Mortars, and also the advantage of using stiff mortar, as compared with the more common practice of Grouting, which is the term used for a very liquid mortar poured over the wall at every one or two eourses, with the object of filling up the joints thoroughly. Besides the evils of drowning the lime, and saturating the brickwork, and tempting the bricklayer to use wet mortar and dry bricks, it is evident from the foregoing considerations that a wall so built would not have as much resistance to shearing in its joints as one well filled up with stiff mortar.

Secondary Parts.-By the secondary parts of walls are meant cornices, string eourses, windowsills, chimneys, and so on.

These are frequently made of stone, although all the remainder of the building may be of brick. The object of a cornice or coping, as far as construction is concerned, is to prevent wet from entering in the top of the wall. It should therefore project over the exposed face of the wall, and be sloped on the top, and hollowed out on the under-side, to prevent the water from running on to the wall; and it-should be itself impervious to water. Stones, when used for such purposes, are generally connected together by metal cramps or stone dowels. A cornice or coping of brickwork should be made of bricks, specially moulded and burnt for the purpose, and set in hydraulic lime. The formation of a cornice by projecting courses (or corbelling) is subject to the same considerations as in footings; the pressures are exerted on the projections in a similar manner, but in the reverse direction. String courses are now seldom used, exeept for purely ornamental purposes, and for these it is only necessary to bed and bond the stones or bricks sufficiently into the wall to give them stable support. For constructive purposes, string eourses are bonding courses in the wall itself, and should therefore go through the wall.

Window-sills are more frequently made of stone than any other of these parts, on aceount of the transverse strain they are subject to: in order to preserve the horizontal line, the sill is generally made in one piece, resting on the piers of the wall at each side, and not on the filling-in part between; otherwise there would be danger of fracture, owing to the unequal settlement of the piers and the intervening part. A window-sill is the coping of that part of the wall it eovers, and should therefore be shaped similarly to a coping or eornice.

If there is no danger of irregular settlement the sill need not be made in one pieee.
To provide a place for the fire and its flue in each room in climates where they are neeessary, part of the wall is made thicker from the foundation; eonsequently, it is desirable to arrange the fire-places in the successive stories so as to come over each other. The space required for a grate for an ordinary room varies from 3 to 5 ft . wide, and about $1 \frac{1}{2} \mathrm{ft}$. deep, and 3 ft . high, according to the size of room and grate; the wall at the back of the grate should be at least one brick thick. The sizes of the piers or jambs, as they are called, depend on the number of flues to be inserted in them; for the flue of each fire, after being gathered into a throat immediately above the fire, is diverted on one side to allow space for the fire-place of the next story above it. A flue, if built in brickwork, cannot easily be less than 9 by $4 \frac{1}{2} \mathrm{in}$., and as there should be at least half a brick between each flue, and one brick on the outside of the outermost flue, the size of the jamb for a given number of flues is thereby determined. It is difficult to give precise rules for the arrangement of fire-places, beeause so much depends on the grate. The problem is to provide as easy an exit as possible for the products of combustion, and yet not more than they require. This is met theoretically by inclining the sides of the fire-place gradually to an opening close above the fire and calculated to be just large enough for the products of combustion; this part of the problem is generally provided for in the grate; it is like the sluice-gate of a water-mill, excepting that air,
being so much more delicate a substance than water, requires much more careful provision for its flow. After passing through the grate, it should be allowed to flow into the open air as regularly as possible: there is, therefore, an advantage in having very smooth sides to the flues; this is generally effected by plastering the inside of them with a special plaster to resist heat; it is common hair mortar mixed with cow-dung. A much better mode has been lately employed, by inserting glazed earthenware pipes in the wall, which gets rid of the objectionable corners of the brick flue, and of its unnecessarily large size. The large size of the flue both decreases the velocity of the stream of hot air, and allows cold air to descend and check the flow of it; the flue should decrease in size slightly as it ascends.

Carrying all the flues up together in one stack, or chimney-breast, as it is called, is both economical, and tends to preserve an equal temperature in them; for the latter reason, which is a most important one, brickwork is better for chimneys than stonework. But for the last few feet above the roof, each flue should be in a separate stack of its own, and as small as possible, for everything that obstructs the wind tends to produce a counter-current in the flue. No two fires should lead into one flue, unless they are always alight together, or a down-draught will occur probably in the one not lighted. No woodwork should be built into the chimney-breast or any part of the chimney for any purpose whatever.

Thickness of Walls.-The thickness of the walls of ordinary buildings, Fig. 2055, is more a practical than a theoretical question. As M. Renaud points out, the theoretical thickness to meet the vertical pressure only would be very small in any case. The rupture of the wall by separation of the joints would almost always take place before the crushing of the stone itself, and that force should, therefore, be provided for, or at least the two must be taken into consideration together. With squared stones the crushing would be the most probable, and in this case generally the safe weight is taken at $\frac{1}{10}$ the breaking weight. In rubble-work the tendency is to rupture of the joints, owing to the irregular pressures; the proportion of the safe weight to the breaking weight
2055.
 should therefore be less.
MI. Renaud quotes the following theoretical rules from Rondelet for the thickness of walls of ordinary buildings :-

For Isolated Walls,

$$
b=12 \sqrt{\frac{\bar{h}}{s}} \frac{\text { where }}{b}=\left\{\begin{aligned}
h & =\text { breadth } \\
h & =\text { height } \\
s & =\text { weight of cub. mètre of masonry. }
\end{aligned}\right.
$$

This rule is evidently based on the assumption that the force acting on the wall is similar to that of the wind, in which case the equation for overturning the wall would be $s \cdot h \cdot b \cdot \frac{b}{n}=p \cdot h \cdot \frac{h}{m}$, where $p=$ force per unit of area of wind, and $n$ and $m$ are coefficients depending on the circumstances in each case.

For Walls of Houses, Fig. 2056,
$\left.\frac{l h}{\sqrt{l^{2}+h^{2}}}\right\}$ where $l=$ span or distance between opposite sides in mètres.
Which appears to be based on the assumption that the pressure on the walls varies as the span of the roof. The height $h$ in this case is to be taken between the floors or points of support supposing them to be of strong construction.

He states the following to be the usual thicknesses of walls in Paris; they are of squared masonry of limestone:-


The interior faces of the walls are always made perpendicular, and the batter or increase is always on the outside.

In London the minimum dimensions for walls of houses in streets is fixed by Act of Parliament, in order to provide for the strength and security from fire of rows of houses with common party walls. But these dimensions must not be taken as guides for military buildings of a permanent character ; in such buildings it is not safe with the ordinary description of brickwork to make any wall less than $1 \frac{1}{2}$ brick thick; and, in accordance with the principle of the Act, an additional half brick should be added to the thickness for every two stories of height. This addition is generally made in England on the inner face of the wall, for the convenience of resting the beams of the floors on them. This is perhaps a better constructive arrangement than the French one, but it must depend upon the circumstances of each case which is adopted.

Footings.-In the process of constructing a wall, the mason or bricklayer first lays the footings on the foundation platform. The footing is an enlarged portion of the wall for the purpose of distributing the weight over the foundation: it is properly a portion of the wall and not of the
foundation, although it is not always easy to draw the line between them. When the pressures pass down through the centre of the wall, the footings may project equally on each side; when otherwise, the footings should be so arranged that the line of pressure shall pass nearly through the eentre of them into the foundation. The size of footings and the mode of forming the increase to the thickness of the wall must depend on the circumstances and the material. For ordinary buildings Tredgold recommends that the extreme breadth of the footing, when the subsoil is clay or sand, be double the thickness of the wall; if on gravel or chalk subsoil, that its breadth to be that of the wall as 3 to 2 .

Supposing the whole pressure per lineal foot on the wall to be equally distributed over the breadth of footing ab, Fig. 2057, then the reaction of the subsoil on the part $b c$ will be equivalent to that proportion of the whole pressure, acting upwards and tending to break the projecting part $b c$ about the section $c d$, which section must be strong enough to resist that transverse strain; in brickwork it is usual to make the projection of a footing for light buildings $\frac{1}{4}$ of a brick in every course, and for heavy buildings $\frac{1}{4}$ of a brick in every two courses. In stonework the proportional projection for a given height of course may be greater, according to the relative transverse length of the stone. The footings should always be made of large stones or of picked bricks, laid in very good mortar, and well bonded, with the object of distributing the pressure as uniformly as possible over the foundations. The foundation platform should, if possible, be in one horizontal plane, and the footings should be equal in height throughout the main
 walls of a building, in order to avoid, as much as possible, irregularity of settlement from unequal heights of wall.

The Damp Course, as it is commonly called, a course of some impervious material to prevent the damp rising from the ground through the masonry into the body of the wall. It is generally placed immediately above the footings, if these project above ground; but the damp course should be, if possible, 1 ft . above the ground. It generally consists of two or three courses of hard-burnt bricks laid in hydraulic mortar. A highly-burnt glazed hollow brick is made for the purpose, the perforations being horizontal, so that a current of air passes through the wall at that point. Perforated bricks are liable to crack under pressure.

## Table of the Pressures on the Footings of Walls.

From Tredgold, Encyclopædia Britannica, and Renaud.
Tredgold's Rule for breadth of walls $\mathrm{W}=\frac{1}{4} f b d$, when $b d=$ area of cross-section of part of wall under consideration, $W=$ safe weights in lbs. $f=$ tabular number $=\frac{1}{4}$ th the splitting or $\frac{1}{8}$ th the crushing force, in lbs. per square foot.

| Name of Building. | Value of $f$. Tredgold. | Pressure per square ft . Tredgold. | $\begin{aligned} & \text { Pressure } \\ & \text { per square } \\ & \text { centime. } \\ & \text { RENAUD. } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Pillar of All Saint's Church, Angers, Forneaux stone | $\begin{aligned} & \text { lbs. } \\ & 110,000 \end{aligned}$ | $\begin{gathered} \text { lbs. } \\ 86,000 \end{gathered}$ | kilos. |
| Pantheon, Paris, pillar Bayneux stone | 62,000 | 60,000 | $29 \cdot 43$ |
| Elgin Chapter House, central pillar, red sandstone .. | .. | 40,000 | .. |
| Piers of St. Paul's Dome, London, squared limestone, Portland | .. | 39,000 | $19 \cdot 35$ |
| Piers of St. Peter's Dome, Rome, coursed rubble .. .. .. |  | 33,000 | $16 \cdot 35$ |
| Keystone Neuilly Bridge, Paris, Sallancourt stone .. .. | 30,000 | 18,000 | .. |
| Party Wall, basilica of Constantine, Rome, coursed rubble faced with brick | .. | .. | $24 \cdot 51$ |
| Palace of the Chancellerie, Rome, columns of rez de Chaussées, |  |  | $34 \cdot 11$ |

General Principles of Wooden Roofs.-Under the head of roofs are included all kinds of coverings of a permanent character to buildings; both the covering itself, which has to keep out the weather, and the framework which has to carry it.

If the covering is quite impervious, it may be nearly flat; but a horizontal beam, or framed girder, is not the most economical mode of spanning a large opening, especially in wood. As far as the framework is concerned, it is more cconomical to arrange it in one or more inclined planes or in a curve; and as few descriptions of covering are quite impervious or can be conveniently built in curves, the inclined planes are the most suitable mode of forming roofs in general. The inclination of the planes and the number of them must depend on the climate, the nature of the covering, and the circumstances of the case. With the same description of covering, supposing it to be only partially impervious, the inclination should vary according to the rainfall and snow of the locality; speaking very generally, it may be said to vary according to latitude; but this would be too uncertain a guide for an architect. There must be a certain inclination for each country, at which a particular kind of covering will keep out the rain and snow. These inclinations for the English climate, for different kinds of coverings, will be given hereafter.

Garbet has stated a very proper general principle to be observed in all roof-frames, of whatever inclination, that the numbers of orders of bearers should be as few as possible. This will be best illustrated by describing the arrangement of ordinary roofs. When covered with slates or tiles, or any similar covering in small flat pieces, the bearers immediately under them must consist either of boards extending continuously underneath, or of battens placed horizontally at the interval of two courses of the slates or tiles. These again must be carried by beams called rafters, placed down the slope of the roof at right angles to the others. This arrangement Garbet would call two orders of bearers ; it is the mode generally adopted in roofs not exceeding 20 ft . span : each pair of opposite rafters are generally tied together, either by a tie-beam at their feet or by a collar-beam about half-way up; from which this has got the name of the collar-beam roof.

When the span exceeds 20 ft ., the bearers would become so large that it would be economical to introduce an additional order of bearers. The ordinary method is to support the common rafters by one or more beams extending horizontally beneath them, called purlins, which themselves are supported on other rafters below them, which latter being placed at considerable intervals, are much stronger, and are called the principal rafters. The common rafters rest against a horizontal beam standing on the foot of the principal rafters, and called a pole-plate, and abut against a horizontal ridge-piece at the top; thus they have little strain on them beyond the transverse strain between the purlins. The purlins are subject to a transverse strain between the principal rafters. The principal rafters are supported by a framework of which they themselves form a part, and which varies according to the span of the roof. This framework is called a roof-truss or a trussed principal, and forms the final and most important bearer of the roof. For spans between 20 and 30 ft ., this truss generally consists of two struts supporting the principal rafters half-way down, and meeting in the centre, where their combined vertical force is suspended by a vertical rod or post called a kingpost, to the vertex of the roof. The horizontal force at the foot of each principal rafter is carried by a tie-beam, which also supports the ceiling, if there is one, and is itself supported by the king-post.

When the span is above 30 ft ., in order to provide intermediate points of support for the principal rafters, two vertical posts, in such case called queen-posts, are introduced each with its struts, the heads of the posts being connected by a horizontal beam, called a straining-beam, which carries the horizontal strains arising from them.

The general principle of such trusses is to carry the strains of the roofs in such a way as to relieve the walls from any but vertical strains.

Garbet would call the above arrangement one of four orders of bearers, namely:-1. Boards. 2. Common Rafters. 3. Purlins. 4. Principal Rafters. The usual interval for such "trussed principals" in ordinary roofs is 10 ft ., and the interval of such purlins is about 6 ft .: with those fixed intervals this form of roof-frame is applicable to a considerable range of spans. It is evident, however, that as the span increases, it will at last be more economical to increase the interval of the principals, and to strengthen the purlins accordingly by framing or "trussing" them : and that for small spans it will be economical to combine the princıpal and common rafters in one, and to dispense with the purlins. The span at which it is desirable to introduce an additional order of bearers, in any such system of framework for constructions, is determined by considerations both of economy and stiffness; for, besides the calculations.for the comparative cost with or without the additional bearer, it is necessary to consider the deflection; and as that varies directly as the cube of the length between the points of support, it may be desirable in some cases, where it is necessary to limit the absolute deflection, to introduce an additional order of bearers before the economical limit has been reached.

The examination of the strains exerted m a queen-post roof will sufficiently explain them for all this class of roofs. The boards or battens are subject to a transverse strain between the points of support afforded by the common rafters, which is made up of the weight of the roof covering and the allowance for pressure of wind (reduced to the vertical direction); they should never have a long bearing, as stiffness is very essential in the bearers of the root-covering. Each common rafter is subject to a transverse strain between its point of support and to a longitudinal compression from the reaction from the opposite rafter, the force causing both these strains is that of the weight of roof-covering and boards, and pressure of wind bearing on the area supported by the rafter. The purlins and pole-plates and ridge-preces are subject in transverse strains arising from the pressure of the common rafters they carry divided proportionally between them; the pole-plate also carries the longitudinal compressile pressure of the rafters, and transmits it to the principal rafters. The principal rafters are not generally subject to transverse strain, except from their own weight, but chiefly to longitudinal compression, because the purlins are generally placed immediately over the points of support of the rafter. Thus, at the junction of the lower strut with the rafter, the pressure of all that part of the purlin and its superincumbent weight that is carried at that point, is resolved intc a compressile strain down the principal rafter, and another down the strut; and at the junction of the queen-post, principal rafter, and straining-beam, the pressure of that purlin, together with the vertical strain from the queen-post, are resolved into a compressile strain down the principal rafter, and another along the straining-beam: and at the upper strut a similar compressile strain results; and at the vertex, the pressure of the ridge-piece and vertical strain from the king-post are resolved into compressile strains down the principal rafters. Thus, the principal rafter is subject to accessions of compressile strains at these points from the vertex to the foot; and each part of it must be calculated accordingly to resist those strains. The struts are subject to compressile strains arising as above mentioned. The king-post and queen-posts are subject to tensile strains arising from the vertical components of the compressile strains.

The tie-beam is subject to the tensile strain which is the horizontal component of the compressile strain down the principal rafter; the vertical component of the same being carried by the walls. Tie-beams are generally made much larger than necessary to resist this strain only, even after allowing for the usual coefficient of safety, because they have generally to carry the transverse strain arising from the ceiling of the rooms below. The beams of that ceiling are generally fixed
to the tie-beams, each of which has therefore to carry all those beams and ceilings between two of the main trusses of the roof. For this reason the depth of the tie-beam is generally made much greater than its breadth. Each of these frames, or trusses, or framed principals, as they are called, are generally put together on the ground and hoisted into their places in one piece. The effect of putting the weight of the roof on this framework will be to deflect it a little out of its original form ; some of the beams will become shortened and some lengthened; this should be taken into consideration in the design of the roof, as it not only throws the strains in different directions to those in the original form, but it alters the slope of the roof and may affect the junction of the eaves with the gutters.

The ends of the tie-beams generally rest on wall-plates or templates, and are sometimes notched on to them. The best position for the wall-plate is in the centre of the thickness of the wall, and the junction of the rafter and tie-beam should come immediately over it; and the most effectual mode of finishing the lower part of the roof is to allow the common rafter to project over the wall, and so carry the rain clear of the building. As the ends of the common rafters rest on the poleplate, the whole pressure from the roof thus falls on the centre of the wall. When a roof is so finished, the lower part of it projecting over the wall is called the eaves: there is therefore an open space between every common rafter, which must be filled up to prevent the wind from entering the space between the roof and the ceiling. This is generally done with woodwork, which is called the eaves' boarding. In any method of finishing the eaves of a roof, the ends of the tie-beams should not be imbedded in masonry, or they will be liable to decay before the remainder of the roof. When there is a parapet or blocking course above the eaves of a roof, then either the parapet must rest upon projecting courses corbelled out from the outer face of the wall, or the wall-plate must be placed nearer the inner edge of the wall; in which latter case the pressure of the roof will not coincide with the axis of the wall. The wall-plate is not unfrequently placed close to the inner edge of the wall, and the junction of the tie-beam and rafter placed on the inner side of the edge; a practice which not only has the disadvantage of directing the line of pressure out of the axis, but subjects that short part of the tie-beam to transverse strain, therehy also causing a tendency to draw the wall-plate off the wall.

The ordinary method of framing these beams together, so as to send the strains in the directions required, will be considered presently; and Tredgold's rules for calculating their dimensions will be given hereafter.

Flat roofs are used in countries where there is little rainfall, as in the southern and eastern coasts of the Mediterranean, probably both from their usefulness and economy. The flat roof is not only the most simple mode of covering the house-a mere repetition of a floor-but adds another story and a garden to it. The following extract from a paper by Sir H. Jones on Maltese houses shows the mode of constructing a flat roof in that island: Beams of red pine from the Adriatic, about 10 by 12 in., are placed horizontally across the walls, at central intervals of about 4 ft . Sometimes a series of arched ribs of stone are built across from wall to wall instead of the beams. On these flat stones of the soft Maltese sandstone, about 4 ft . long, 10 or 12 in . wide, and 3 in. thick, are placed, butting close together without mortar; on these a concrete of red argillaceous earth and small stones, and called "Forba," is laid, with a slight slope on the upper surface to carry off the rain that falls: this forba is kept moist and beaten with rammers. When dry, there is spread over it a layer of cement $\frac{1}{4}$ in. thick, composed of four parts lime, three parts puzzolana, or of five parts lime and three parts pounded earthenware. This is spread in a liquid state, and then beaten with rammers and worked with trowels, and kept moist until it has set. The beating is done by women kneeling, with hand boards.

In countries where stone suitable for this purpose cannot be procured, timber is used instead, or a combination of wood beams and flat tiles, or wood beams and concrete of a quick-setting cement is used. Asphalte is sometimes substituted instead of cement for the exterior coating in this country when flat roofs are made; there is some danger in using it, on account of the expansion and contraction of the framework below. It is necessary in all flat roofs to make the framework very stiff-that is, to avoid any considerable amount of deflection, which would break the impervious coating of cement or asphalte; consequently, it would be desirable to introduce an additional order of bearers, or to provide in some other way for stiffening the structure at a smaller span than in an ordinary wooden floor. For the same object of stiffening the framework, it should be connected with the walls as firmly as practicable; the ends of the beams should be framed into the wall-plates or templates, which should rest on the wall as near the centre of its breadth as practicable. When a masonry parapet is made round the walls, great care should be taken in the construction of the open gutters at the sides to carry off the roof-water, and prevent it getting into the walls; frequent outlets through the parapet should be provided. Solid stone forms the most effectual gutter : and next to that, perhaps, a concrete of hydraulic cement.
$G$ Gothic and Arched Roofs.-In England roofs are generally made in one plane from the ridge to the eaves; but in France and some other countries, it is common to make them in two planes; this is partly for the architectural effect, but chiefly for the economy of gaining an additional story in the roof. The general method of arranging the roof is the same in almost all cases; there are the principal trusses at intervals, the purlins, the common rafters and boards: it is only the framing of the truss that varies. In the principal truss of the Mansard roof (as that of two planes is called, from the architect who first employed it extensively in Paris), besides the main rafters, which might be made to balance each other without ties, there are tie-beams from each angle to the next but one, and connected together at the crossing; these ties do unt carry the whole horizontal strain of the roof, like that of the English truss, but leave a portion of it to be carried either by the walls, or by the strength of the bolt and beams at the crossing of two such thes.

In some of the old Gothic wooden roofs, the frame of the principal truss was made without any direct ties, chiefly for the sake of the effect when the timbers of the roof were visible, but also because the high pitch of the roof caused the horizontal strains to be less. 'The principal rafters
were gencrally supported by a collar-beam high up, and at one or two points lower down by vertical posts resting on the ends of horizontal pieces projecting from the point of support next below, the projecting points of junction of these vertical and horizontal picces being connected by curved struts: the series of these curved pieces being carried from the walls on each side up to the collarbeam in the centre, they formed an imperfect tie to the roof-imperfect because it depended on so many joints, and because, if perfectly jointed, it resisted deflection by the strength of the joint, and not by the beam itself. The lowest of these curved pieces, being carried to a point considerably lower in the walls than the eaves, brought the general line of pressure of the roof to a lower point on the wall, and thus reduced the necessary thickness of the wall to meet it. Hampton Court Palace contains a good specimen of these roofs; those at the old palace at Eltham, and at Westminster Hall, are also remarkable examples.

When the span of a roof exceeds 50 ft., the queen-post roof-truss becomes heavy, and uses a large proportion of timber in the dead weight of the bearers: the polygonal or curved truss is one of the forms which has been proposed to avoid this ineffective use of timber in very large spans. In it a polygonal or curved rib of wood is formed, to which the tie-beam is suspended, and from which the purlins are supported: the curve of this rib should be such that a line of pressure will pass through it, thus putting it entirely in compression throughout its length; the tie-beam and suspension-bars will then be in tension. Dobson states that Philibert de Lorme was one of the first architects who proposed a polygonal rib of wood, which he constructed of two or more thicknesses of plank, the pieces of which were in equal lengths, and their ends abutted against each other; each thitkness of plank thus forming a polygonal rib; the two thicknesses were bolted together.

Walls of Framed Work.-Walls of wooden buildings generally consist of vertical pieces, placed close enough together to support the boarding or plastering with which they are covered, and resting on a horizontal piece at bottom, commonly called a ground-plate, and connected at the top by a wall-plate. When the ground-plate is supported continuously by a wall, it only performs the part of an ordinary plate-that of distributing the pressures equally; and the uprights, or posts or studs, as they are sometimes called, are so many wooden columns, carrying each its proportion of the vertical pressure. No piece of qualrilateral framing can be put together so firmly as to resist completely some small distortion of shape from lateral pressure ; therefore, it is most desirable in every wooden wall, as in every piece of framework, to have some inclined pieces framed unto the structure, by which the pressures can be resolvable into triangles. These inclined preces, which are called struts or braces, are sometimes only battens fastened across the studs. The corner-posts are generally larger, as they have to withstand a double lateral pressure. It would make a stiffer, though a more expensive wall, if the studs were placed farther apart, and, consequently, made of large cross-section, and an intermediate order of bearers for the boards was introduced. In America a wall is sometimes made entirely of boards in two or more thicknesses, placed at right angles or diagoually to each other. When a wooden wall has to span an opening, as is frequently the case in the interiors of houses, in partition walls covered with lath and plaster, it becomes a very deep-framed girder, and should be framed on the principle of the queen-post truss, in which the braces would represent the principal rafters, and the top plate would represent the strainingbeam, and the bottom plate would be the tie-beam, which should be supported from the top plate and braces by the vertical studs; in such a framewrork the studs below the braces would be in tension; those above the braces in compression. The bottom plate might in such case support part of the weight of a floor or ceiling. In arranging the openings for doors and windows in wooden walls, great consideration should be paid to the position of the braces; the studs on each side of the opening should be made stronger than ordinary studs, in order to form the framework of the door or window; and with a careful arrangement of the braces in connection with these frames, these openings may be made to add to the stability of the wall.

Scarfs.-In all searfs and joints in woodwork, it is an important principle that the bearing parts should have as large a surface as possible, in order to save the fibres from injury. For the same reason, timbers in framerwork should be cut as little as possible. Also, that the inevitable expansion and contraction of the wood slould be borne in mind in the form of the joint.

The following notes on scarfs and joints are extracted from Tredgold's Carpentry :-

1. Longitudinal sarf to resist tension only, such as for some tie-beams, king-posts, and so on.

Fish-joint, Fig. 2058. -In this the two pieces of timber abut on each other, end to end, and an iron or wooden plate is fastened on each side by bolts passing through the beams; the tensile strain is thus passed through the bolts to the fibres of the timbers. The slrinkage of the timbers loosens this joint, and the pressure of the bolts injures the fibre; the total area of bolt-sections should be at least equal to $\frac{2}{2}$ the area of the section of beam.

Indented Scarf, Fig. 2059.-To determine the depths of indents and length of scarf, the tensile

2059.
 strengtb of $b c$ should be equal to the compressile strength of $a c$ and to the shearing strength of $c d$. Therefore, when the compressile and tensile resistances of the wood are equal, $b c$ should be $\frac{1}{2}$ the depth of the whole beam.

In fir and other straight-grained wood, $c d=16$ to $20 c b$.
In oak, ash, elm, and such woods, $\quad c d=8$ to $10 c b$.
In fir, and so on, a scarf with bolts only, the whole length of scarf $=6$ times breadth of beam.

In oak, " $", \quad " \quad=2$
2. Longitudinal joint to "resist compression: "̈s for pillars, struts, and so on.

Fish-joint, Fig. 2060.-If four fishing-pieces are used, one on each side of the joint, and a piece of hard timber is inserted between the ends of the pieces, to distribute the pressure equally, an effective joint can be made without injuring the fibres.

Indented Scarf, Fig. 2061. -Similar calculations can be made for this, as for the joint, to resist tension: a key or double wedge of hard wood is inserted at $a$ to ensure a fair bearing; they should not be driven too tight, or the fibres of the beam may be injured. No bolts or plates are required for it; but a tongue or mortice and tenon at each end is necessary, to prevent side shifting, as at $c$, Fig. 2059.

Plain Scarf.-This joint is to be recommended, as it does little injury to the fibres, and the bearing surfaces are large and perpendicular to the strains: but it requires bolts and plates, Fig. 2061.
3. Longitudinal joint to resist transverse strain, as
 in some tie-beams and girders.-This joint is recommended. The depths of indents and lengths of scarfs may be calculated as for tensile strain. It is necessary to have a plate over the joint on the upper and lower sides, as at $a b$, and bolts through the beams; at $d$ are the hard-wood wedges, Fig. 2062.

Straining-beam.-A horizontal beam may be connected with and supported by a vertical beam (as in the case of straining-beams in a roof) in the manner shown, Fig. 2063.


Joints.-4. Transverse joints, horizontal, as in joists, purlins, and all such beams resting on or framed into others, and subject to transverse strain.

Framed Joint.-This is applicable to horizontal beams, which are to be framed in between two others, in order to avoid increasing the total depth of the bearers, such as the bridging of joints in a framed floor. A projection is formed on one beam, called the tenon, and a hole to receive it cut in the other, called a mortice. The mortice should be cut in the neutral part of the beam-that is to say, in the middle of the depth. The tenon should be as near the bottom as practicable of the other beam. The form in Fig. 2064 is recommended for combining these objects with other requirements in the framework. The tenon is $\frac{1}{6}$ the depth of the beam in thickness, and at $\frac{1}{3}$ the depth from the lower side. The object of the bevelled part above, and of the shoulder below, is to strengthen the bearing surface of the tenon.

It is almost impossible to make a tenon fit so well into a mortice that it will replace the original wood in the hole; and even if it did at first, the shrinking of the wood would soon loosen it. Therefore, double tenons are not recommended in carpentry, on account of the difficulty of making both of them to bear equally.

Notching, Fig. 2065.-When the other arrangements of
 the framing admit of it, it is always better to connect two horizontal beams together, which are subject to transverse strain, by notching-that is, by resting one across the other, and cutting a kind of tenon in the lower one, and a corresponding
mortice in the upper one. There is both a practical advantage in this, in being able to use long timbers, and a theoretical one, in getting a better bearing, and in having the upper beams virtually fixed at each end, instead of merely supported, as in the former method.

Cross-framing, Figs. 2066, 2067, as in wall-plates and such beams, which have to be connected together and are not subject to transverse strain.

The beams are halved into each other, as it is called, half the thickness of the beam being cut out of each. Sometimes the form of this halving is dovetailed, as in Fig. 2068, which is objectionable, because the shrinking of the timber causes the joint to become loose; for this reason no dovetarled joints are to be recommended in carpentry. The better form for the halving is shown in Figs. 2066, 2067.
5. Transverse joints, perpendicular, Figs. 2069 to 2071 ; such as posts or studs into plates.-The object of every system of joints is to reduce all the pressures into the direction of the axes of the pieces.

The ordinary mode of connecting two beams, the one perpendicular and the other horizontal, is by cutting two shoulders in the end of the perpendıcular piece, leaving a tenon or tongue between them, which fits into a corresponding mortice or hole cut in the horizontal pier. As the only object in the tenon and mortice is to prevent lateral motion, Tredgold recommends a short tenon having a thickness of about one-quarter that of the timber

It is evidently of great importance that the tenon and mortice should be cut square and accurately; on account of the difficulty of ensuring this, Tredgold recommends an angular-shaped or a curved joint, as shown in Figs. 2070, 2071.
6. Inclined joints, Figs. 2072 to 2074 ; such as principal rafters with tie-beams.-The resistance of a joint is always most effectual where the abutment one piece rests against is perpendicular to the strain down that piece. Thus, in the figure, if the strain down the rafter is in the direction $c b e$, and $b a$ is one of the abutting surfaces, then, if $a c$ be drawn perpendicular to $b a$, and $b d$, the other abutting surface, be made parallel to $a c$, then in the triangle of forces $a b c$, if $b c$ represent the whole strain, $a c$ will represent the strain on $b a$, and $b a$ will represent the strain on $b d$ : thus the two components of the whole strain will be perpendicular to the two abutting surfaces. Tredgold recommends that $b d$ should be rather more than half the depth of the rafter. There should be a tenon at the under-side of the rafter, and a mortice in the tie-beam, to prevent the beams slipping over each other; the thickness of the tenon should be about $\frac{1}{5}$ that of the beam. As the shrinking of the wood, which is sure to occur after the framework is up, will have the effect of lowering the slope of the rafters, and thus bring a greater pressure about the point $a$ of the joint, that part of the joint should be left a little open, as shown in the figure, in the original construction of it.

Tredgold recommends either of the accompanying forms for this joint, in which the tenon is made on the tie-beam, and the mortice on the rafter; it has the advantage of showing the whole of the workmanship of the joint more completely to view than the other method.
7. King and Queen Posts, Figs. 2075, 2076.-The upper end of the principal rafters may be framed into the king-posts in a similar manner to the lower end into the tie-beam. But in this case the joint, when originally made, should be left slightly open at $a$, as the effect of a settlement from shrinking will be to bring a greater pressure on that part. Or, the upper end of the kingpost may be enlarged and cut with a shoulder at right angles to the direction of the rafter. In both these cases there should be a small tenon and mortice to prevent the beams slipping over each other sideways. The lower ends of the struts may be connected with the king-post by either of the above methods; if the king-post is in two pieces, a very good joint may be formed with the principal rafters, by letting them abut against each other between the two parts of the post, as in Fig. 2075, the two parts being bolted together above and below the junction.

The struts and principal rafters may be connected by a joint similar in principle to that of the rafter and tie-beam. The purlins, pole-plates, and ridge-pieces, are generally notched into their respective beams.

The following practical rules by Tredgold for determining the size of the beams in wooden roofs, of the description of framework before mentioned, are based on the theoretical calculations for the resistance of beams to deflection. He apparently considered that the amount of deflection in a framework is of as much importance as the absolute strength; then assuming that for security the deflection of any beam ought to vary inversely as its length, and starting with some fixed arbitrary deflection for a given beam, and assuming that the arrangement and intervals of the beams and the loads upon them are constant, he obtained from these data and his own judgment a set of constant coefficients for the different beams for fir wood and oak wood, in terms of their dimensions, span of roof, and so on. These rules generally give large dimensions compared with those for absolute strength.

Principal rafters, assumed central interval $=10 \mathrm{ft}$.

$$
d=0.96 \frac{l^{2} \mathrm{~L}}{b^{3}}\left\{\begin{array}{l}
b d=\text { breadth and depth of beam in inches, } \\
l=\text { length of beam in feet } \\
\mathrm{L}=\text { span of roof in feet }
\end{array}\right.
$$

which is virtually the theoretical equation for a long beam subject to compression in the direction of its length.

Straining-beam.- $d=0.9 \sqrt{l \sqrt{\bar{L}}}$, which is also the same theoretical equation, $b$ being taken constantly as $=0.7 d$, which is an arbitrary assumption.

$$
\text { Struts.- } d=0.8 \sqrt{l \sqrt{\overline{\mathrm{~L}}}}, b \text { being taken constantly }=0.6 d .
$$

King-posts, $-b d=0 \cdot 12 l \mathrm{~L}$.
This rule for a beam under direct tension, is evidently based on the same law that the elonga-

2076.

tion should be inversely as the length of beam: and this gives of course much larger dimensions than are required for absolute strength.

Quecn-posts.-bd=0.27l $\mathrm{L}_{1}, \mathrm{~L}_{1}=$ length in feet of that part of tie-beam carried by the queen-post.
Tie-beams.-d $=1.47 \frac{l_{1}}{\sqrt[3]{ } \sqrt{b}} ; l_{1}=$ length in feet of longest part of tie-beam unsupported.
This is virtually the same as the theoretical equation for the deflection of a beam subject to transverse strain; in this case, apparently, the tensile strain on the tie-beam is expected to counterbalance to some extent the deflection caused by the transverse strain.

Common rafters.- $d=0.7 \frac{l_{11}}{3 \sqrt{\bar{b}}} ; l_{11}=$ length in feet of bearing between one purlin and another.
Common rafters should be at least 2 in. thick to hold the nails of the laths or boards.
Purlins. $-d={ }^{4} \sqrt{\mathrm{D} l^{3}{ }_{111}} ; \mathrm{D}=$ distance in feet of purlins from each other.
$b$ is assumed constantly $=0.6 \mathrm{~d} . \quad l_{111}=$ length of bearing of purlin in feet.
These two last are virtually the same as the ordinary theoretical equations for deflection of beams subject to transversc strain.

Roof Coverings.-The object of all roof coverings is to keep out the wind, rain, and snow, and to keep an even temperature inside as far as practicable.

A roof covering of small pieces is generally heavier, more difficult to keep dry, and requires a steeper slope than one of large pieces, but the material itself is cheaper.

In a roof covering, the crown or meeting of the planes at the top is technically called the Ridge; the meeting of two planes forming a salient angle down the slope of the roof, as frequently occurs at the end of a building, is called a hip, and such a roof is said to be hipped. The meeting of two planes forming a re-entering angle down the slope of the roof, as occurs at the junction of one roof at right angles to another, is called a valley; the lower edge of the roof, where it meets the wall, is called the eaves.

Slates form the most effective and durable roof covering of any stones or tiles. They are used all over England from the facility with which they are obtained. The strata of roofing slate are found in all the Silurian series of rocks to which they belong. They are quarried by blasting, and are split and cut into sizes at the quarry. As it is convenient to have slates all one size in one roof, they are cut to fixed dimensions according to the size of the blocks obtained from the quarry; consequently the larger slates are more expensive. The best slate for roofing is of a light sky-blue colour, and gives a clear bell-like sound on being struck; dark blue or blackish slate absorbs moisture and decays more rapidly.

The following, taken from Hurst's Tables for Engineers, are the sizes and the technical names of the slates in ordinary use for roofing in England:-


[^2]latter are soon destroyed by rust; the partial protection which is sometimes given them by dipping them in boiled linseed oil, does not preserve them much longer, and a coating of tin is an uncertain protection. Slates for ridges and hips are sometimes cut out of thicker pieces than ordinary; they are generally in two pieces, and are joined together on the roof with slate mastic, and form very effective and durable coverings.

Large slabs of slate are frequently used for special roofs, and for cisterns and other similar purposes. They are expensive and require great care in fixing, especially in forming the water-tight joints.

Tiles are now made in England in great variety of forms. They are made in almost all districts where clay for brickmaking is found; but they should be made of a more plastic clay than that for ordinary bricks, otherwise they are liable to lose their shape in the burning. The preservation of the shape is indeed the only limit to the size of tiles; the larger the tile the lighter the roof covering; large well-burnt tiles would be almost as effective, light, and durable a covering as slates, though requiring a steeper slope: and if a cheap method of glazing them should be discovered they will probably supersede slates to a great extent.

Plain tiles are the oldest and commonest form of tile in England. They are flat and about 10 in . by 6 in ., and $\frac{1}{2} \mathrm{in}$. thick, and weigh $2 \cdot 13 \mathrm{lbs}$. ; they are generally laid on luths of fir about $1 \frac{1}{4} \mathrm{in}$. wide and $\frac{3}{8}$ in. thick, nailed to the common rafters and therefore horizontal : two holes are formed in the tile, near the upper edge, through which they are fastened with wooden pins to the laths. The overlap of one course over the next is generally from 6 in . to 8 in .; rows of laths are fixed at intervals to give the required overlap, between each of which an intermediate row is placed : the tiles are bedded in mortar, which should of course be hydraulic: the tiles in one course are laid with their sides touching, the next course is laid with these joints occurring over the spaces between the joints of the course below.

There are flat tiles used in France 12 in . by $9 \frac{1}{2} \mathrm{in}$.
A square of plain tiling requires 700 plain tiles; 1 bundle of laths, or 500 ft . lineal; 600 nails, 4 lbs. to the $1000 ; 4$ cub. ft. of mortar; and 1 peck of wooden pins, or $\frac{1}{2}$ bushel.

Pantiles were introduced into England from Holland; they are curved in transverse section, with a kind of lip on one side to cover the joint with the next tile in the same course. They measure about 14 in . long and 7 in . to $9 \frac{1}{2} \mathrm{in}$. across the chord of the curved part, and about $\frac{5}{8} \mathrm{in}$. thick, and weigh $5 \frac{1}{4} \mathrm{lbs}$. They are made with a tongue projecting from the centre of the upper edge to the rear; by which they are hooked on to the laths. The overlap of one course over the next is from 3 in. to 4 in., consequently they form a much lighter covering than plain tiles. The laths are from 10 ft . to 12 ft . long, $1 \frac{1}{2} \mathrm{in}$. wide, and 1 in . thick. The tiles should be pointed on the inside of the horizontal and vertical line joints with "hair mortar," that is, common mortar with a small portion of ox hair mixed with it. With pantiles it is usual to employ special tiles to cover the ridges, hips, and valleys.

The ridge and hip tiles are curved, and are about 12 in . long and $9 \frac{1}{2} \mathrm{in}$. across the chord; they are laid in mortar without any overlap, and are fastened with nails or hooks. Valley tiles are curved and triangular-shaped, and are about $10 \frac{1}{2}$ in. by 12 in .; they overlap, are laid in mortar, and fastened with wooden pins like plain tiles.

A square of pantiling requires 180 pantiles; 1 bundle of 10 ft . laths, or 120 ft . lineal; 150 nails, 10 lbs . to the 1000 ; and 6 cub . ft. of hair mortar.

Italian Tiles.-The system of roof covering that has been in use all over the Mediterranean from the earliest known periods, has been that of the ridge and furrow tiling; one set of curved or channelled tiles being laid with the concave side uppermost, to form a series of channels down the slope, and another set laid with the curved side uppermost, covering the joints of the former, and forming a series of ridges between the channels. This system is not so effective as plain tiles in countries exposed to heavy winds and snow, because the number of irregularities in the surface of the roof afford so many points for the wind to act on and for the snow to accumulate in. Sometimes the channel and ridge tiles are both the same, or about the same shape, as is the case in Chinese roof covering. M. Renaud mentions the following usual form now in use in Italy:-The channel tiles (tegoli) are flat, and about 16 in . long, 10 in . wide at the lower end, and 13 in . at the upper, with a ledge or rim down each side; the ridge tiles (canali) are curved, and about the same length, and $6 \frac{1}{2} \mathrm{in}$. wide at the upper end and 9 in . at the lower; the overlap is about 3 in . On the common rafters of the roof (which are at about 1 ft . central interval) are laid common flat tiles, and on these the ridge and furrow tiles are laid. The proper slope for them is from $15^{\circ}$ to $27^{\circ}$, and the weight of a square of 100 sq . ft. is 1800 lbs .

A triple pantile (as it is called) is made by Browne, of Bridgewater ; it is $16 \frac{1}{2} \mathrm{in}$. by 14 in ., and 1 in . thick, and weighs $7 \frac{3}{4} \mathrm{lbs}$.

Thatching is a name given to a kind of covering over roofs: it is generally made of wheaten straw, laid on lathing and rafters, which may be of the same strength and placed the same distance apart as for a common slated roof; but in country places, where thatching is mostly used, the rafters are generally formed of the branches of trees of from 3 in . to 6 in . in diameter ; the slighter they are the better, provided they are sufficiently strong, as the lighter the roof is the less strain there is on the walls: of course, if the rafters are stout, they should be placed farther apart than slight rafters; and if the rafters are far apart, the lathing must be stronger, otherwise the thatching will bag, or lay in hollows between the rafters.

The straw is laid on the lathing in small bundles called hellams, until it attains a thickness of from 12 in . to 16 in. ; it is fastened to the rafters with young twigs and rope-yarn.

A good pitch for a thatched roof is $45^{\circ}$, or, as it is technically called, a true pitch : if the pitch is made less, the rain will not run off freely; and if a greater pitch than $45^{\circ}$ is used, the straw is found to slip down from its fastenings.

The Thatcher's Tools are :-
A common stable fork.-This tool is used to toss the straw up together when it is wetted, preparatory to its being made into bundles for use.

A thatcher's fork, Fig. 2077.-This is a branch of some tough kind of wood, cut with two smaller branches proceeding from it, so as to form a fork, as shown in the diagram; the joint of the two branches is generally strengthened by a small cord, to keep it from slipping when it is used. A small cord is fastened by one end into one of the ends of the fork, and a loop is spliced on the other end of the cord; this loop is made to pass over the other end of the fork, and to fit into a notch cut to receive it, as shown in Fig. 2077.

This tool is used to carry the straw from the heap, where it has been wetted and prepared, up to the thatcher on the roof, where it is to be used.

A thatcher's rake, Figs. 2078 to 2080.-The handle should be of ash or some tough wood, made not round but square, so that it may be grasped firmly without fear of its slipping
 round in the hand: the arrises may be slightly rounded off, so as not to hurt the hand. It will be seen by referring to Fig. 2079 that a crook is formed in the handle; the reason for this will be explained when we come to speak of the manner of using the different tools.

The use of this tool is, after the straw is laid, to comb it down straight and smooth.
A thatcher's knife, or eaves' knife. -This tool is similar in shape and make to the reap-hook, except that it is larger, and not curved so quickly.

The use of this tool is to cut and trim the straw to a straight line at the eaves of the roof.
The thatcher also requires a knife shaped something like a bill-hook, to point the twigs used for securing the straw.

A half-glove or mitten, of stout leather, to protect the hands when driving in the 2081. 2082. smaller twigs, called spars.

A long flat needle, Figs. 2081, 2082.
A pair of leather gaiters, to come up above the knees, to protect his knees and shins when kneeling on the rafters.

A sharp grit-stone to sharpen the knives.
The Timbering and Lathing necessary for a Thatched Roof.-As before stated, the
rafters for a thatched roof may be of round timber, such as the branches of trees, and young trees, of from 3 in . to 6 in . in diameter, placed not more than 14 in . from centre to centre, but sometimes the raflers are of sawn timber : in that case they should be cut about the same scantling as for a slated roof, not as for a tiled roof.

The lathing in a thatched roof being very liable to rot, it should be split out of heart of oak, or some other equally durable wood: the laths are about $1 \frac{1}{4} \mathrm{in}$. wide, and $\frac{1}{4} \mathrm{in}$. to $\frac{3}{4} \mathrm{in}$. thick, and are nailed on the rafters about 8 in . apart in a horizontal direction, just the same as for a tiled or slated roof.

If the laths are placed farther apart than 8 in., the straw is apt to bag or sink down between them; the rain lodges in the hollows, and of course soon rots the straw.

An eaves' board about 7 in . wide is required to start the first part of each course of thatching upon.

A description of the manner of executing a Thatched Roof.-The rafter and eaves' board being fixed, and the lathing nailed on in rows at the prescribed distance apart before mentioned, -

As much straw is taken as it is thought will be required for the whole roof, which may be got at by estimating a square to take from $3 \frac{1}{4}$ to $3 \frac{3}{4} \mathrm{cwts}$. of wheaten straw : care should be taken to keep the fibres or stalks as parallel to each other as possible. As each truss of straw is opened, it is spread out and wetted, using about 3 or 4 gallons of water to each truss. The straw is then tossed over and mixed together in one great heap with the stable fork, so that every part may get an equal portion of the water. If the weather is fine and dry, the straw may be used directly; but if the weather is damp or rainy, the straw should be allowed to lay for a day or so to drain, and be once more turned over.

The reason for wetting the straw is to make it lay close, and to enable the thatcher's labourer more easily to draw the stalks out parallel.

The thatcher and his labourer being now ready to commence, the labourer spreads as much of the straw on the floor as will make a bundle 12 in . wide and 4 in . thick; the labourer then stooping down, with his left hand draws the straw, little by little, to his feet, and while doing so, with his right hand draws out any loose straws that may be lying crosswise : by this means he gets a compact bundle of straw from 3 ft . to 4 ft . long, according to the goodness of the straw, and all the stalks are parallel. This bundle is called a "hellam." The labourer having placed four or six hellams crosswise in his thatching fork, he carries it on his shoulder up to the thatcher on the roof, in the same manner as a bricklayer's labourer carries a hod of mortar: the fork is secured on the roof by a small peg and a piece of string.

The thatching is now laid in courses 3.0 wide, beginning at the right end of the roof, so that the thatcher works from right to left. The courses are laid parallel with the rafters, and not parallel with the lathing (as is the case in slating and tiling). Care must be taken at starting the eaves to have a good firm body of thatch, letting the straw hang over, to be afterwards trimmed with the eaves' knife to a straight and good-looking edge. A row of three hellams is placed on each succeeding lath in the course, and each row of hellams is secured to the rafters with a young tough twig, called a ledger; it is about 4 ft . long and 1 in . in diameter: each row of hellams is also secured to the row underneath it with three split twigs, called spars: the spars are about 2 ft . long, and eight can be split out of a branch 2 in . in diameter ; they are pointed at both ends, and are then doubled in two, and the thatcher gives them two twists round in his hand, in the same
manner as a rope is twisted: this gives the spar a splintery surface, and enables it to hold on when driven into the straw.

The thatcher has a leather glove on his right hand : and keeping his hand flat or open, he gives the spar two or three smart blows, sufficient to drive it into the straw, and the leather serves as a protection to the hand. The spars must be soaked in water for some hours before they are used, in order that they should not break in the doubling up.

The ledjer is a tough twig, about $4 \cdot 0$ long and 1 in . thick, as before described: one end is pointed, and driven or rather pushed 6 in. under the outside rafter of the course: it is then brought over the top of two rafters, and over the top of the hellams, and then secured to the inside rafter of the course with about 8 ft . of rope-yarn, by means of the long flat needle, thus holding down the row of hellams, and preventing them from slipping off the roof. In speaking of the outside and inside rafter of a course, it is meant by the outside rafter, the rafter that is farthest from the thatcher ; and by the inside rafter the one that is nearest to him ; and thus the inside rafter of one course becomes the outside rafter of the next course.

The thatcher gives each course, as it is laid, a combing down with his rake, to get out the loose straws : he then takes a bucket of water, and throws it right down the course, and gives the straw a good beating with the back of his rake, to break any stubborn straws and to make it all lay close: he then finally gives it another combing, and after that smooths it down with the back or flat side of his rake, and it is finished.

It will be seen by referring to Figs. 2078 and 2080, that a crook is formed in the handle of the rake. The reason for thus crooking the handle is to keep the thatcher's hand from contact with the straw, and thereby save his knuckles.

The ridge and hips are managed thus :-The thatcher, in doing one side of his roof, takes care to leave a good length of screw hanging over and past the ridge. As he finishes the top of each course on the other side of the roof, he bends down the tops of the first side, and covers them over with the last row of hellams on the last side, bending these last in their turn down over the other side of the roof.

The ridge is then secured on each side with three rows of bands or spars, placed end to end, and each spar is secured with three other spars to thatch.

In the case of the hips, there are no bands of spars, but single spars, 12 in . apart, are bent crosswise over the hip, and secured with three other spars, as before. The eaves are also secured with two rows or bands of spars.

Wheaten straw thatching, done as here described, will last in our climate from fifteen to twenty years. Oat straw, about eight years.

Shingles, or wooden slates, are made from hard wood, either of oak, larch, or cedar, or any material that will split easily. Their dimensions are usually 6 in . wide by 12 or 18 in . long, and about $\frac{1}{4} \mathrm{in}$. thick. They are laid in horizontal courses of 4 or 5 in . gauge, nailed upon boards, the joints broken, commencing with the eaves' course. The ridge is secured by what is called a ridge-board, or a triangle of inch stuff of 6 or 8 in . each side. In America, where this roof is common, the mechanics have a special tool for shingling, called a shingle-axe, with a hammer at the back.

Metal Coverings.-Metal coverings are generally laid in large sheets or plates, and as they are for the most part composed of materials of great tenacity, one of the intermediate orders of bearers can generally be dispensed with, and the sheets of metal can be laid on the rafters or purlins. Metal coverings should be fastened in such a manner as to admit of expansion and contraction, and therefore not with nails as a general rule; also, because the nail-holes, if exposed, soon enlarge with rust and form leaks.

Iron.-Cast-iron plates are seldom used, except when the roof consists of a reservoir or tank, in which case it is frequently made of cast-iron plates bolted together by means of flanges. Cast iron is frequently employed in roofing, for eaves' gutters, and rain-water pipes, and is one of the most effective materials that can be used for these purposes. Both gutters and pipes may be made as thin as they can be efficiently cast; the gutters may be of any cross-section required to correspond with the other mouldings; for an ordinary roof they should not be less than 5 in . deep in this climate. With that depth they may be laid nearly horizontal, as it is an advantage to have some water always in the gutter. The lengths or pieces should be connected by flanges and bolts, to allow of the introduction of fresh packing. The connection of the gutter with the wall should be very firm, and such as to allow for its expansion and contraction. With such a gutter a downpipe or means of escape for the rain should be provided at about every 50 ft . lineal of gutter ; this should not be less than 4 in . in internal diameter, and should have a larger head with a grating over it to prevent leaves or other matter getting into it. Wrought-iron plates are used both flat and corrugated. With flat plates the mode of forming the longitudinal joints (down the slope) mentioned by Renaud is the most effective, namely, by bending the two edges of the adjacent plates over a roll of wood extending down the roof, so as to avoid nailing. Corrugated wroughtiron plates have been much used of late years in this country. They are made in sheets about 12 ft . long and 6 ft . wide, and in thicknesses from about No. 25 to No. 15 of the Birmingham wiregauge (that is, from about $\frac{2}{50}$ to $\frac{1}{14}$ ); the corrugations are from 2 to 5 in . across. It is only necessary to support medium-sized corrugated iron at intervals of about 6 ft ., hence the framework of the roof can be simplified; the corrugations being placed so as to form ridges and furrows extending down the slope of the roof, the overlap of the sheets at the sides forms of itself a nearly water-tight joint; it is necessary, however, to fasten the sheets with nails along the top and bottom of each sheet to horizontal purlins or battens, in order to prevent them being lifted by the wind. These nail-holes are sources of decay and leakage.

No thin sheets of wrought iron will last many years without some artificial coating, and no artificial coating at present in use will effectually prevent oxidation, without frequent renewal. With thick plates not liable to any disturbing causes, the coating of oxide forms an effective
covering in itself, for it will not penetrate far into the iron and resists further change; but the liability to decay from rust of thin plates, especially when exposed to the action of any acid, has checked the application of wrought iron as a roof covering. A coating of zinc is applied to such plates for this object, and preserves them for several years, but wherever the coating is broken through, both the zinc and the iron decay rapidly; for this reason no holes or cuttings should be made after the zinc has been applied, and a coating of paint is frequently desirable in addition to the zinc.

Lead is the most durable covering for roofs of all the metals. Its malleability requires that it should be supported on boards, and as it must be of considerable thickness to retain its shape, it forms a heavy covering. There are two kinds of lead used in roofing, cast and milled. The former is the harder of the two, and is preferred for flats, cisterns, gutters, and tanks; the latter being more malleable is preferred for hips, ridges, and flashings. Both kinds are made in sheets about 6 ft . wide, and the cast from 16 to 18 ft . long, and the milled from 18 to 25 ft . long, and it is generally designated by the weight per square foot: that for flashings being not less than 5 lbs.; for hips, ridges, and valleys not less than 6 lbs ; for gutters, flats, and cisterns, not less than 7 lbs . a sq. ft. 5 lb . lead is about $\frac{1}{12}$ in. thick, and 7 lb . about $\frac{1}{8}$.

As the expansion of lead is considerable with increase of temperature, the joints must not be fastened together by solder or by any other means: when a joint is required to be in the direction of the slope, it is made by a roll of wood as above mentioned for iron plates; when the joint is across the slope, it is made by what is called a drip, which is a small vertical fall arranged in the boarding over which the two edges of the lead sheets overlap. Lead on flats, that is roofs nearly horizontal, should be laid in widths not greater than half the width of the sheet; on gutters and flashings it should be in lengths not greater than half the length of the sheet. Gutters and flats should have an inclination of at least $\frac{1}{4} \mathrm{in}$. in a foot or $\frac{1}{48}$, that is when the cross-section of the gutter is nearly flat, as is frequently the case with lead gutter. When the sides or ends of gutters or flats are formed by walls, the lead should be turned up against the walls of about 6 in . high, and be covered by a separate piece of lead called a flushing, the upper edge of which is let into a mortar joint in the wall, and the lower edge covers the top of the gutter lead about 3 in . The joints of flashings, and of hips and ridges, are lapped joints; this joint is suitable only for such narrow pieces of lead in vertical or inclined planes. In a ridge, the lead is moulded over a wooden roll fixed to the ridge-piece, and laid over the roof covering on each side; its own weight is sufficient to keep it down without any nails. In hips there is a tendency in the lead to slide down : this might be prevented by the top of every piece being nailed down with ordinary nails, which would be covered by the lap of the piece above it.

In water cisterns and tanks lined with lead the joints must of necessity be made of solder to ensure their being water-tight.

Solder used by the plumber is composed of equal parts of lead and tin; it is run into the joint in a molten state, and smoothed down by what is called the grosinj iron, and finished off by filing and scraping. The joints of the lead are first scraped clean and covered with borax, which easily melting forms a coating to prevent oxidation while the soldering is in process.

Copper is in itself very durable as a roof covering, but it is generally used in such thin sheets, on account of the expense of it, that it is liable to become ineffective as a covering before the material is worn out. It is used in England in sheets weighing about 16 oz . a sq. ft.; and in Paris in sheets measuring about $3 \frac{1}{2} \mathrm{ft}$. by $4 \frac{1}{2} \mathrm{ft}$., and weighing up to 28 oz a sq. ft . It is not considered necessary in Paris to cover sheets of that thickness with paint or other coating, as the coating of oxide which soon forms on it is a sufficient protection. The mode of fastening copper sheets is similar to that recommended for lead: on account of the thinness of them, they should be laid on boards.

Zinc is frequently used for roof coverings on account of its cheapness as compared with lead and copper. It is not so durable as either of them, but being stronger than either it can be used in thin sheets without boards immediately under it; it is used in England in sheets weighing from 12 to 20 oz . a sq. ft., and in Paris in sheets about 6 ft . long and 18 to 30 in . wide, and weighing from 20 to 40 oz . a sq. ft. The thick sheets are said not to require any other coating beyond that of the oxide naturally formed.

Iron Roofs.-The facilities now available for the rolling of wrought iron into bars of various cross-sections, enable it to be used with efficiency and economy in the framework of buildings. The metal can be disposed in the cross-section in such a manner as to give the maximum of strength with a given quantity of material, and to give considerable stiffness at the same time, and its elastic force is much greater than that of wood. Therefore, beams of wrought iron can be used with efficiency and economy in larger spans and bearings than of wood: hence, a framework or roof of iron can be made of simpler construction than one of wood: a smaller number of orders of bearers can be employed, and the principal frames or trusses can be more simple and have greater bearings. The general arrangement of a wooden roof has been followed in iron roofs up to a span of about 50 ft .; there are principal trusses, but they are sometimes 15 and 20 ft . apart; the purlins are consequently stronger, and are sometimes framed, and in roofs that are covered with iron or zinc plates, these are frequently laid on the purlins direct. The facility of forming joints in wrought-iron work has caused other forms of trusses to be adopted, which bring the strains more directly on to the bars composing it. The principle of one of the most common of these trusses is, that the beam or bar representing the principal rafter is supported at one or more points by short struts at right angles to it, the ends of which are connected by a contivuous bar or rod with both ends of the rafter; the whole strain which passes down the struts is thus transferred to the rafter, compressing it at each end in the direction of its length: owing to the inclination of the rafter there will be an unbalanced force at the lower end, which must be met partly by the wall or column, and partly by a tie-bar or rod connecting the lower ends of the two rafters or some other intermediate points on the rafters. When this tie-bar is placed at an intermediate
point, a cross strain is thrown upon the principal rafter near that point, as in the case of a collarbeam truss.

When the span of the roof is very large, a curved rib or rafter is frequently used, in which the line of pressure of the forces acting on the roof is kept within or nearly within the rib, and the horizontal force at the springing points is provided for by a tie-bar connecting them. This tiebar being of comparatively small cross-section for its length, is supported by king or queen rods from the rib; sometimes it is curved, or rather polygonal, consisting of straight pieces between the supporting rods, forming a convexity upwards with the object of gaining height in the centre; in all such cases there is a horizontal force not taken up by the tie-bar, which has to be carried either by the rib or by the walls.
'I he rafter or rib in iron roofs has generally a $T$ cross-section with the flange uppermost, for the convenience of fastening the upper parts of the structure to it; this, however, is evidently not the most effective cross-section, and consequently large ribs are generally made with an I section. Both sections can be rolled in one piece up to 10 in . in depth and $\frac{1}{2} \mathrm{in}$. thickness of metal, and to a length of 30 ft ., without difficulty; above that size they must be built up of flat and angle plates riveted together. Struts are made of wrought iron of $T$ cross-section, and sometimes of two $T$ pieces bolted together flange to flange, forming a + cross-section: sometimes they are of cast iron, in which case they can be made of the best theoretical section, both transverse and longitudinal; to resist compression. Tie-bars are generally rods of a circular cross-section when small, or flat bars when large, any additional required strength in any part being generally obtained by using additional bars of the same cross-section. The same method of obtaining additional strength is adopted generally in the rafters or ribs, in which the thickness of the top flange is generally increased as required, by adding additional flat plates to it. Purlins are generally made either with an $L$ cross-section or a bridge ( $(\Omega)$ cross-section, in both cases for the purpose of fastening other parts to them; for which purpose also the vacant space in each is sometimes filled in with timber. Large purlins are made of timber trussed with iron, or of framed ironwork entirely, and in these cases are sometimes framed in between the principal ribs to gain height and stiffness.

Provision should be made in all large iron framework for expansion and contraction. This is generally done by allowing one end of a truss or frame to be unfixed and free to move horizontally; and to reduce the friction, the bearing surface of the end is sometimes made to rest on iron rollers turning in an iron frame and working in oll: sometimes the end is suspended by or fixed to a short piece fixed to an iron support resting on the wall, which from the action of the two centres of motion admits of a small motion of the whole frame. With short iron beams, such as the girders for small bridges, it is considered sufficient to bed them on lead without other fastening, their own weight being sufficient to keep them in their places.

The bearing surfaces of ironwork are so small that it is impossible to frame them together in any manner depending on those surfaces, as in woodwork. There are two methods commonly adopted for connecting iron bars. 1st, by large bolts, which are used when the framework consists of a comparatively few pieces of large cross-section. It is an advantageous method when the bolts are so large that their efficient manufacture can be depended on. 2nd, by rivets, which are used when the pieces consist of comparatively thin plates, and are connected by a large number of small bolts called rivets. The joints in 1ronwork are always the weakest part of the structure: no joint has ever been used, except welding. of which the strength is at all equal to that of the solid part of the pieces joined : then, the manufacture of small bolts cannot be depended on, consequently at the parts of the framerork where the greatest strain occurs, the material is weakest: and as those are the points on which oxidation is most likely to cccur, it may be expected that the joints of ironwork will give way in course of time before the solid parts of the bars themselves. Therefore when lolts are so large as to become bars, it is advantageous to use that method; otherwise there is an adrantage in riveting, because there is a greater number of individual connections at one joint, and a rivet bemg hammered in red hot is a more effective connection than a bolt fixed cold, the pieces lie closer together, and upon the whole there are generally fewer separate joints in the whole structure when rivets are used.
2083.


Bolts which are subject to a shearing strain, as in Fig. 2083, should not have to bear more than 4 tons to a square inch of each section liable to be sheared. If fixed as in Fig. 2083 it is considered as having six shearing sections, and would therefore bear 24 tons per square inch of cross-section of bolt.

Boits which are subject to direct tensile stram, as in Fig 2084, should not have to bear more than 5 tons to a square inch on the cross-section at $a b$, or 4 tons a square inch on the cross-section of which $c d$ is the depth, and the circumference of the bolt is the length, which is subject to shearing strain. Consequently, if the bolt is of equal strength throughout, $c d$ must be equal to $\frac{1}{3}$ rd $a b$.

Screw boits, which are subject to direct tensile strain, as in Fig. 2085, should have heads of double the depth of common boltheads, becanse it is estimated that half the resistance to shearing is lost by weakening the metal by the thread of the screw. Con-
 sequently, if the bolt is of equal strength throughout, the height of the head or of the nut should be equal to the diameter of the bolt.

The diameter of the nut is generally twice that of the bolt itself; but evidently the part $c q$, Fig. 2084, should not be subject to a greater pressure per square inch than the safe crushing strain of the metal.


Links, that is to say, two or more flat bars connected together by a bolt passing through their ends, and subject to tensile strain, should have those ends enlarged, as in Fig. 2086, so that the sum of the areas of the cross-sections $\alpha c$ and $b d$ shall be equal to the area of the cross-section of the body of the link. The end is also subject to a shearing strain on the sections $a e$ and $b f$, the areas of the cross-sections at each of those two points should therefore be calculated so as to have a strain of not more than 4 tons a square inch on them. Also, the compressile strain on the surface $a g b$ should not exceed 5 tons the square inch.

Rivets.-The dimensions of rivets and of the plates at the joint may be calculated by the same rules as for single bolts. If it is a joint subject to tension, as in Fig. 2087, the effective strength of the joint and of the plate is the resistance of the cross-sections $a b$ and $c d$ to tension, and of the cross-sections be and cf to shearing. If it is a joint subject to compression, as in Fig. 2088, the effective strength is the resistance of the section $g i h$ to compression. Hence, in a tensile lap joint the size of the rivets should be as small as possible, and the sections of the parts $a b c d$ as large as possible; and in a compressile lap joint the size of the rivets should be as large as possible.

Lap joint is the name given to a riveted joint when the plates overlap each other. In a single rivet lap joint, as in Fig. 2089, the whole tensile or compressile strain being divided amongst the spaces between the rivets determines the interval of them. And the whole shearing strain being divided amongst the sections $a b, c d$, \&c., determines the amount of overlap. Fairbairn considers that the strength of such a joint under tension is only 0.56 of that of the solid plate of the same general cross-section.
2087.


In a double rivet lap joint the amount of overlap and the intervals between the rows of rivets both ways, and the size of the rivets, are all determined by the above considerations, and by the rules for bolts. Fig. 2090 shows the joint recommended by Humber for tensile strains

Fig. 2091 shows the joint he recommends for compressive strains.

In practice the diameter of the rivets is generally made a little more than the thickness of the plate, and the interval is from 2 to 4 times the diameter, according to the closeness of the joint required.

The practice in H.M. Dockyard at Chatham, in the construction of iron ships, is to use rivets rather larger in diameter than the thickness of the plate, and at intervals from 2 to 4 times the diameter. Thornton states that a water-tight joint can be formed with single riveting at intervals of 4 diameters; double riveting is commonly used, the first row being placed at a distance of at least one diameter (of rivet) from the edge of the plate, and the second row at about 3 diameters from the first. These rules determine the length of what is called the butt-plate, or fishing-piece. The rivets in the second row are placed directly opposite those in the first row, and not diagonally opposite the spaces. In all exterior plates the outer rivet-holes are countersunk and the rivets hammered flush.

Bolt-nuts are generally made in depth equal to the mean diameter of bolt, and india-rubber washers of about $\frac{1}{3}$ that thickness are used with bolts through thick plates.

The experiments in the testing-house of Chatham Dockyard show that if the total area of the rivets is equal to the area of the cross-section of plate at one row of rivet-holes, the strength of the rivets to resist shearing will be greater than that of the plate to resist tension; also, that the punching of holes in steel or hard iron injures the metal round the holes, so that the plate is considerably weakened at that part. Rivets with hammered heads are more liable to leak than with heads rounded in a swage, and the latter than countersunk conical heads: countersinking should always be drilled; and rivets fixed cold are tighter than those fixed hot.

The following extracts from the Regulations of the Society of Underwriters at Liverpool for Iron Ship Building show the practice in that trade for riveting plates :-

Sixteenths of an inch.

| Thickness of plates | .. | .. | 5. | 6. | 7. | 8. | 10. | 12. | 13. | 14. | 15. | 16. |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| Diameter of rivets | .. | .. | 8. | 10 | 12. | 13. | 14. | 15. | 16. | 18. | 19. | 20. |

Rivet-heads should be thin, and rivet-necks, under the heads, should be bevelled to fill the countersink formed in punching. All riveted seams and butts to be laid up quite close, so as to prevent the introduction of the thinnest knife used for trying riveted work.

Every hole requiring to be countersunk must be countersunk quite through the plate. Each rivet to fill the hole and to be laid up close round the head, and when finished to be flush and fair, neither projecting above nor sinking below the surface of the part through which it passes.

All double riveting to be in parallel rows, or what is termed chain riveting. All butts to be double riveted. Breadth of lap in seams and butts, for double riveting, to be $5 \frac{1}{2}$ times the size of the hole punched. Rivets to be 4 diameters apart, from centre to centre, longitudinally in seams and butts. Rivets in framing to be 8 times the size of the hole punched, apart.

Iron decks to hare their butts treble riveted amidships.
Extract from Lloyd's Rules.-Rivets not to be nearer to the butts or edges of the plating, or of any angle iron, than a space not less than their own diameter, and not to be farther apart from centre to centre than 4 times their diameter, or nearer than 3 times their diameter. In single riveting the overlap is not to be less in breadth than 3 times the diameter of the rivets. No rivet to be less than $\frac{5}{8} \mathrm{in}$. in diameter.

Table of the Weight of Roof Coverings and the Slope for then sutitable for the English Climate, from Tredgold.


The weights here given include the covering and the immediate bearers underneath, but no rafters or other framework. An allowance for the pressure of wind and snow must be made in addition. Tredgold recommends that the former should be calculated at 40 lbs . a square foot of horizontal pressure on a vertical surface.

Floors.-Under this head we include all kinds of floors and pavements of stone and wood, both the framework and the floor covering. The technical builder sometimes includes under the name flooring only the boards or covering; but it is evidently more correct to treat under one head all the parts of those platforms which form the living places in houses, of whatever material they are.

The same general primciples of framing apply to all floors of wood framing as were applied to roofs; being simplified in the case of floors, because they are almost always flat. The problem in floors of this class is to provide the smallest number of bearers that will carry an effective floor covering with the greatest economy and stiffness.

Single Floors.-The simplest description of wooden floor, when there is only one order of bearers supporting the boards, is technically called a single floor, and the bearers are called joists. Tredgold recommends that single joisting should not be applied to a bearing above 15 ft ., on account of the deflection, which then becomes so great as to be inconvenient. As a joist has only to bear perpendicular transverse strain, the deeper it is the better; but such beams have a tendency to buckle or turn over on their sides, and to counteract this, they should be strutted against each other ; such strutting may be composed of short pieces of wood nearly the depth of the joists, and fitted in at
right angles to them, and moderately tight, and nailed, not morticed. Tredgold recommends that there should be a row of strutting when the bearing of the joists is more than 8 ft ., and two rows of struttings to bearing above 12 ft ., and an additional row for every additional bearing of 4 ft . He mentions an experiment made by Professor Robison to illustrate the strength of single floors of two models, one representing a single and one a double floor, both 18 in . square, and containing the same quantity of wood. The former broke with a load of 487 lbs ., the latter with a load of 327 lbs.

Joists are generally laid at a central interval of 1 ft ., when covered with boards, because floor boards are not generally more than $1 \frac{1}{2}$ in. thick; and with a much greater bearing than 12 in . they would deflect considèrably.

The minimum thickness for a joist is 2 in ., to allow hold for the nails of the boards without splitting the timbers.

They are generally laid on the wall-plates, and not notched or framed. The wall-plates extending all round the walls are very frequently laid in a kind of groove left in the brickwork; and, therefore, the ends of the joists enter into the walls, a practice which tends to cause decay in the wood, besides weakening a thin wall considerably. The more effective method is to corbel out the wall to form a support for the wall-plates, and so keep the joists clear of the maiu wall. In ground floors this is accomplished by a set-off increasing the thickness of the wall.

Ceilings that are covered with lath and plaster, which is the method commonly used in England, are supported by ceiling joists, which are notched and nalled at right angles to the joists or beams of the floor above. The ceiling joists should not bave more than 10 or 12 in . central interval, otherwise the lath and plaster would be liable to break from its own weight. In a single floor, in order to check the passage of sound from one room to the other, it is usual to make every third or fourth joist about $1 \frac{1}{2}$ in. deeper than the others, and to nail the cellug joists to these only, thus diminishing the points of contact between the floor boards and celling. Tredgold recommends that no ceiling joist should have a greater bearing than 10 ft .

The ceiling joists should not be less thail 2 in. thick, on account of the lath nails.
Framed Floors.-The method most recommended by Tredgold and others for introducing a second set of bearers to stiffen a floor when the breadth or spail exceeds 15 ft ., is by placing large beams across the breadth at suitable intervals, and notching the joists min to these. These large beams are called girders. Tredgold assumes the central interval to be generally 10 ft ; but that must vary according to circumstances. One advantage of the framed floor is that the ends of the girders can be supported easily by corbels projecting from the wall, a template being introduced under each to distribute the weight. If, however, the ends of the girders are let into the main wall, an open space should be teft round them for ventilation, covered with a flat stone or arch to carry the wall above. Girders should have a bearing from 9 to 13 in . on the wall. Such floors of two orders of bearers are technically called double floors.

When the breadth of the floor is above 30 ft . it is difficult to procure timber of the necessary size for girders, and the deflection of them would be considerable: hence arose the practice of trussing or framing wooden girders, on which great ingenuity was expended before iron was used, with the object of reducing the total depth of the truss as much as possible, to keep down the depth of the whole framework of the floor. Tredgold considered that no trussing which is contained within the depth of the solid beam itself can be depended on, on account of the injury to the fibres of the main beam by the framing; also, that the practice of cambering a beam, that is, so fixing the framework that the main beam without any load on it assumes a curved form, convex towards the load, so cripples the fibres that the additional resistance obtained by it is apparent rather than real. The method generally recommended for floors of such great breadth before iron was used was to place large trussed girders at about 10 ft . central intervals, and to frame in other girders between them at about 6 ft . central interval, and to notch the joints on these last, the object of framing the intermediate girders between the others instead of notching them on being to save total depth in the floor. These intermediate girders are technically called binding joists, and the common joists are in this case called bridging joists, and the whole is called a framed foor. The ceiling of the room below such a floor is supported by common ceiling joists fixed to the binding joists of the floor, or to others fixed to the girders for the purpose. But Tredgold considers it would probably make a stronger floor, though of greater total depth, to place girders at 5 ft . interval, and omit the binding joists altogether.

He recommends that a wooden floor should be laid with a slight camber upwards (about $\frac{3}{4}$ in. in 20 ft .), and especially ceiling joists, with the object of allowing for settlement from shrinking.

The following practical rules for the sizes of the beams of ordinary wooden floors are taken from Tredgold's Carpentry. They are most of them founded on the same assumption he adopted in the case of roof beams, namely, that the safe deflection to be allowed in floor beams should vary inversely as the length between the points of support; and as mathematically the deflection is proportional to $\frac{l^{3}}{b d^{3}}$, Tredgold's law reduces the formula for practical purposes to $\frac{l^{2}}{b d^{3}}$, in order that the stiffness may be directly as the length. Then by assuming a given deflection to be allowed in a given floor, and a fixed method of arranging the beams, he obtains a constant coefficient for each beam applicable to all similar floors.

Common or Single Joists. $-d=2.2 \sqrt[3]{\overline{l^{2}}}\left\{\begin{aligned} \text { where } d & =\text { depth } \\ b & =\text { breadth }\end{aligned}\right\}$ of beam in inches,, the central interval of joists being assumed to be 12 in.

$$
\text { Binding Joists.- } d=3 \cdot 42 \sqrt[3]{\frac{\sqrt{2}}{b}} \text {, the central interval of joists being assumed to be } 6 \mathrm{ft} \text {. }
$$

Girders.-d $=4.2 \sqrt[3]{\frac{l^{2}}{b}}$, the central interval of girders being assumed to be 10 ft .

## Bridging Joists.-The same rule as for common or single joists.

Celling Joists.- $d=0.64 \frac{l}{\sqrt[3]{b}}$, the central interval of joists being assumed to be 10 to 12 in .
In this case the formula is the same as that for deflection; hence Tredgold has apparently assumed that the actual amount of deflection should be the same in all ceiling joists.

Wall Plates.-


Floor Coverings.-Almost the only kind of covering used in England for floors of wooden frames work is of boards. The thickness of floor boards is a practical rather than a theoretical consideration : they are destroyed chiefly by wearing away, and for the economy of replacing them comparatively thin boards and close bearers are better than thick planks and open bearers, as in the deck of a ship. They are seldom thicker than $1 \frac{1}{2} \mathrm{in}$.; this is a necessary thickness for barrackfloors which are soon worn away: common house floor boards may be 1 in . and $1 \frac{1}{4}$ in. thick. The narrower in breadth the boards are the less danger there will be from shrinking after they are laid; to guard against this, the floors of important rooms are laid with boards, about 6 in . wide, technically called battens; more common floors are laid with yellow deal boards, 9 in. wide. It is evidently of great importance that they should be of straight-grained, clean and sound and wellseasoned timber ; and to provide for this as far as practicable, they should be brought on the ground and wrought and stacked at the commencement of a new building, and if possible a new floor covering should be laid in its place without being nailed down for at least a year. As a part of the strength and elasticity of the covering depends on the length of the boards, the more completely the ends of them break joint in the several courses of boards the better for this object; therefore, it is advantageous to lay the boards along the room and not across it, as in the former case the joints can be more efficiently broken and the whole floor laid without cutting the timber to waste so much as the latter would require.

The upper face and all the edges of floor boards are wrought, that is, planed to a smooth even surface.

There are four ways practised in laying floor boards, in each of which each board is laid and fastened separately.

1. Square edqed.-That is when the boards are simply laid close together without any connection between two adjacent boards, and are fastened by one or two nails driven straight through the boards into the joists; there is therefore a line of nails over every joist. The longitudinal joints run in straight lines from end to end of the room; the transverse joints, formed by the ends of the boards, are broken as much as possible. In this method the slightest shrinkage of the boards leaves an open joint to the ceiling of the room below.
2. Rebated.-That is when a rebate is cut on one longitudinal edge of the boards, and a corresponding but reverse rebate on the other longitudinal edge, the adjacent boards thus overlapping; the nails may be driven obliquely through the rebates of the outer face before the next board is laid, by which no nails will be visible. This is called edge nailing.
3. Rebated and Filleted.-That is when a similar rebate is cut on each edge, on the under-side, leaving between two boards laid together a square groove below, which is filled with a small piece called a fillet. The rebate in this case need be only half the width of that in the last case. From the facility of making this jöint and of repairing it, it is generally adopted in barrack-floors. The boards are generally face nailed, that is nailed straight through the breadth into the joists, with two nails to each crossing of a joist. But they may be both face and edge nailed, one nail being driven into the face and one into the edge at every crossing of a joist.
4. Ploughed and Tongued.-That is a narrow deep groove is cut in each longitudinal edge of each board, and a thin strip of hard wood or of hoop iron is inserted into the outer edge of the first board, and is pressed into the groove of the next board. The boards are generally face nalled. Sometimes dowels or pins with corresponding holes in the edges of the boards are used instead of continuous tongues, and then the boards may be edge nailed.

Warchouse floors require thicker floor covering than ordinary floors, on account of the heavy traffic and liability to shocks; consequently, the bearers or joists may be farther apart. On account of the wear, oak or some hard wood is frequently used for these floors.

French floor coverings are generally of ornamental parqueterie or inlaid woodwork, the climate not requiring the constant use of carpets as in England. The parqueterie is laid on ordinary rough boards on framework, and as a large bearing surface is desirable with this kind of covering to prevent injury to the inlaid work by deflection as much as possible, the joists in French floors are generally broader than in English floors.

Fireproof Floors.-One of the greatest preventatives to the spread of fire in buildings is lime; a wooden floor well cased in mortar on all sides will resist a considerable action of fire; but the difficulty of thoroughly casing timber in mortar and of preventing its decay has checked the application of it to these purposes. The introduction of cast-iron beams was the first step towards thoroughly fireproof floors, for they can be made of such size and shape as to be convenient and effective for the support of brick arches between them. Many floors have been made, and some are still made with cast-iron girders and brick arches between them. The limit to the interval and consequently to the size of the beams or girders is that the brick arch shall be just strong enough to carry itself and any load that may come upon it, and that the total quantity of iron in the floor shall be the least possible. With arches of one ring of common bricks, that is $4 \frac{1}{2} \mathrm{in}$. deep, a span
of 6 or 7 ft . is found to be most effective. The use of cast iron admitted the employment of girders of uniform depth, which is advantageons in floors, the requisite strength in the different parts of the length of the girder being obtained by varying the size of the bottom flange.

The exposed ironwork on the under-side of this kind of floor causes a condensation on it from the moisture in the atmosphere of the room below, which not onIy rusts the iron, but forms lines of droppings on the floor below each girder. A more serious defect is that the exposed ironwork becoming readily heated by fire is very easily cracked by the sudden contraction caused by throwing water on it; the fall of several floors from this last defect led to the more general use of wroughtiron girders.

The facility with which wrought iron can now be rolled into various forms of bars suitable for these constructions has tended to increase the use of them in floors generally. The same advantages of elasticity and strength and favourable arrangement of metal in the cross-section, which rendered the application of wrought iron so efficacious in roofs, tells equally in its application to floors. It is seldom necessary with wrought iron to use more than single joists; sufficient stiffness can generally be obtained by increasing the depth of joist or girder, or by using box-girders. As rolled-iron joists or girders must necessarily be of the same size thronghout, there is a waste of metal in them. They are now rolled solid direct from the rolls of all forms of rectangular flanges up to about a foot in total depth. When they are of double flange or I section, they can be used as girders to support the wooden joists of common floors, or as girders to carry brick arches turned between them and resting on the bottom flanges. Another method of using them, which is now commonly adopted for fireproof floors in England is that generally called Fox and Barrett's plan. The iron joists in this method are placed at about 2 ft . apart, and small fillets of wood about 1 in . or $1 \frac{1}{4} \mathrm{in}$. square are laid across between the joists resting on the bottom flanges, and with intervals of $\frac{1}{4}$ to $\frac{1}{2} \mathrm{in}$. between them - a temporary platform is erected close under the bottom flanges of the joists, and a fine concrete made with good lime is poured in from above, passing through the intervals between the small fillets and forming a key or tongue below. The total thickuess of the concrete must depend on the circumstances of each case: sometimes the concrete is filled in up to the level of the top flanges of the iron joists, and small wooden joists are laid in it to carry the floor boards of the room above; sometimes the concrete is laid no thicker than is necessary to resist ordinary fire, and to stop the conduction of sound, for which purposes probably $t$ to 6 in . is sufficient, the small wooden joists of the upper floor are in this latter case carried on the upper flanges of the iron joists. To complete the fireproofing on the under-side when the concrete has set, the temporary platform is removed from below and the under-side of the floor is plastered, the projecting tongues of concrete below the wooden fillets forming a key to the plaster, small pieces of wood being fastened to the under-side of the lower flanges of the iron joists to form a key for the plaster over them.

There are several other methods of forming these iron and concrete floors; the general principle of them is all the same. In all the object is to have a thickness of several inches of good cement and to cover the under-side of iron and woodwork with plaster. The coating of plaster underneath both checks the spread of fire and prevents the condensation and oxidation on the ironwork. It is advantageous to have a quick setting lime for the concrete, as the deflection of the framework caused by its weight may affect the efficiency of the floor; when once the lime has set, the concrete adds to the strength and stiffness of the floor by forming a solid block between the joists.

Sometimes it is desirable, either for purposes of traffic or for better security against fire, to cover the floor of the upper room with stone. In these cases the concrete is generally filled up to the top flanges of the iron joists, and a layer of finer concrete or mortar is laid over it, and the stones or tiles are bedded in that.

These descriptions of floors might be advantageously applied to the construction of flat roofs, as they can be increased to a considerable strength and used for spans or bearings up to 30 ft . Above that span it is desirable to introduce another order of bearers, which wonld generally consist of framed or plate girders of wrought iron. The intervals between the iron joists are sometimes filled up with tiles made expressly for the purpose, both flat and curved; sometimes with thin iron plates. Curved iron plates are recommended by W Fairbairn, who has written upon these floors, instead of the brick arches between cast-iron girders, the upper side of the plate being filled with concrete. The ends of the iron joists should be laid mpon a continuous stone wall-plate extending round the walls: a bearing of from "to 12 in on the wall-plate is sufficient for ordinary floors, and it is only uecessary to bed the iron joist on some slightly elastic material, such as lead or some resinous compound, to distribute the pressure uniformly; the friction on the bed will be sufficient security against motion. The ends of large girders should be laid on lead plates resting on stone templates, or on cast iron beds expressly prepared to receive them, and bolted down in such a manner as to allow for expansion and contraction. Very large girders should have one end bedded on iron rollers, as in the case of large roof ribs, to allow for expansion and contraction.

A further advantage of these floors is that pipes or other arrangements for ventilation can be readily provided for in between the iron joists in the course of construction; or, if box-girders are used, they could be employed for the same object.

Maltese Floors.-A good fireproof floor is commonly made, in Malta and other places in that part of the Mediterrancan, of the same construction as the flat roofs, before mentioned. Arched ribs of stone are built across the room from wall to wall at about 4 ft . interval, and having a horizontal surface at top; these carry the ceiling stones, which are, consequently, about 4 ft . long, and are 10 or 12 in . wide and 3 in . thick; they are laid close together without mortar; then stone chippings are laid over them for a depth of about 2 in .; then the flooring stones of the room above are laid; these are from 18 to 24 in. square and 3 in . thick, and jointed with fine hydraulic mortar.

It must be remembered that in Malta there are great facilities for oltaining suitable stones for
this work, and that the stone, when dry and scraped and covered with a coat of warm oil, soon becomes hardened and polished.

Pavements.-In ground floors and in yards and other similar parts of buildings, the flooring is frequently of stone, on account of the nature of the traffic. Such floorings, in England, are generally made of large thin square stones, commonly called flag-stone. These stones are in size from 2 ft . square upwards, and from 2 to 4 in . thick. They are generally laid on a substratum of a few inches of concrete, having a bedding layer of fine concrete on the top, and jointed with mortar. Where there is wet of any kind to be dealt with, the lime of the concrete and mortar should be hydraulic. Careful attention to these matters in the laying will save unevenness in the wear, and fracture of the stones afterwards. If the floor is intended to carry off water at any time, the stones should be rubbed on the face and tooled on the edges, and laid, of course, to a slope in the required direction. Tooled-faced stones may be used for ordinary dry floors, and quarry-faced stones for exterior pavements.

Thick paving tiles are now frequently used for ground floors; they should be laid on a bed of mortar on a substratum of concrete and jointed with fine mortar. From the small size and softer material of the tiles, they do not generally form so durable or even wearing a floor as flag-stones.

That which is technically termed paring in London is what is used for heavy carriage traffic, and is made with small hard stones, about 9 in . square and 3 in . thick, set together on edge; the stones are dressed slightly to a wedge form, and as the roadway is generally laid with a slight curve, they form a rude kind of arch. Where no hard substratum exists, they should be laid on from 6 to 9 in. of concrete, the stones should be set dry, as close as possible, and grouted with coarse liquid mortar. Granite and siliceous stones form the best paving of this description; limestones wear too smooth.

Pavements of Stables.-The requirements in the flooring of a stable are that it should be as level as the necessary drainage will admit of, hard and not too smooth, and impervious to water. Hardburnt bricks made in a machine so as to be uniform in shape, and chamfered on the upper edge so as to give a rough surface, form perhaps the best flooring for a stable. They should be laid on a few inches of concrete, according to the substratum, and jointed with cement. Granite pavingstones, cut carefully to shape and with an even upper edge, form a good and durable pavement ; they should be laid on concrete and jointed with cement.

The pavement of each stall should be laid with a slight slope from each side towards the centre of the stall, where there should be an open drain with a slope to the rear into a longitudinal drain running along the outside of the heel-posts. The best construction for these open drains is to cut a broad shallow channel from 6 in . to 12 in . wide in a solid stone, and lay these stones very carefully to the required slope of the drain. There should be no covered drains inside a military stable.

The terraza floors used in Italy at the present day are made in the following manner ;-1st coat; a concrete consisting of common lime $\frac{1}{4}$, sand and fine gravel $\frac{3}{4}$, laid 6 in. thick and well beaten with wooden rammers; after two days in that climate, it is suffioiently dry for the next coat.

2nd coat; a terraza, consisting of pounded brick or tile $\frac{1}{6}$, common lime $\frac{2}{6}$, sand $\frac{3}{6}$, of the consistency of mortar, laid $1 \frac{1}{4} \mathrm{in}$. thick, well beaten with a light flat rammer. After two or three days it is hard enough for the next coat.

3rd coat; ; a similar terraza, but with the grit of broken stones instead of sand in it, laid on like a coat of plaster with a trowel. After this has been laid for one day, a layer of small hard broken stones is pressed into it; these stones should be of some substance that will take a polish, and be of uniform size (they are passed through a gravel screen) of about a walnut; these being afterwards rubbed to a smooth even surface with some smooth hard stone, form a kind of mosaicwork; the stones are frequently selected by colour, and laid in the third coat to a rough pattern. They should be moistened with oil or water till hard set.

The French use these concrete floors both on the ground floor and upper floors: in the latter case on boards and joists. When on the ground floor, it is made with beton $0^{\mathrm{m}} \cdot 15$ to $0^{\mathrm{m} \cdot} 20$ thick: when on boards. it is $0^{\mathrm{m}} \cdot 5$ thick, and is made with plaster of Paris.

Asphalte floors are now commonly used in England for light traffic. The asphalte is formed and laid very similarly both for floors and roof coverings; but for floors the mixture is coarser and thicker. When applied to a floor, a foundation of common coarse concrete is laid 3 or 4 in . thick, according to the traffic on this, when set, a thin layer of fine concrete is laid with a very even surface, and when this is quite dry, the cakes of asphalte are melted, and a layer of it, about $\frac{1}{2}$ in. thick, is poured on with ladles and reduced to an even surface by hand with a small board. The whole area is laid in breadths of about 3 or 4 ft . at a time, each ladleful being enough to cover that breadth of 3 or 4 ft . for a length of about 18 in . The liquid asphalte is kept in its place by strips of wood along the edges; thus there are series of joints or seams all across the area covered, which are parts liable to cracks and leaks; therefore, in floors and roofs where it is a matter of importance to keep out the damp, it is desirable to have two layers of asphalte, the joints of the upper layer occurring over the spaces of the lower layer. For light foot traffic, or for the protection of the asphalte against the sun, coarse sand is sprinkled over it while it is soft; for heavier traffic, it is necessary to mix grit or broken stone with the asphalte before laying it on, and to lay it 1 in . thick and upwards.

It is very necessary that the layer of fine concrete should be quite dry, otherwiso the hot asphalte will convert the moisture into steam, and cause blisters on its surface.

The disadvantages of an asphalte floor are, that its general low temperature makes it a cold floor, and by condensing the moisture of the room on it, a damp floor; and that it is liable to crack and become broken in holes, and is difficult and expensive to repair.

Tar Pavement.-This is a mixture of tar, chalk or lime, and broken stones or sand. It is a concrete more or less fine, according to the traffic of the roadway on which it is to be laid, and in
$3 \times 2$
which tar and lime or chalk take the place of the lime aud sand of ordinary concrete. It is too soft, especially in hot weather, to be applicable to heavy carriage traffic; but for light carriage traffic and foot traffic it makes a good and tolerably durable road covering. For carriage traffic the stones should be about the size used for macadamizing; they should be coated with or soaked in tar, and then mixed on the road with the tar and lime or chalk. A coating of sand should be strewed over the surface while the tar is soft, and the road should be well rolled. In forming a pathway, a layer of the tar pavement made with small broken stones should be laid down first, and over that a layer made with sand; the surface should be strewed with sand and well rolled.

Weight of Floors.-Tredgold gives the following Table of the weight of ordinary wooden floors:-

|  |  |  |  | lbs. a square of 100 sq . ft . |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Single-joisted, without counter-flooring | .. | .. | .. | .. | 1260 to 2000 |  |  |
| Double-framed floor, with counter-flooring | .. | .. | . | 2500,4 | 4000 |  |  |
| Wooden partitions | .. | .. | .. | .. | .. | .. | .. |
| .. | 1480, | 2000 |  |  |  |  |  |

It is evident that these weights are intended to include the ordinary moving weight which the floor of a common living room would have to support; for it has been ascertained by experiment at Chatham that the weight of soldiers standing in the ranks, armed and accoutred, is about 56 lbs . a square foot, which is as much as the floor of an ordinary living room would be exposed to. The weight of men unarmed and packed as closely together as conveniently practicable, and which is the heaviest moving weight a floor could be subject to, except, perhaps, of some exceptionable carriage, is 110 lbs , a square foot. The weight of corn stacked on a floor in bulk and 10 ft . high, is 500 lbs . a square foot, and that is the heaviest weight the floor of a general storehouse is ever likely to be subject to, for 1 it is assumed that such articles as guns, shot, and heary gun-carriages and platforms would be placed on the ground floor of a storehouse. Corn, however, is seldom stacked more than 4 ft . high.
loors, Windows, and Stairs.-The great difference between the principles of the framing of large timbers required in roofs and floors, and the joining of small pieces to form doors and windows, and such like fittings, is that in the former the main object is to direct and resist great strains; in the latter, the main object is to make close-fitting joints and smooth surfaces. This difference between the two systems has caused this trade in England to be divided into two branches-the carpenter proper, and the joiner; the former dealing chiefly with heavy framework and rough timbers, the latter with fine woods in small pieces fitting nicely together. The joiner uses finer kinds of wood, more carefully selected and better seasoned.

In all kinds of joiners' work one of the chief objects is to reduce the framing into narrow pieces of wood, so that the work may not be sensibly affected by the shrinking; for timber in joiners' work, however well selected, is almost certain to contract when exposed to the constant dry warm atmosphere of a living room, and the chief shrinkage is laterally, and not longitudinally. Therefore panels of framing for doors, and the like, should not be more than 15 in . wide and 4 ft . long. Woodwork in joinery is connected together generally by mortices and tenons. The tenons and mortices should be very truly made, or there will be a danger of one of them splitting when the parts are brought together; tenons should be about $\frac{1}{4}$ the thickness of the timber, and their width should not exceed five times the thickness, otherwise they are liable to bend or warp: therefore, with wide pieces of timber, two tenons and mortices should be made. Sometimes pieces of wood in joinery are keyed together with small wedge-shaped pieces of hard wood, as in the curved ribs of small arches. Tredgold recommends screw-bolts in preference. Sometimes pieces are framed together with dovetailed joints, as in the sides of boxes: the dovetailed mortice and tenon-joint, which is disadvantageous in large framework, is useful and effective in joinery.

Doors.-The door and the door-frame are two distinct parts of this house-fitting. The doorframe consists essentially of four pieces, two vertical pieces, called stanchious or posts, and a top sill or lintel, and a ground sill. For exterual doors, the posts are generally of solid timber cut with a rebate on the inner faces for the door to shut against; the top sill or lintel is almost always of solid timber, as even if it has no superincumbent weight to carry, which it should not have, the stability of the door depends much on the bonding of the top sill into the wall. The ground sill is generally of hard wood or stone, in order to withstand the wear of the traffic. The framing together of these four pieces is done in the manner of ordinary carpenters' framing, and the timber is generally wrought or planed. It is fixed in the reveal constructed in the wall to keep the wind and rain from passing between the frame and the wall; hence an external door always opens inwards, an arrangement suitable both for convenience and defence. The frame of an internal door may be made in the same manner as that of an external door, and set in a reveal in the wall; but it is usual in ordinary houses to line the whole of the door opening in the wall with wood for the sake of appearance, and the vertical and top pieces of this lining are used as the posts and top sill of the door-frame; hence an internal door-frame is a kind of box, the pieces of which are dovetailed together, and are thick enough to allow of a rebate being cut in them for the door to shut against. As the efficiency of a door-frame depends more on its stiffness than its strength, the scantlings should never be very.small; probably $3 \times 3 \mathrm{in}$. is the smallest cross-section that should be given to any solid door-frame; and a barrack solid door-frame should be $4 \times 4 \mathrm{in}$. An internal door should be flush with the wall of the room it leads into, and should open into the room.

The door itself is composed of a frame of four or more pieces, consisting of two vertical, called styles, one horizontal at the top and bottom, called the top rail and bottom rail, and one or more intermediate horizontal pieces, called the lock rail and frueze ralls, and sometimes an intermediate style to reduce the breadth of the panelling. According to the mode of filling in this framework, the door recenves its technical name. The rails are framed in between the styles with common
mortices and tenons, driven up tight with little wooden wedges. The intermediate styles are framed in between the rails. The filling in is sometimes of boards or battens nailed against the framework, or let in flush with the framework on one side; this method is generally used with common doors, as for barrack-rooms, and is called a framed and battened door. Sometimes the filling in is of thin panels of wood morticed into the frame with a continuous groove all round; this is used with doors of more important rooms, and is called a frumed and panelled door. It is the strongest description for ordinary purposes. In large battened doors a diagoual brace is sometimes introduced, extending from the upper outer corner to the lower inner corner.

The thickness of the framework determines the strength and weight of the door; 2 inches is sufficient for the framewori of barrack-room doors, and $1 \frac{1}{2} \mathrm{in}$. for common living-room doors. The widtly of the framework is more a question of appearance. Tredgold says it is commonly abont $\frac{1}{3}$ that of the panel. The lock rail is generally wider than the others.

The mouldings and ornamentation of doors are questions almost entirely of appearance, and not of construction. Sometimes the panelling is flush with the framework on one side, and therefore sunk on the other; sometimes a double panelling is inserted, so as to make them flush ou both sides; sometimes the edges of the sunk part are square, and sometimes moulded. That moulded woodwork, which extends round the door-frame or door-opening, and which is commonly called the architrave, is chiefly for effect, its only constructive use being to form a stop or finish to the plastering of the wall, covering its junction with the door-frame.

Panclling.-There are several other fittings in the interior of houses which are made by the joiner, such as skirtings, door and window linings, shutters, wainscotting, and so on. Most of these are constructed of a framework filled in with boards, and therefore the principles of the construction of doors apply to almost all of them. A skirting is the kind of plinth or base, geuerally of wood, which extends round the walls of a room at their junction with the fioor; its constructive object is to cover the junction of the floor with the walls, but for the sake of effect is commonly enlarged much beyond the requirements of that purpose. It consists commonly of a wooden board framed to battens fixed to the wood bricks built for this object into the wall. The battens are called grounds. They should be carefully fixed, as on them depends the accuracy of the finished work. The boards of the skirting, as in the case of all panel-work, should be so fixed as to allow for expansion and contraction without splitting. Tongueing and grooving is the best method for this object; one side of a skirting board may be fixed, and the other let in with a groove and tongue.

In barrack-rooms a skirting is frequently made with a flat wrought-iron bar fastened to the floor; this would not be sufficient in rooms the walls of which are liable to be injured by the feet of the inhabitants.

Window and door linings and shutter's and wainscotting are constructed on the same principle as doors, with framing filled in with panelling. The linings and wainscotting are fixed to grounds, the shutters are hung to the window or door frames. Wainscotting is the term applied to the lining of walls with woodwork, because when it was generally used, before plastering was much adopted, the commoner kind of oak, called wainscot oak, was used for it. The lining of walls and other parts of houses may be made with boards laid like those of a floor. The framework, when there is a masonry wall, need consist only of battens fixed to that wall at intervals of about 2 ft . The principles of laying floor boards apply also to lining boards; they should be narrow, and grooved and tongued or filleted, and it is more convenient and economical to lay them horizontally than vertically. With walls composed entirely of wood-framing the vertical posts should be at about 1 ft . apart, in order to prevent as much as possible the warping of the boards; the boards should be of well-seasoned timber, to reduce as far as practicable the inevitable shrinking which takes place in timber exposed to the warm dry air of rooms. The exterior covering of such wooden walls is frequently made of boards; but in order to carry off the rain, it is necessary to lay them horizontally and with a small overlap. This is done either by cutting a kind of rebate in the lower edges of the boards, the shoulder of which fits on to the upper edge of the board below, or by bevelling the inner face of the lower edges, and the outer face of upper edges, so as to avoid a great projection. An overlap of $\frac{3}{4} \mathrm{in}$. is sufficient if the timber is good and properly fixed.

Windows.-There are two ordinary methods of arranging the opening of windows, requiring two kinds of construction.

1st. The sliding sush, in which the window slides vertically up and down in its frame, being counterbalanced by two weights, one on each side, moving in boxes made in the frames and connected with the windows by cords passing over pulleys at the top. This is the method commonly used in England, and is the most effective one in climates of much wind and rain. There is a window-frame, just as there is a door-frame, for the windows to work in, and this is made of four pieces, like a door-frame, two side picces, a top sill, and a bottom sill. The side pieces, instead of being solid, consist of the boxes before mentioned, or cases as they are called, made of thin boards, the sides of which project slightly towards the window, forming a kind of groove for the window to slide in. The top and bottom sills are cut with a rebate for the window to shut against, and the bottom sill is weathered-that is, sloped on its upper surface, to carry off the rain. The bottom sill is generally of hard wood, on account of its exposure to wet, it rests on the stone sill. The frame is made by the carpenter, and fixed in its place by the mason or bricklayer.

The window, or sash, as the joiner calls it, is made like a door, of a frame of four pieces, two styles and two rails: they are put together on the same principles as in a door; the intermediate pieces to hold the panes are called sash-bars; the vertical sash-bars extend continuously from top to bottom; the horizontal bars are framed in between them. The bars and frame-pieces are cut with a rebate on the outer sides, forming a shoulder against which the glass is laid; the under-side of the bottom rail is bevelled to fit the weathering of the bottom sill of the windowframe.

When the window is in two separate pieces, or is hung double, having an upper and a lower
sash, each piece is hung separately to a pair of counterbalancing weights; the upper sash slides downwards to open, and the lower sash slides upwards, the upper one being placed outside the lower one for this object partly. The meeting rails of the two sashes are cut with a bevel on the inner and outer edges respectively, in order that they may fit quite closely when shut.

The size of the panes or intervals of the sash-bars are determined by the most convenient and economical sizes of glass which can be obtained. The pulleys of the counterbalancing weights are generally made of brass, and the weights of lead, and the cords of a small white rope called sash-cord. The only other fastening required is one connecting the two meeting rails together, which should be such that when open it shall not interfere with the movement of the sash; it is partly for convenience of arrangement of this fastening that the upper sash is placed outside the lower one.

2nd. Casement Windows.-The second method of opening windows is that generally used on the Continent, of hanging them with hinges to the frame to open like a door. It is very difficult to keep out the weather with this method. The window-frame is in this case solid, and is cut with rebates on the side pieces and sills for the windows to shut against, otherwise the frame is made on the same principles as a door-frame. The window is generally divided in two parts or leaves vertically, like a folding door, in order to reduce the breadth of the moving part. Each part is made of a frame of four pieces with sash-bars; but in this case the horizontal bars should extend continuously through, and the vertical bars should be framed in between them. Several different methods have been proposed of forming the rebates and the junction of the two centre styles, with the object of effectually keeping out the rain and wind; the most effective appears to be to make a curved groove in one part, and a corresponding curved tongue or projection on the other, so as to fit close to each other when shut.

Besides the hinges, a fastening similar to a door lock and bolts are required for this window.
Another method of arranging the opening of a window, common in factories, is by swinging it on two horizontal pivots in the side styles, a little above the centre of their height, so that it is opened and shut by means of two lines, one from the top rail and one from the bottom, the upper part opening inwards and the lower part opening outwards. This is, therefore, an advantageous plan for windows out of ordinary reach of hand. The horizontal sash-bars in this window should extend continuously across, and there should be a centre rail to hold the two pivots. It is easier to make this window water-tight than the casement window, because a rebate forming an effective stop can be put in the inner side of the top part and the outer side of the bottom part. It is not applicable to large windows, on account of the strain from the mode of hanging.

The weight and strength of a window depends much on the thickness of the woodwork, as in the case of a door ; $1 \frac{3}{4} \mathrm{in}$. is a suitable thickness for an ordinary barrack-window. The sash-bars are generally the same thickness as the frame.

Skylights.-This is the name given to windows formed in the roof of a house, and more or less corresponding to the slope of the roof. The difference between that and the ordinary vertical window is, that being on a slope the framework must be stronger to resist the greater transverse strain, and that the glass must be fixed in such a manner as to allow the rain to run off. Skylights in general are not made to open; when they are required to open, it adds considerably to the difficulty of construction. In order to carry off the rain, all the sash-bars should lie down the slope of the roof, and the glass panes should overlap each other like slates, without any cross-bars. The panes are kept in their places by the friction and putty in the grooves cut for them in the side sash-bars, and by small metal clips suspending the bottom of one pane to the top of the one below it, the bottom panes of all being sustained by nails or clips to the bottom rail. To connect the sides of the skylight together, iron rods or bars are passed through the bars horizontally. The frame of a skylight is fastened to the beams of the roof, and should project 3 or 4 in. above the roof covering. The styles and rails of the skylight itself should project over the frame, and have a deep rebate cut in or a projecting piece fastened on to the outer edge to close over the frame. A lead flashing should be laid round the outside of the frame and under the roof covering on all sides. When the skylight is required to open, the woodwork should be stronger and the rebates or projecting stops deeper, and a strip of lead should be laid round the styles and rails hanging down over the lead flashing on the sides of the frame.

The panes should be so wide that the sash-bars will necessarily require to be of considerable depth to carry their weight, and thus become in fact descriptions of rafters; by which arrangement an additional system of bearers will probably be saved. They should also be of a size that will allow of an economical division of the sheet of glass of the ordinary dimensions of which the particular glass used is made, and not too large to be expensive in renewing.

Dimensions of Doors and Windows.-Doors.-The minimum width for a door of a living room is that through which a man can conveniently pass; that is to say, about 2 ft . There are few internal doors less than 2 ft .9 in . in width; those of the soldiers' barrack-rooms in Brompton Barracks are 3 ft .7 in .; they are too large for one man and too small for two, and form very heavy doors. Those of the single officers' quarters are 3 ft .3 in . wide, and are large and heavy doors. When doors are required to exceed 3 ft . in width, it is better to hang them in two leaves, meeting in the centre, or folding, as it is technically termed, in order to reduce the weight of the door. The weight of a door, besides being a question of convenience, is often a cause of injury from the strains caused by it. The exterior doors of the soldiers' houses in Brompton Barracks are 3 ft .10 in . wide, and huny folding. They are sufficiently wide to allow two men to pass, which should be the case with all exterior doors of soldiers' quarters. The minimum height for a door of a living room is 6 ft . 3 in ., but barrack-room doors should be 7 ft . high, to allow for the passage of soldiers with their chacos on. Those in Brompton Barracks are 7 ft .7 in . high. The exterior doors of barracks should be 8 ft . high, to allow soldiers to march in or out with bayonets fixed.

The following extract from Chambers on Decorative Architecture and from Renaud show the dimensions of doors, recommended by those authors on the double grounds of convenience
and beauty of proportion. The dimensions of doors are measured from out to out of the door itself.

General dimeusions for breadth of doors ;-

(From Rickman's 'Gothic Architecture '):
Gothic (decorated) doors, height to crown equal $2 \frac{1}{4}$ times the breadth.
(From Renaud, 'Cours d’architecture );
Breadth.

|  |  | Below. | Above. | Height. |
| :--- | :--- | :--- | :--- | :--- |
| Doors of the Erecthæum Temple, Athens | .. | $2 \cdot 44$ | $2 \cdot 35$ | $5 \cdot 21$ metres. |
| Palazzo Massini (Rome, Peruzzi, 1500$)$ | .. | $2 \cdot 170$ | $2 \cdot 170$ | $4 \cdot 64 \quad "$ |
| The heights of doors vary from | 2 to $2 \frac{1}{2}$ the breadths. |  |  |  |

Windows.-The width of a window between the frame is generally a question of appearance and of the distribution of the light in a room ; the maximum limit, as far as construction is concerned, being that which can be conveniently opened.

The following extracts from Chambers and Renaud give the principal considerations, as far as appearance is concerned, for the dimensions of windows.

Dimensions of Windows.-Palladio agrees with Vitruvius that the height between the floor and the ceiling should be divided into $3 \frac{1}{2}$ parts, and that the height of the window should be two of those parts and the brealth $1 \frac{1}{6}$ part.

Vitruvius says the breadth of windows should be from $\frac{1}{4}$ to $\frac{1}{5}$ the breadth of the room, and the height $2 \frac{1}{6}$ times the breadth.

James Morris' rule for window-space ;-The total area of light $=\sqrt{ }$ cubic content of room.
Chambers' rule $;-$ Breadth of window $=\frac{1}{8}$ (breadth + height of room).
The window-sill should be at such a height from the floor as will enable a person to lean on it. The breadth of the windows in upper stories should be the same as those of the lower stories. The breadth of the interval between windows should be from 1 to 2 times the breadth of windows; the end interval should be at least equal to breadth of window. Palladio says the jambs (or reveals) should be from $\frac{1}{6}$ to $\frac{1}{5}$ breadth of window.

From Renaud;-
Windows in the Louvre, Paris, P. Lescot .. .. .. $\quad 1.955 \quad 4.557$ mètres.

From Rickman;-Gothic decorated windows; height to crown equal $2 \frac{1}{3}, 2 \frac{1}{4}$, and $2 \frac{1}{5}$ times the breadth.

The heights of windows vary from 2 to $2 \frac{1}{2}$ times the breadths.
These rules give windows of greater width than is convenient for opening in one piece, but the practice in modern houses is to employ the large widths based no doubt on these rules for the sake of the appearance, notwithstanding the inconvenience. In the casement window the effect of the great width is quite taken off by constructing the window in two leaves or folding, thus forming a broad dividing style in the centre. The casement window also admits of the opening part being further reduced in size, by placing a rail or transom, as it is called, at a convenient height from the bottom, dividing the window into two parts; the upper part being fixed or opening independently of the lower.

It is an important consideration in barracks to make windows of a size convenient for opening, and for construction and repair; and although it is probably more effective to introduce light into a room through as large area as possible, the purposes of ventilation require a distribution of the openings throughout the walls of the room, and the convenience of the men also requires a general distribution of the window-space. Further, it is recommended by the Sanitary Commissioners that windows should extend as nearly as construction admits to the ceiling, for the sake of ventilation. All these considerations taken together are in favour of employing narrower windows than those usual in modern English buildings, and of obtaining the necessary area of light opening, by increasing the height, at the loss of the ordinary proportion between breadth and height recommended by Chambers and Renaud.

In hospitals and places where light and ventilation are of more importance, the necessary area of light opening might be obtained by a row of small windows near the ceiling, thereby allowing the lower ordinary windows to be reduced to the ordinary proportion between brcadth and height.

Stairs.-The following are the technical names for the parts of stairs;
Flight is the term for one continued series of steps without any break.
Landing is the level flat between two flights.
Tread is the horizontal surface of a step.
Riser is the vertical part between two steps.
Winders are the winding steps round a curve when there is no landing.
Stone Stairs.-The most simple arrangement for the construction of stairs is to build two parallc.? walls, to carry the ends of the stones forming the steps, and if the width of the stairs is great, tc build intermediate parallel walls underneath the steps to support them. In this case the stones are subject to scarcely any transverse strain if they are properly laid. It frequently happens, however, that the arrangements of the house prevent the employment of two parallel walls; in such cases one end only of the stone step is let into the sidc wall, leaving the other end unsupported,
excepting that the outer and lower edge rests on the inner and upper edge of the stone below; this, theoretically, gives a support to each stone from bottom to top of the stairs; practically there is a certain amount of transverse strain even when extraordinary care is taken with the laying. To ensure a good bearing of the upper stones on the under, the bearing edges are cut with a bevel, which Renaud recommends should be $\frac{1}{3}$ the height of the step; he also recommends the bearing of the step on the wall to be $0 \mathrm{~m} \cdot 20$ for ordinary steps. When, on account of the arrangements of the house the steps wind round to change their direction, the theoretical support of each on the step below is still greater, and becomes complete when the steps form a continuous spiral stair round a common centre. In this latter case, the inner euds of the stones are sometimes cut with cylindrical heads, which, resting ou each other, form a continuous column from bottom to top. Sometimes the inner edges are cut short of the centre, so that they do not rest vertically over each other, but form a kind of inner spiral, leaving a small well-hole in the centre from top to bottom.

In barracks of more than two stories, the steps of the lower story should be of stone, on account of the greater wear upon them. It is desirable that all stairs in barracks should be of stoue in order to be fireproof. The stone should not be a limestone or any stone that will wear to a very smooth surface; such steps, after they become slightly worn, are daugerous for men to descend quickly.

Wooden Stairs.-Stairs may be constructed of wood in the same manner as of stone; but for the sake of economy it is usual to take advantage of the elasticity of the material and construct them as follows :-

Two or more strong beams are fixed at the slope determined on for the stairs, framed into the beams of the floor below and into those of the floor above, or into some beams specially provided if there should be a landing. The sloping beams are called bearers or strings. Their upper edges are cut into vertical or horizontal notches to correspond with the risers and treads of the steps; sometimes boards are fixed to the bearers parallel to and projecting above them, and the notches are cut in them instead of in the beams themselves. The steps are formed with boards, a vertical board for the riser and a horizontal board for the tread of each step; these are framed together with a special tongue and groove joint; the tread-board projects over the riser and is finished with a rounded head or nosing. The tread-board is also framed with the riser of the step next above it. The treads of barrack-stairs slould be made of hard wood, and be at least $1 \frac{1}{2}$ in. thick to withstand the wear; the risers should be at least 1 in . thick: with such dimensions the bearers of an ordinary stair may be about 4 ft . apart.

When there is a landing at some intermediate part between two floors, horizontal beams are fixed in the wall, to carry it and the bearers of the stair ; these beams are like the joists of a floor, and are covered with boards in the same manner. When there is no flat landing, but the steps wind round a central well-hole to change the direction of the stairs, a horizontal beam to carry each winding step is fixed into the wall, and framed into one or more vertical posts about the well-hole: these posts are not necessarily supported from the floor below. Or sometimes a curved and winding bearer is specially constructed for this part of the stair. The treads of this part are necessarily wider at the outer end than at the inner, according to the degree of winding. When no wall is close enough to be available, the joists of the landing must be supported by vertical posts from the floor below.

Handrails.-A convenient height for the handrail of a stair is about 3 ft . from the surface of the treads. The upper surface of it should be semicircular and about $2 \frac{1}{4} \mathrm{in}$. diameter; it should be continuous without break of any kind from top to bottom of the stairs. The Balusters which support the handrail are sometimes also intended to fill up the space between it and the stairs, so as to prevent any one falling through. When for the former object only, as is generally the case in barracks, the fewer balusters there are the better, as they are very liable to injury and so cause expense either to the public or the soldier in repair'; for this reason it is better to have a few strong posts well framed into and connected by iron straps with the bearers of the stair. In private houses where the balusters are generally required to fill up the space, the ordinary practice is to make them square wooden bars of small size, and to place iron balusters of the same size at intervals to strengthen the whole structure. But in all public buildings, especially in military buildings, it is desirable to use balusters of a much larger size, and more firmly fixed to the stairs, and at just sufficient interval to prevent children falling through.

Dimensions of Stairs.-Treads and Risers.-The proper angle of inclination for a stair, that is, the height of the riser and breadth of the tread, is that which will enable a person to ascend it with the least fatigue. The theoretical rule for determining this originated apparently with the French architect Blondel, and is based on the supposition, that as there is a certain length of pace which is least fatiguing to a man on level ground, there is also a certain interval between the rungs of a vertical ladder which can be ascended with the same ease. Assuming this proposition and assuming the average length of pace and vertical interval of rungs, this equation will then give the breadth of tread for any assumed height of riser in a stair.

$$
h=\mathrm{H}-\frac{\mathrm{H}}{\mathrm{P}} p\left\{\begin{array}{c}
\mathrm{H}=\text { horizontal distance of each step. } \\
\mathrm{P}=\text { vertical height which can be } \\
\text { ascended with equal ease. }
\end{array}\right.
$$

M. Renaud assumes the length of the pace of an ordinary man on level ground to be $0 \mathrm{~m} \cdot 64$; and that he can ascend a vertical ladder with equal ease when the steps are $0^{\mathrm{m}} \cdot 32$ apart. Consequently by this rule the breadth of any step plus twice its height ought always to be equal to $0^{m \cdot 64}$. This rule is however not true either theo;etically or practically. M. Renaud says that in Paris experience has restricted the height of steps between $0^{n_{i}} \cdot 11$ and $0^{m} \cdot 19$, which by the preceding rule gives for the breadth $0^{\mathrm{m}} \cdot 42$ and $0^{\mathrm{m}} \cdot 26$. Low steps are wot absolutely an adivantage, because there
are more of them to rise to a given height. When the total height is considerable the latter dimension is preferable; when it is a stair of one flight only, the former is better. In ordinary dwellinghouses the heights are generally between $0^{\mathrm{m}} \cdot 16$ and $v^{\mathrm{m}} \cdot 17$, and the breadths consequently between $0^{\mathrm{m}}-32$ and $0^{\mathrm{m} \cdot 30}$.

The above dimensions correspond very nearly with those used in England in ordinary houses. In Brompton Barracks, Chatham, in the soldiers' stairs, the risers are $7 \frac{1}{4} \mathrm{in}$. and the treads are $10 \frac{1}{2} \mathrm{in}$., which latter is too narrow for a man to descend rapidly with safety; $12 \frac{1}{2}$ in. would be better, though it might make the ascent a little more laborious. The breadth of the tread must be measured from nosing to nosing of step, and the height from surface to surface of tread.

Width of Stair.-The minimum width for the stair of an ordinary house is that which will admit of the passage of a man, that is to say about 2 ft . Renaud mentions a spiral stair of which the diameter was $1^{\mathrm{m}} \cdot 20$. The ordinary width for a barrack stair is that which will allow two men armed to pass each other conveniently ; the barrack stairs in Brompton Barracks are $4^{\prime} 2^{\prime \prime}$ wide, which is not sufficient for two men to pass; 5 ft . would probably be enough. The maximum width for a stair is a question rather of effect than of convenience. The width should be measured inside the balusters.

Size of Flight.-It is not desirable or generally convenient to hare a stair between two floors in one continuous flight or even in one straight line with an intermediate landing: considering conrenience and effect together it seems desirable to limit the height of one flight to about 7 ft ., and to avoid having two flights in one direction.

Lath and Plaster.-The plaster used for covering the walls of buildings is a mortar composed of various kinds of limes or cements, and various descriptions of sand, mixed in various proportions and generally with a little hair or some such material to give it a little more elasticity. It is laid on by hand with a trowel in several thicknesses of about $\frac{1}{8}$ to $\frac{1}{4} \mathrm{in}$. each, and either on the bare masonry wall or on a special screen of lathing made for it, to either of which it adheres by entering into and keying itself in the joints and openings and by its adhesive quality. With some variations in the materials and mixing, it is used for exterior and interior work and for ceilings. That material which is more properly called plaster, or plaster of Paris, namely, the sulphate of lime obtained by burning gypsum is commonly used for mouldings and ornamental work; several cements are now made in England for that same branch of the work.

The constructive advantages in the use of plaster for covering walls and ceilings are chiefly connected with dryness and cleanliness, otherwise the chief object of it is for the effect of a smooth surface and as a preparation for ornamental painting. For the purpose of assisting to keep the interior of the rooms of a house dry, it is advantageous to employ lathing, which being detached from the masonry of the walls forms a lining distinct in itself and not liable to the effect of any moisture which may be in the walls, and which would in time destroy the plaster if placed on the walls themselves, or if the plaster should be made with hydraulic lime would tend to separate it from the walls and always keep it damp and cold.

The following extracts from the article on Building, by Hosking, in the Encyclopredia Britannica, and from other sources, explain the ordinary method of forming a lath-and-plaster lining to a wall or ceiling in England; -

Different kinds of Plaster used.-Coarse stuff is composed of a mortar made of equal parts of lime and sand and clean long ox hair; 1 lb . of hair to 3 cub. ft . of mortar is the usual proportion. The hair should be as long as it can be procured and free from grease and dirt, and as fincly scparated in the mortar as possible. Nothing but clean sharp sand should be used with this lime and hair in the composition of this any more than of building mortar. Fine stuff is a mortar made of fine white lime exceedingly well slaked with water, or rather formed into a paste in water to make the slaking complete: for some purposes a small quantity of hair is mixed up with it. Finc stuff very carefully prepared, and so completely macerated as to be held in solution in water, which is allowed to evaporate till it is of sufficient consistence for working, is called putty, plasterers' putty. Gaujed stuff is composed of $\frac{3}{4}$ of this putty and $\frac{1}{4}$ calcined gypsum or plaster of Paris; this must be mixed in small quantities at a time, as the gypsum causes it to set rapidly. Common stucco is composed of $\frac{3}{4}$ clean sharp sand and $\frac{1}{4}$ lime. Busturd stucco is composed of $\frac{2}{3}$ fine stuff, without hair, and $\frac{1}{3}$ of very fine clean sand.

All kinds of limes and cements may be used for plaster; with respect to the quantity of sand they will bear, sec the article on Lime and Mortar. Where the plastering is intended to hold water, as in a tank, it is evidently necessary that a hydraulic lime should be used.

Lathing.-Before the plasterer begins to lath a ceiling he proves the under face of the joists by the application of a long straight-edge, and brings them all to one horizontal plane. This is done by nailing on strips of wood, and is called firring. If it be a framed floor it is tolerably sure to be straight ; when the ceiling joists are fastened to the binding joists of the room above, nothing of this kind is necessary. It is an important point to be attended to in plastering on laths, and in ceilings particularly, that the laths should be attached to as small a surface of timber as possible, because the plaster is borne not so much by its adhesson to the wood, but by the keying of it betwcen and behind the laths. Under a single fioor in which the joists are necessarily thick a narrow fillet should be nailed along the middle of the whole length. Plasterers' luths are narrow strips of some straight-grained wood (generally fir in England), split with an axe to give roughness of surface, and cut into lengths of 3 or 4 ft ., about 1 in . broad, and $\frac{3}{16}$ or $\frac{1}{4} \mathrm{in}$. thick. These are single laths; double laths are $\frac{3}{8}$ in. thick. Lath nails are either wrought, cut, or cast, varying in length to suit the single or double laths: cast nails are commonly used in England. The laths are laid in courses or bays, and these should break joint over the whole surface of the ceiling; the rows of laths are about $\frac{3}{8} \mathrm{in}$. apart, and there is one nail in the centre and one at each end, which latter also sccures the ends of the adjacent laths in the same row.

In lathing on walls the bonding of the bays of lathing is not of so much importance, nor is the breadth of the timbers behind the lathing, because the toothing which the thickness of the lath
itself affords to the plaster is sufficient to support it vertically. The laths for walls may be weaker than those for ceilings; weak laths in a ceiling are sure to produce inequalities by sagging to the weight attached to them. When the lathing is on a wall, the timbers to which it is fixed, being supported by the wall, need only be battens of $1 \frac{1}{3}$ to $2 \frac{1}{2}$ in. thick, which are fixed to wood bricks in the wall, their object being both to support the lathing and to provide a space between the plaster and the wall, so that they shall not come into contact.

Laid and set is the technical term for two coats of plaster on laths. The first coat is a thick coat of coarse stuff of a consistency thin enough to pass between the laths and form a key behind, and stiff enough not to fall apart in the operation, a contingency which not unfrequently occurs in practice, in consequence of the workmen using thin mortar in order to avoid the extra labour of working stiff mortar through the latter. It is put on with a trowel, and when sufficiently dry is swept with a birch broom to roughen its surface. The secoud coat is a thin coat of fine stuff, and is put on with a trowel with the assistance of a hog's-bristle brush to keep the surface of the second coat wet while the operation is going on. If the first coat has become very dry it must be wetted, or the second coat in drying will shrink from it.

Plaster, float and sct is the term for three coats of plaster on laths. The first or pricking-up coat is of coarse stuff pui on with a trowel to form a key behind the laths, and about $\frac{1}{4}$ or $\frac{3}{8}$ in. thick on the laths: while it is stili noisi it is scratched or scored all over with the end of a lath in parallel lines 3 or 4 ib . apart, the scoring, bemg made as deep as possible without exposing the laths; the rougher the edges are the beiter, as tue object is to produce a good key for the next coat. When the pricking-up coat is sufficiently dry bct to yield to pressure in the slightest degree, the second coat or floating is put on. The floating is of fine stuff with a little hair mixed with it; ledges or margins, 6 or 8 in . wide, and extending across the whole width of a ceiling or height of a wall, are made at the angles and at intervals of about 4 ft . apart throughout : these must be made perfectly in one plane with each other with the help of straight-edges. These ledges are technically called screeds. They form gauges for the rest of the work, and when they are a little set the spaces between them are filled up flush, for which a derby float or long straight-edge is used. The screeds on ceilings ought to be levelled, and those on walls plumbed. When the floating is sufficiently set it is swept with a birch broom for the third coat or setting. The third, or setting coat, should be of plasterers' putty if the ceiling or wall is to be whitened or coloured. If it is to be papered, the third coat slould be of fine stuff, with a little hair in it. If it is to be painted, the third coat should be of bastard stucco trowelled. Trowelled stucco should also be hand-floated. In this operation the stucco is set with the trowel in the usual manner, and brought to an even surface with that tool to the extent of 2 or 3 yds . The workman then takes the hand-float in his right hand, and rubs it smartly over the surface, pressing it gently to condense the material. As he works the float he sprinkles the surface with water from. the brush in his left hand, and eventually produces a texture almost as fine and smooth as that of polished marble.

Plaster of Walls.-The process of plastering on the naked brick or stone wall differs in little except in name from the mode on laths. The first coat is called rendering, and differs ouly from the first coat for laths, in the quantity of hair, which may be less, and in the consistency of the mortar, which may be more plastic, because in a moister state it will attach itself more firmly to the wall: the wall itself should be wetted before the rendering is applied. The second coat is called setting, and is the same in every respect as the setting coat on laths. In three-coat work the first, or rough rendering, should be made to fill up completely whatever crevices there may be in the work behind it, and be incorporated with it as much as possible. Its surface should be left rough, but it is not scratched or lined as the similar coat on laths is. For the second coat, or floating, screeds must be formed, as before described. The consecutive processes are exactly the same as on laths, both for the floating coat and for the setting coat. In almost every case in which plastering has to be floated, the workman finds a guide for the feet of his wall screeds, in the narrow grounds which the joiner has previously fixed for the skirting. From these he plumbs upwards, and makes his work perfectly flush with them.

In plastering a wall with common stucco (its use is mostly for outside work), the dust is first removed from it by brushing, and it is then well wetted. If the wall to be stuccoed be an old one, or one of which the joints have been drawn, the mortar of the joints must be chipped, or even raked out, and the bricks picked, to expose a new and porous surface to the plastering before brushing and wetting. The wall is then covered with stucco in a fluid state, applied with a broad and strong hog's-bristle brush, like common whitewashing. When this is nearly dry the stucco must be laid on as in common rendering. When the work is to be floated the process is nearly similar to that in floated plastering. Screeds must be formed at the highest and lowest extremities of the wall, and be returned at the angles, putting the whole surface into a sort of frame; inner screeds must then be made at every 3 or 4 ft. apart over the whole surface, and the interstices filled in as before. As the work is made good it must be well rubbed with the hand-float to compress the material and produce a hard and glossy surface. Preparations for cornices and other projections should be previously made by bricks or tiles projecting from the brick or stone work, forming a core on which the mouldings are sun with moulds in stucco. No plaster of Paris should be used for external work, as the wet will dissolve it. When the stucco is dry it may be painted in oll colours, or coloured in distemper.

Rendering in Romar. cement is executed almost exactly in the same manner as stucco rendering, culy it is laid oa the saturated wall directly without the preliminary operation of roughing in, or washing the surface with a solution of the material. The same process is also followed in floating this cemeni, and wath the same exceptions. A quick-setting cement like this is far preferable to common lime for mouldings. Roman cement may be painted in oil or coloured. In the latter case the colour should be mixed with dilute sulphuric acid instead of size.

Rough cast is a cheap and useful covering for external walls which are well protected by eaves. The surface is first roughed in cr readered with lime and hair. When that is dry another coat of
the same material is added, laid as evenly as it can without floating; and as soon as a piece of 2 or 3 yards is executed the workman lays on it an almost fluid muxture of fine clean gravel and lime. This is immediately washed with any ochreous colour, and the whole dries in one mass. The lime in rough cast should be a strong lime.

Mouldings and Ornamental Work.-If a moulding which is to be made in plaster does not project beyond the plane of the wall more than 2 in . it will be sufficient to make a foundation for it in lime and hair after the first coat of plaster. If any one part of the moulding should project more than that, a row of nails, 6 in . apart, and driven into the plaster, will be sufficient to support it. But if the general mass of the moulding project more than 2 in . a rough form of it must be made by brackets of wood fixed at intervals, and cut roughly to the outline of the moulding, and covered with laths. The first, or pricking-up coat, must be laid over this form. The second coating of the moulding, if interior work, should be made of gauged stuff, and laid as follows:-A mould, or piece of board with the section of the moulding cut out of it is made to move along the line of the moulding guided by two battens, one above and one below, fixed temporarily to the wall, and so as to have a space of about $\frac{1}{4} \mathrm{in}$. between it and the first coat of plaster. One man lays on the gauged stuff in almost a fluid state with an angular trowel; another works the mould backwards and forwards over it until it takes the form of the mould, the superfluous stuff being swept off by the action. If the whole height and projection of a moulding be too large and heavy to be executed at once in this manner, it can be done in parts, one at a time.

This method evidently applies only to plain mouldings of the same continuous section. Enriched mouldings can be commenced in the same manner, and the space for the enrichment can be left vacant in the mould, and completed afterwards by hand. The angles formed by two lines of moulding meeting from different directions must also be finished by hand. Such joinings are termed mitres, and the workman uses what is called a joining tool for them. Such enriched parts of a moulding as cannot be executed in the above manner are generally cast in plaster of Paris in moulds. These moulds are made of plaster of Paris or wax or gutta-percha from wooden models of the design. These cast parts of the moulding, when set and trimmed, are fixed in their places in the moulding with plaster of Paris, if there is any projection to support them, or with white lead or iron cement if they have to depend entirely on the cement. Such ornamental castings as are too large to be trusted to cement alone must be fixed with screws to woodwork behind them specially provided for the purpose.

Hosking says the most general cause of the decay of stuccoes and cements on external walls is the impurity of the materials. If the sand is quite clean and the lime good of its kind, and the work be well hand-floated and trowelled, particularly on the upper surfaces of projections, where wet is liable to penetrate, the plastering, with common attention to the painting of it, will last as long as anything of the kind can be expected to last.

In repairing plastering, the surface should first be well washed to remove the dirt. The cracks and fractures are then repaired with new plaster, and when the new work is quite dry the joinings are scraped to produce an even surface, and the whole is re-coloured or whitened once or twice. Stuccoed walls that have been painted must be rubbed with pumice-stone to take off the old paint before they are repainted.

Table of Materials in Plastering required to cover Superficial Yards as under.


Painting Woodwork.-The useful object of painting materials used in construction is to protect them from the action of such causes of decay as heat, gases, moisture, \&ce by covering them with an almost impervious and a very durable coating; but probably the origin of house painting is due to purposes of ornament rather than use. The most effective known composition that combines both objects is a mixture of white lead and linseed oil. The mixture of white lead and oil to a consistency that can be readily used with a brush, when spread on any material sinks into it, and dries in a few days, forming a covering durable under ordinary circumstances, and impervious to ordinary damp. It can be mixed freely with almost all colours, and is sufficiently economical in England to be applicable to houses generally.

The linseed oil is the material in this mixture which forms the durable coating. This oil, in common with a few other vegetable oils and with some resinous matters, possesses the property of drying, after it has entered the surface of the material, into a resinous compound, which thus fills up the pores of a material like seasoned wool with a substance similar to the natural resin, and so prevents further decaying action going on. The diyer, as it is called, is some oxidizing substance, such as litharge, added to expedite the drying process, and a solvent, such as spirits of turpentine, is added to dilute the mixture, and which soon evaporates again. The white lead gives body and opacity to the mixture. It combines readily, and to a certain extent chemically, with the oil into a creamy compound, drying into a saponaceous substance.

White lead is the carbonate of that metal, obtained in this country from the metallic lead. It is a fine white powder, weighing about 400 lbs . a cubic foot. There are other powders of the same appearance, which, being cheaper, are sometimes used to adulterate the white lead with, such as whiting, sulphate of baryta, Paris white and zinc white. As there are none of these so durable or useful as white lead, it is essential that it should be pure. They can be detected either by the specific gravity or by resolving the powder back into metallic lead, which should weigh $\frac{1}{10}$ less than the powder, or in the case of the sulphate of baryta, which is the most usual adulteration, by nitric acid, which will dissolve the lead but not the baryta. White lead combines more readily and effectively with oil than any other of these pigments.

Linseed oil, in its natural condition, dries slowly to the resinous varnish required for coating materials, and rather injures the brilliancy of delicate colours by its strong amber colour. As the operation of drying is an oxidizing process, the addition of good oxidizing agents expedites the drying of paints. Boiled linseed oil also causes it to oxidize quicker; but, as it makes it thicker it is not so suited for in-door or delicate work. Linseed oil can be clarificd by mixing it with an acid, such as oil of vitriol. It must be well washed with water to get rid of the acid.

Driers.-The ordinary oxidizing agent used is litharge, or oxide of lead. Oxide of manganese is also used, and is a quicker drier than litharge. These driers generally contain some inert matter, such as chalk or sulphate of baryta, as an agent, as well as the acetate as an oxidizer.

The proportion of litharge generally used is about $\frac{1}{4} \mathrm{lb}$. to a gallon of oil. The oil is then called a drying oil, and can be made clear and colourless by leaving it exposed to the air in shallow vessels for two or three days.

Solvents.-Some substances are required to dilute the mixture of white lead and oil, which would not otherwise penetrate into the pores of the material. This solvent should be light and easily evaporated.

Spirits of Turpentine (commonly called turps) is generally used for this purpose; it is a solution of resin in spirit, and evaporates perfectly. The best test for its purity is by evaporating it; it deteriorates by keeping. Some natural drying oils could be used as solvents, but they generally contain foreign substances, which will not dry and which cannot be easily got rid of.

Pigments.-The object of adding all pigments to the drying oil is to give it opacity and body and colour.

They should be always of a permanent character, and are almost all mineral. Besides white lead the following are used in house painting.

Patlinson's White is an oxy-chloride of lead, and not a carbonate. It has not so much body as white lead, but it mixes freely and is not readily discoloured.

Sulphate of Baryta has not so much body as white lead, in the adulteration of which it is much used; it is durable, and forms the basis of several whites employed by house painters: it is heavier than white lead.

Zinc White is the oxide of zinc. It is cheaper than, but not so durable as, white lead ; it is light, and not opaque, and is readily attacked by salts, therefore is not durable and is easily acted ou by wet and acids in wood. It does not dissolve so readily as white lead, and therefore requires more oil. For, as long as it lasts, it keeps its colour better than white lead and is not so easily adulterated; it takes longer in drying, and when adulterated is liable to change colour. Lead dryers should not be used with zinc white: sulphate of manganese is the best drier.

Chromes or Yellows.-All consist of oxide of lead and chromic acid: when boiled with lime the mixture is lightened in colour; when boiled with acid it is darkened; when boiled with saltpetre orange colour is formed. Chromes combine well with oils, and are therefore the best of all yellows.

Prussian Blue is made from animal refuse (such as horns, blood, \&c.), burnt with potash and iron at a high temperature.

Smalt Blue is oxide of cobalt, and is made by roasting cobalt ore and mixing silica with it, and the fusion of it with sulphur produces blue smalt.

Ultramarine Blue was formerly made from lapis lazuli and was very expensive: it is now made artificially and of better colour by fusing together carbonate of soda, silica, alum, and sulphur, and then washing the compound and fusing it again, and washing it again, and then heating it with sulphur, which gradually colours it.

Greens are almost all obtained from copper. Verditer is oxide and carbonate of copper: other greens contain arsenate of copper. They are durable colours.

Brunswich Green is made of Prussian blue and chrome, mixed with carbonate of lime or sulphate of baryta to dilute it. If the chrome has an acid in it the colour fades: therefore, to test it, the powder should be exposed for a fortnight to a strong sunlight.

Red Lead is oxide of lead : on account of its durability it is frequently used as a priming or first coat on ironwork; but care should be taken that no salt is present, otherwise a chemical action commences, blisters are formed, and the lead is reduced to the metallic condition.

Oxide of Iron paints.-These are the most effective and durable paints to use on iron, as they have no tendency to change or affect the surface of the iron. There are several preparations of them. Grant's Black is made of shale containing oxide of iron. The purple brown oxide is a hydrated
peroxide of iron. The Torbay paint is a protoxide of iron. But the most effectual coating that can be used for heavy ironwork, such as thick plates, is the natural surface fresh from the rolls or hammer; wrought iron, after such treatment, has always a coating of magnetic oxide on its surface, which is hard and thoroughly attached to the body of the iron, and which as long as it is left unbroken, will prevent any further action on the iron.

In painting woodwork, the surface must be first prepared by counteracting the effect of anything that may prevent it from becoming identified with the material. Thus, in painting pinewoods of any kind, the resin contained in the knots which appear on the surface must be neutralized, or a blemish will appear over every knot; this is done with two or more coats of red lead ground in water and mised with size laid over the knots; this is called killing the knots. A preparation under the name of patent knotting is also extensively used; it is composed of shellac, naphtha, and perhaps some other drying agent. A single application of the mixture will effectually kill the knots, and as it dries almost instantly it is greatly esteemed by painters as a substitute for the red-lead mixture. All nail-holes (the heads of nails having been punched in) and other defects must be stopped, or filled up with putty or wood. The surface of the wood is then rubbed smonth with sand-paper or pumice-stone.

In laying on the paint, the white lead and oil and dryer are mixed to the consistency of thick cream; this is done either on a flat stone by hand or in a mill; the necessary colouring matter is mixed with it. The brush should be held at right angles to the face of the work, so that only the ends of the hairs touch it, for thus the paint is forced into the pores of the wood and distributed equally over the surface. Painting, when properly executed, will not present a shining, smooth, and glossy appearance, as if it formed a film or skin, but will show a fine and regular grain, as if the surface were natural, or had received a mere stain without destroying the texture. Before the paint is applied the wood should be free from moisture of any kind and seasoned, or it will at the least prove useless, and probably injurious. Dampness or moisture or unseasoned substances in woods, stopped in or covered over with paint, will, under ordinary circumstances, tend to their destruction.

New woodwork should have four coats of paint. The first coat, which is called the priming coat, need have very little, if any, of the final colouring matter in it. After priming, all nail-holes or other superficial defects are carefully stopped up before the next coat is applied. The remaining coats are laid on as the previous coats become dry, which is generally in about two days. When the painting, after a lapse of four or five years, requires to be renewed, two coats only are usually applied.

For fine work, each coat should be carefully rubbed down with pumice-stone or glass-paper and dusted before the next coat is added.

Puintung on Plaster.-Plaster, being more absorbent than wood, requires a greater number of coats to saturate the exterior face, and that the first coat should be thinner. The plaster, being quite dry and hard and well sized with the common thin glue size, the first coat consists of white lead diluted with linseed oil to a thin consistency, with the addition of a small quantity of litharge : the oil is entirely absorbed, thereby hardening the plaster to the depth of about $\frac{1}{8} \mathrm{in}$. The second coat is also thin, in order that the plaster may be thoroughly saturated. The third coat is thicker, and contains a little turpentine, with some of the colouring pigment. The fourth coat is as thick as it can be used, equal parts of oil and turpentine being employed, and sugar of lead as the dryer. A finishing coat is frequently added of pure white lead diluted with spirits of turpentine only. A small quantity of japanner's gold-size is sometimes used as the dryer, and the proper pigment to give the required colour is added; this coat, from its drying without any gloss, is called the flatting coat.

The flatting coat is also sometimes put on woodwork.
Distemper is the name given to paint composed of white lead and other colouring matters ground in water instead of oil and mixed with size to make it set; it is, in fact, a water-colour. It can be used in the same manner as oil colour, but will not stand exposure to rain as the latter will. Sometimes whiting is used instead of white lead.

Whiting.-This material is pure chalk, reduced by levigation to a fine powder. When mixed with size in water it is used to cover plastered ceilings and walls of common rooms, and sometimes for external work of common buildings. The proportions recommended by Vanherman are 12 lbs . of whiting to 2 quarts of double size, the whiting to be covered with cold water for six hours and then mixed with the size and left in a cold place till it becomes like jelly in appearance, and then, but not till then, it is fit for use. The colouring matter should be first ground in water and then added to the whiting before the size is put in. About 1 lb . of this composition will cover 6 yards.

Size is the glue extracted from animal tissue: it is used by itself as a priming coat sometimes, and sometimes as a varnish.

Anti-corrosive paint, as it is called, is made of equal parts by weight of whiting and white lead with half the quantity of fine sand, gravel, or road-dust, and a sufficient quantity of colouring matter. This mixture is made in water and can be used as a water-colour; but it is more durable to dry it in cakes or powder after mixing, and then use it as an oil-paint by grinding it again in linseed oil. The preparation of oil recommended for this purpose is 12 parts by weight of linseed oil, 1 part of boiled linseed oil, and 3 parts of sulphate of lime, well mixed. One gallon of this prepared oil is used to 7 lbs . of the powder manufactured as above mentioned.

Painting old Work.-If the work is much soiled it should be washed with soap and water and scoured; if foul with smoke and grease, it should be washed with lime and water. It should be then rubbed with pumice-stone; then the first coat of the new colour (corresponding to the second of new work) may be laid on; when it is dry, the stopping should be done, and the whole rubbed with glass-paper.

To remove old paint from painted work, a solution composed of the following ingredients is used,
namely:-soft soap $\frac{1}{2}$, potash 1, and quick-lime $\frac{1}{2}$ or $\frac{1}{3}$. The soap and potash are first dissolved by boiling in water : the lime is then added, and the whole applied, while hot, with a brush, care being taken that all portions of paint to be removed are covered with the solution, which must be left on from twelve to twenty-four hours, after which the whole of the painting (no matter how many coats) will easily be removed by washing with hot water. Paint may also be removed by burning, but this is not to be recommended except for very plain and common work.

White Copal Varnish.- 4 oz . of copal, $\frac{1}{2} \mathrm{oz}$. of camphor, 3 oz . of white drying oil, 2 oz . of essential oil of turpentine. Reduce the copal to powder, mix the camphor and drying oil, then heat it on a slow fire and add the oil of turpentine and strain it.

Mastic Varnish.-A quarter of a pound of mastic melted over a slow fire with a pint of essential oil of turpentine, and strained.

Common Colours, in house painting, are considered to include lampblack, red lead, and the common ochres. Superior colours include blues, greens, rich reds, pinks, and yellows. White is a common colour, except when flatted.

Whitewash.-This is the name given to a mixture of common lime and water, which is frequently used for coating the interior walls and ceilings of barracks and such buildings. It is mixed to a thin consistency, and laid on with a large flat brush. It does not last long, and rubs off to the touch and will not stand rain; but being cheap and easily applied and healthy, it is a very useful preparation for the purpose. It will not adhere to very smooth surfaces, such as wrought timber, paint, \&c.

Silicate of Soda.-A solution of silicate of soda has been found by Abel, when applied like paint to wood, to give it a very considerable protection against fire, as well as to form a hard coating durable for several years; it can be used with the ordinary colours like distemper.

Directions for covering Timber with a Coating of the Silicate of Soda and Lime as a Protective from Fire.-Materials employed.-The silicate of soda must be in the form of a thick syrup of a known degree of concentration, and is diluted with water when required for use, according to the prescriptions given below.

The lime-wash should be made by slaking some good fat lime, rubbing it clown with water until perfectly smooth, and diluting it to the consistency of thick cream. It may be coloured by admixture with mineral blacks, ochres, \&c.

Treatment of the Wood.-The protective coating is produced by painting the wood, firstly with a dilute solution of silicate of soda; secondly, with a lime-wash; and lastly, with a somewhat stronger solution of the silicate.

The surface of the wood should be moderately smooth, and any covering of paper, paint, or other material, should be first removed entirely, by planing or scraping.

A solution of the silicate, in the proportion of one part by measure of the syrup to four parts of water, is prepared in a tub, pail, or earthen vessel by stirring the measured proportion of the silicate, first with a very small quantity of the necessary water until a complete mixture is produced, and then adding the remainder of the water, in successive quantities, until a perfect mixture in the requisite proportions is obtained.

The wood is then washed over with this liquid, by means of an ordinary whitewash brush, the latter being passed two or three times over the surface, so that the wood may absorb as much of the solution as possible. When this first coating is nearly dry, the wood is painted with the lime-wash in the usual manner.

A solution of the silicate, in the proportion of one part by measure of the syrup to two parts of water, is then made as above described, and a sufficient time having been allowed to elapse for the wood to become moderately dry, this liquid is applied, upon the lime, in the manner directed for the first coating. The preparation of the wood is then complete. If the lime coating has been applied rather too thickly, the surface of the wood may be found, when quite dry after the third coating, to give off a little lime when rubbed with the hand. In that case, it should be once more coated over with a solution of the silicate of the first-named strength.

Tuble of Materials in Painting required to cover Superficial Yards as under.

| 1st coat | $\left\{\begin{array}{l}10 \text { lbs. white lead } \\ 4 \text { pints linseed oil } \\ 2 \text { oz. litharge } \\ 1 \text { oz. red lead }\end{array}\right\} 25$ sup | ial yards. |
| :---: | :---: | :---: |
| 2nd coat | $\begin{cases}10 & \text { lbs. white lead } \\ 2 \frac{1}{2} & \text { pints linseed oil } \\ 1 \frac{1}{2} & \text { pint spirits of turpentine } \\ 2 & \text { oz. litharge }\end{cases}$ | 40 superficial yards. |
| 3rd and subsequent coats | $\left\{\begin{array}{l} 10 \text { lbs. white lead } \\ 2 \text { pints linseed oil } \\ 2 \text { pints spirits of turpentine } \\ 2 \text { oz. litharge } \end{array}\right.$ | 50 superficial yards. |

For coloured paints, the last two coats have the colour added to the composition in the proportion of 1 ll . to 2 lbs . for every 10 yds . of surface to be painted; and the quantity of white lead is reduced in proportion.

Glazing.-There are three kinds of glass used in England for glazing windows:-1st, crown glass ; 2nd, sheet glass ; 3rd, plate glass.

The constituents of the three kinds are nearly the same, but the latter, from its mode of manufacture, is made in larger and thicker plates, and is much more perfect and expensive.

Crown glass is made in circular dises blown by hand; they are about 4 ft . diameter, and the
glass averages about $\frac{1}{15}$ in. thick. Owing to the mode of manufacture there is a thick boss in the centre, and the glass is throughout more or less striated in concentric rings, and frequently curved in surface, and thicker at the circumference of the disc. Consequently in cutting rectangular panes out of a disc there is a considerable loss, or at least variety in quality: one dise will yield about 10 sq. ft. of good window glass, and the largest pane that can be cut from an ordinary disc is about $34 \times 22 \mathrm{in}$. The qualities are classified into seconds, thirds, and fourths.

Sheet glass is also blown by hand, but into hollow cylinders about 4 ft . long and 10 in . diameter, which are cut off and cut open longitudinally while hot, and therefore fall into flat sheets. A more perfect window glass can be made by this process, and thicker, and capable of yielding larger panes with less waste. Ordinary sheet glass will cut to a pane of $40 \times 30 \mathrm{in}$., and some to $50 \times 36 \mathrm{in}$. It can be made in thicknesses from $\frac{1}{20}$ in. to $\frac{1}{2}$ in.

Plate glass is cast on a flat table and rolled into a sheet of given size and thickness by a massive metal roller. In this form, when cool, it is rough plate. Ribbed plate is made by using a roller with grooves on its surface. Rough and ribbed plate are frequently made of commoner and coarser materials than polished plate, being intended for use in factories and warehouses. Polished plate is rough plate composed of good material and afterwards polished on both sides, which is done by rubbing two plates together with emery and other powders between them. Plate glass can be obtained of almost any thickness from $\frac{1}{8} \mathrm{in}$. up to 1 in . thick, and of any size up to about $12 \times 6 \mathrm{ft}$.

In the glazing of a window the sizes of the panes, that is to say, the intervals of the sash-bars, should be arranged, if practicable, to suit the sizes of panes of glass which can conveniently be obtained, so as to avoid waste in cutting; this consideration is of more consequence in using crown and sheet glass than with plate glass. But in barracks, where the soldier has to pay for broken glass, the panes should never be large nor of expensive glass. The woodwork of the sash should receive its priming coat before glazing, the other coats should be put on afterwards. With crown glass, which is sometimes curved, it is usual to place the panes with the convexity outwards. When the glazier has fitted the pane to the opening with his diamond, the rebate of the sash-bar facing the outside of the window, he spreads a thin layer of putty on the face of the rebate and then presses the glass against it into its place, and holding it there, spreads a layer of putty all round the side of the rebate, covering the edge of the glass nearly as far as the face of the rebate extends on the inner side of the glass, and bevelling off the putty to the outer edge of the rebate. The putty is then sufficient to hold the pane in its place, and hardens in a few days.

The glass should not touch the sash-bar in any part, on account of the danger of its being cracked from any unusual pressure, there should be a layer of putty all round the edges. This precaution is especially necessary in glazing windows with iron or stone mullions or bars.

Putty.-Glaziers' putty is made of whiting and oil. The whiting should be in the form of a very dry fine powder; it should be specially dried for the purpose, and passed through a sieve of forty-five holes to the inch, and then mixed with as much raw linseed oil as will form it into a stiff paste ; this, after being well kneaded, should be left for twelve hours, and worked up in small pieces till quite smooth. It should be kept in a glazed pan and covered with a wet cloth. If putty becomes hard and dry, it can be restored by heating it and working it up again while hot. For special purposes white lead is sometimes mixed with the whiting, or the putty is made of white lead and litharge entirely.

Paperhanging.-Decorative paper for covering the walls of rooms is manufactured in pieces, which are, 12 yds. long and 20 in . wide.

The walls of rooms which are to be finished in a superior manner are generally plastered three coats, and upon the plaster when quite dry a coating of what is called lining paper should be laid to ensure a smooth surface. The decorative paper is laid on this. The paste used by paperhangers is made of flour and water and a little size or glue; alum also is added to paste to make it flow or spread more freely without losing any of its tenacity or sticking quality. Sometimes a common thin canvas is used instead of lining paper, and sometimes instead of plaster, in which latter case battens should be fixed against the walls to fasten the canvas to and prevent it from touching the walls. Canvas is an unsatisfactory substitute for plaster, in consequence of its expanding and contracting according to the hygrometric state of the atmosphere.

In renewing old paper, if the old paper merely requires cleaning, it can be done by first brushing it well, then rubbing it with stale bread crumbs, and then with a dry linen cloth. If the old paper cannot be cleaned it should be taken down and a coating of size laid on the walls, preparatory to a coat of paper : or a coat of size may be laid on the old paper, and a coating of whiting and size or distemper over that. See Bond.

COOLER. Fr., Bac, Bac-refroidissoir ; Ger., Kühlschif, Kühlstock.
Wort-coolers and Refrigerators.-After being drawn from the hop-back, the wort has to be cooled down to the temperature at which it is to be pitched or placed in the fermenting tun. This temperature varies somewhat in different cases; but it may be taken as averaging from $54^{\circ}$ to $64^{\circ}$, and therefore, allowing for some loss of heat in passing through the hop-back, traversing-pipes, and so on, the temperature of the wort has to be reduced about $150^{\circ}$. This reduction of temperature is effected sometimes by exposing the wort to the air in shallow vessels or coolers; sometimes by passing it through a refrigerator, or apparatus in which water is used as a cooling agent.

Cooler's are shallow vessels, generally about 6 in . or 8 in . deep, made of wood, iron, or copper. Wooden coolers are most frequently met with, probably on account of their cheapness, but they are open to many objections. They are usually made of Dantzic deals about $1 \frac{1}{4} \mathrm{in}$. thick, the boards being pegged to the joint-pieces with wooden pins. The coolers should be laid with a slight inclination towards the point at which the wort is drawn off, and the boards forming them should be planed as smooth as possible, so that they may be more readily kept clean. Too much care cannot be paid to the cleanliness of coolers, and to ensure it they should be frequently washed with lime water. If the coolers are not in almost continual use, it is advisable to keep thern corered with water in the intervals when they are not required, as the pores of the wood which
have been opened by the action of the hot wort are thus to a great extent prevented from absorbing air, which would, when the next gyle was poured on, come in contact with the wort, and be apt to cause a creaming of the surface, or incipient fermentation, generally called the fox. Wooden coolers, also, if allowed to get dry between the times of the wort being poured on, cause a considerable loss by absorption.

To avoid the objections to those of wood, coolers are made of iron or of copper. Coolers are placed so that their under sides are exposed to the air as well as their upper sides, and the cooling effect is thus increased. This arrangement should be adopted in all metal coolers

At Truman's there are two very fine copper coolers, which have been put up under the directim of King, the engineer to the brewery. These coolers are each 110 ft long by 25 ft wide, their weight the square foot being about $3 \frac{1}{2}$ lbs., and they are supported on joists, the under-sides being freely exposed to the cooling influence of the air. The wort is not allowed to rest on these coolers, but is run over them in a thin stream to one of Morton's refrigerators, which completes the cooling process. 'These coolers are capable, under ordinary cireumstances, of cooling about fifty barrels of wort an hour from boiling-point to a temperature of $110^{\circ}$; and as the combined surface of the coolers is $5500 \mathrm{sq} . \mathrm{ft}$., this corresponds to work $=\frac{(212-110) \times 50 \times 360}{5500}=\frac{102 \times 50 \times 360}{5500}$ $=333 \cdot 8$, or-allowing for the wort being rather below boiling-point when delivered on to the coolers-say about 300 pound-degrees a square foot of surface the hour. This is a very high result, and is partly due to the wort being kept in motion over the coolers, and being thus kept in a state of circulation; and partly to the fact of the coolers being made of thin copper, thus rendermg the bottom cooling-surface very effective.

Figs. 2092 to $209 \pm$ represent the standard form of refrigerator made by Morton and Wilson. 'The wort enters at $a$, passes over and under the tubes $b b$, in the direction of the arrows, and finally passes out at $c$.


The cold water is admitted at $d$, passes through the tubes in the direction of the arrows across the current of the wort, and escapes at the opposite end by the pipes shown at the top on the lefthand side of Fig. 2092. Caps, $f f$, connect the tubes together at their alternate ends, by which means a continuous passage is formed from end to end. These caps may be hinged to the ends of the tubes, so that they can be readily removed for cleaning the inside of the latter; but this is only necessary where the water is impure, and tends to leave a deposit inside. The tubes are
formed of four separate pieces for strength, these pieces being united, so as to form apparently a single flat tube. The tubular surfaces and casing are formed of copper surrounded by a wooden frome $g$; the caps are of gun-metal tinned. The spaces between the tubes are drained from the worts remaining after the brew by means of the continuous valve arrangement $h$, which is opened and shut by the lever $i$.

Figs. 2095 to 2097 exhibit further improvements. In these figures $a$ is the casing to which is attached the tubes $6 b b$, each of which is a flattened tube formed by the union of three round

tubes, the spaces between the circles being filled up. These tubes are connected at their alternate ends by the caps $c c c c$. The wort is distributed on the external surfaces of the tubes $b b b$, from the trough $e$, placed immediately above, and the bottom of which is perforated, or covered with metallic cloth. The worts percolate through the bottom of the trough, and, falling upon the upper tube, flow round it and descend to the second, and, in the same manner, fall from tube to tube until they are received by the trough $f$. The water enters at $g$, and passes in the direction of the arrows from tube to tube, finally escaping at $h$. The whole tubular surface is surrounded by an outer casing $o$, hinged at $i i$, and secured at $j j$. Vertical air tubes $k k$ are attached to this outer casing, these tubes having horizontal air tubes $l l l$, extending from them on each side. The latter air tubes are perforated, and currents of attemperated air are driven into $k k$ by a fan or other arrangement, and diffused throughout the casing which encloses the tubular coolingsurface.

The refrigerator, Figs. 2098 to 2100, constructed by Pontifex and Wood, is very complete. Like Morton, Pontifex and Wood employ flat tubes, traversed by the water; but instead of passing the wort alternately under and over them, they cause it to follow the course shown by the arrows marked on the plan of the apparatus. It will be seen from Fig. 2098 that each end of the casing

of the refrigerator is touched by the alternate tubes only, so that a zigzag channel is formed for the passage of the wort. The tubes are, as we have said, traversed by the water; and one of the
main peculiarities in the refrigerator is the manner in which the tubes are connected together so as to form a continuous channel. The required connection is effected at their alternate ends by castings, which form water bridges, beneath which the wort can pass The form of these bridges, or connecting castings, is shown in Figs. 2099, 2100. The stream of water flcws in the opposite direction to that of the wort, and the course of the current is shown in Fig. 2098 by arrows. The casing of the refrigerator is of wood, and its ends are made to slope slightly inwards towards the bottom, so that a tight joint can be readily made between them and the ends of the tubes. These ends are formed by brass castings, each of which carries a piece of indiarubber projecting from a groove formed in it, this india-rubber bearing against the end of the casing and making the joint tight. The tubes are all fastened on their upper sides to a frame of wood which is hinged to the side of the trough or casing, so that they can be readily raised for cleansing. Each tube is, like those employed by Morton, formed of three or


Section on line CD.
2100. four separate tubes or pieces joined together so as to present the appearance of a single flat tube. This plan is resorted to in order to prevent the tubes from bulging in the event of their being supplied with water under pressure.

Another form of refrigerator is that which we illustrate, Figs. 2101 to 2104. This refrigerator was designed by Joseph Stirk, the engineer of Messrs. Allsopp's Brewery at Burton-on-Trent, and Bycroft, of Burton. In this apparatus, the flat pipes employed are arranged somewhat as in Pontifex and Wood's refrigerator; but instead of their being connected by water bridges, so as to form a continuous series, each pipe is independent of the others. It will be seen by the transverse section, Fig. 2104, that each pipe is so divided by partitions that the current of water is made to traverse its length four times. The pipes do not extend across the entire width of the apparatus, but are fixed to the opposite sides of the casing alternately, so as to leave a passage which is traversed by the wort in the same

2102.
 manner as in Pontifex and Wood's refrigerator. The water is supplied to, and led from, the flat tubes by pipes which extend along each side of the refrigerator, these pipes being connected with the flat tubes by branches furnished with cocks, as shown in Figs. 2103, 2104. The object of this arrangement is to enable any one of the flat tubes to be removed for cleaning or repairs without interfering with the action of the refrigerator. It will be seen from the arrows on Fig. 2104 that the water is supplied by the lower pipe on each side to the lowest compartment of each flat tube, and after traversing the length of the latter four times, it escapes into the upper pipe. This refrigerator from its construction affords great facilities for repairs; but it is open to one objection-that is, a great portion of the water used for cooling can be but slightly heated, and can therefore do but
 little work. This will be understood when it is considered that each transverse tube acts independently, and the water passed through each tube cannot therefore be raised to a higher temperature than that of the wort with which that particular tube is brought into contact. Thus the water passing through the tubes near the end at which the wort escapes will probably not be heated to more than from $60^{\circ}$ to $70^{\circ}$, whilst near the end at which the wort enters it can be raised to within a few degrees of the initial temperature of the wort. The result of this is that a greater proportion of water will be required to cool a given quantity of wort than if there was a single
continuous current of water flowing in an opposite direction to that of the wort. To a certain extent this coil can be remedied-although it cannot be entirely overcome-by so regulating the openings of the cocks connecting the transverse tubes with the supply mains, that those tubes which are nearest the end at which the wort enters may receive the largest supply of water, the supply being gradually diminished towards the other end of the refrigerator. See Attemperator. Brewing Apparates.

COPPER. Fr., Cuivre ; Ger., Kupfer; Ital., Rame ; Span., Cobre.
Pure copper is of a light reddish-brown colour and of a high lustre. It is one of the mosu ductile and malleable metals. Sheets and wires may be formed of it with the greatest facility. Its fracture is similar to that of tin, or wrought iron. After hammering, its appearance is silky and its lustre seems increased. Its specific gravity when cast is $8 \cdot 85$, in wire $8 \cdot 93$ to $8 \cdot 94$, in sheets 8.95. Copper fuses at $1922^{\circ}$ Fah., and absorbs oxygen from the air when that is accessible, so as to reduce its specific gravity to $8 \cdot 7$ or $8 \cdot 8$. It may be welded when pure. Heated to fusion it absorbs oxygen and oxidizes the surface, and becomes covered with a black crust; by a strong heat in the muffle it may be converted into suboxide altogether. Heated to a white heat, it burns with a light-green flame. In dry air, copper is unchangeable; in moist air and in that containing carbonic acid, sulphuretted hydrogen, or other acids, it becomes dark green and assumes a bronze colour.

Copper ores form an extensive class of minerals, which it is difficult to distinguish by mere ocular inspection. However, at all copper veins oxides more or less green are found on the surface, which, in connection with other marks, form a sure indication of the presence of copper ore.

Native Copper.-This occurs in crystals disseminated through rocks, usually massive, in the form of scales; and compact masses ramifying the rock in all directions. It is found in beds, veins, and detached masses and grains, in solid rock and imbedded in loose soil. Most of the copper-ore veins contain metallic copper. Native copper is distributed over the whole surface of the globe, but nowhere is it found more generally and in larger masses than in the United States. It occurs in the greatest abundance at Lake Superior, near Kewenaw Point; at the Ontanawgaw River, and other localities of that region. Masses of native copper, of 80 tons weight, have been excavated in the Cliff Mine at Lake Superior. The copper occurs here in trap or sandstone rock, or near their junction, in the form of injected veins.

The usual copper ores are sulphurets and oxides; the former are more abundant than the latter. Copper is also found combined with arsenic, selenium, antimony, iron, silver, and acids.

Sulphuret of Copper. Whis occurs in various forms. Copper glance is one of the varieties frequently met with in copper-ore veins. Its specific gravity is $5 \cdot 5$, lustre metallic, colour and powder black or lead-grey, fracture conchoidal. It occurs frequently massive, but also granular and in fine powder. When pure it consists of $77 \cdot 7$ copper, $\cdot 91$ iron, 20 sulphur, and some silica.

Copper Pyrites, or yellow copper ore, is the most common sulphuret used in the smelt-works. It is rather light; its sp. gr. $4 \cdot 1$ to $4 \cdot 3$, colour brass-yellow; it is subject to tarnish in the air, and is then iridescent. It forms a greenish-black powder, of sharp edges. It always contains much iron, and is on that account highly esteemed in the smelt-works. Its composition in crystals is $34 \cdot 40$ copper, 30.47 iron, $35 \cdot 87$ sulphur, and sometimes a little quartz. It is often largely mixed with iron pyrites-in fact, so far that the latter fills the vein-and there are either only traces, or but a small percentage of copper ore in the mixture. Copper pyrites is the principal ore of the English smelt-works, as well as those of America, along the Atlantic coast. The bulk of copper is manufactured of this ore. Although copper pyrites is found in great profusion, the ore is always poor; it does not often yield more than 12 per cent., and frequently the body of a vein does not often contain more than 2 per cent. of copper. When it can be brought at reasonable prices to the smeltworks it is valuable, for it is much liked in the furnaces. It yields its copper with great facility, requiring but little labour and the use of little fuel. The contents of copper in an ore of this kind may be estimated by an experienced person on mere inspection. A bright yellow colour and softness indicate a rich ore ; a dull yellow, or pale yellow, and great hardness, are indicative of a poor ore. Copper pyrites is readily distinguished from iron pyrites by its inferior hardness-it may be cut by a steel point or a knife; this is not the case with iron pyrites, which will strike fire with steel, but not so that of copper. Spangles of this ore are distinguished from those of gold by their brittleness.

Grey Copper.-This is a variety of sulphuret of copper, which, on account of its interesting composition and its good behaviour in the furnace, is much liked by the smelter. It occurs massive, granular, in a fine powder, and also crystallized. It is of a steel-grey, often iron-black colour; its sp: gr. is $5 \cdot 1$, and it is rather soft and brittle. The composition of this ore varies greatly, but on an average it contains from 25 to 40 per cent. of copper, from 20 to 30 of sulphur, and nearly as much antimony. This forms the bulk of the ore; but it contains besides arsenic, zinc, silver, quicksilver, lead, platinum, and other metals.

Oxide of Copper.-Red oxide of copper is hardly used as an ore. It occurs as an accidental admixture with other ores-particularly with native copper. 'It is of a cochineal-red colour, occasionally crimson-red, or various shades of red. It occurs in the form of a powder, granular, massive. and crystallized. Other varieties of oxide of copper, such as the black cxide, are of no practical interest.

Silicate of Copper.-This occurs chiefly as an accidentaladmisture of other ores, and is a constant companion of them. It is green, varying from the emerald-green of the dioptase to the sky-blue of the chrysocolla; when impure, it is brownish or of an earthy colour. It is most frequently translucent, not often opaque. Its sp. gr. is 2 to $2 \cdot 2$. The ore contains frequently carbonic acid.

Carbonate of Copper.-Malachite, green carbonate of copper. This is similar to the silicate of copper. It is an ore which accompanies other copper ores. As an ore of copper it is of little consequence, however rich it may be, because not much of it is known to exist. Its composition is $71 \cdot 82$ protoxide of copper and 20 carbonic acid, $18 \cdot 18$ water.

Besides these ores of copper, there are sulphates, phosphates, arseniates, chlorides, and others, all of which are of little practical interest; they are companions of other copper ores, and occur only in small quantities.

Alloys of Copper.-Of all other alloys, those of copper are of most interest. Copper alloyed with arsenic is extremely white, similar to silver; but it is brittle and hard. With zinc it forms brass; and the amount of the respective metals determines the variety of this alloy. Pure copper does not form close and compact castings. Instead of pure copper, about 99 of copper and 1 zinc is considered pure cast-copper. Zinc is introduced by adding about 2 oz. of brass, poor in copper, to every pound of copper. This quantity may be varied from $\frac{1}{2} \mathrm{oz}$. of brass to 3 oz . for every pound of copper. Gilding metal consists of 1 oz . to $1 \frac{1}{4} \mathrm{oz}$. of zinc to 1 lb . of copper; it is of a bronze colour. Red sheet is 3 oz . of zinc to a pound of copper. Manheim gold, pinchbeck, 3 oz . to 4 oz . of zinc to a pound of copper. Ordinary brass of a red colour, for being soldered, contains 6 oz . of zinc to a pound of copper; 8 zinc, 16 copper, is a fine brass. Any proportion between 50 zinc, 50 copper, and 37 zinc, 63 copper, will laminate well and make good sheets. Common brass is 50 copper, 50 zinc. Solder may be made by melting brass, and casting it through a broom or faggot of brushes, into a tub of water. Or, the whole metal may be cast into iron moulds in the form of small cubes, of about 1 lb . or 2 lbs . each. When these are gently heated, nearly to melting, they may be broken up into small fragments by a smart blow of a hammer after placing the hot metal on an anvil or a thick cast-iron plate. It is stated that 50 copper to 52 or 58 zinc forms a dark-coloured metal, which on dipping forms a gold-coloured metal-mosaic gold. Zinc 32 to 16 copper forms a bluish-white, brittle metal, which may be pounded in a mortar. Zinc 8 and 1 copper forms a white metal little differing from zinc except in tenacity; this alloy is stronger than pure zinc.

Copper and zinc appear to mix in all proportions, and the extremes of both assume the characters of the principal metals. The red colour of copper is blended by the white of zinc to all shades from red to white. In forming brass by melting the two metals together, a heavy loss of zinc, which varies from $\frac{1}{10}$ to $\frac{1}{2}$, is always experienced. The usual plan of smelting brass is to melt the copper in a blacklead pot first, dry and heat the zinc near to the melting-point, and drop it gradually, in small pieces, into the copper, when the latter is not hotter than barely to continue fluid. The Editor of the present work found, by experiment, that the zinc should be added when cold but dry. When the surface of the hot metal is covered by fine charcoal, which is prevented by renewal from burning, the smallest loss of zinc is sustained. Tombac consists of 85 copper, 15 zinc; prince's metal, 75 copper, 25 zinc ; fine brass for turning, 66 copper, 32 zinc, and 2 lead.

Copper and tin form another most interesting series of alloys; 20 copper and 1 tin is a flexible, tenacious alloy, gocc for nails and bolts; 9 copper, 1 tin, was ancient bronze- 7 to 1 is hard bronze; the addition of a little zinc improves this article. Soft bronze, which bears drifting, rolling, and drawing, is generally composed of 16 copper to 1 tin; 12 copper to 1 tin is metal for mathematical instruments; 8 to 1 , bearings for machinery; 9 to 1 , a very strong metal; it may be considered the most tenacious of this series. Copper 5 to 1 tin, is very hard, crystallized, good for hard bearings in machinery. A soft metal for bells is formed of 3 tin, 16 copper; $7 \mathrm{tin}, 32$ copper, is for Chinese gongs and cymbals; 1 tin, 4 copper, is for house bells; 9 to 32 , large bells. Speculum metal ranges from 1 tin and 2 copper to equal parts of both metals. Ordinary bronze is 78 copper, 17 zinc, $2 \cdot 5$ tin, $2 \cdot 5$ lead. Large bells are cast of 80 copper, 6 zinc, 10 tin, 4 lead. A very fine large bell consisted of 71 copper, 26 tin, 2 zinc, 1 iron. A good average bell composition is 75 copper, 25 tin. 90.5 copper, 6.5 tin, 3 zinc, is an imitation of gold; 91.4 copper, 5.5 zinc, $1 \cdot 4$ lead, $1 \cdot 7$ tin, composes bronze for large statues. Copper 80 , tin 20 , is common statue bronze; 92 copper, 8 tin, is bronze for medals; 85 copper, 14 tin, 1 iron, is the composition of ancient weapons. Copper 62, iron 6, tin 32, is the composition of ancient mirrors.

The melting together of tin and copper is less difficult than that of zinc and copper, because tin is not so liable to evaporate as zinc, and little metal is lost. The appearance of the alloy may be improved by covering the melted metal with about one per cent. of dried potash; or, which is better still, a mixture of potash and soda. This flux has a remarkable influence on the colour, and particularly on the tenacity of the alloy. The former becomes more red, and the latter stronger. The scum forming on the surface by this addition ought to be removed before the metal is cast. Tin and copper are liable to separation in cooling; this can be prevented, at least partly, by turning the mould containing the fluid metal, and keeping it in motion until it is chilled.

The ancients manufactured their tools of copper, and hardened them as we harden iron. This art appears to have been understood over the whole world, for the Asiatic nations, Africans, and Europeans, as well as the American Indians, knew how to render copper hard. The copper of these ancient people was always impure, very likely in consequence of the composition of their ores. Their bronze-metal contains always more or less tin, lead, zinc, arsenic, silver, and gold. The hardening extended frequently through the body of the metal, but generally it was confined to the surface.

A remarkable difference is perceptible between the alloys of copper and those of iron in respect to hardening. Iron alloys, and most others, become hard on being heated and suddenly cooled, while copper alloys become softer by such an operation. Compression has a similar effect on these alloys, as on all other metals-it renders them hard.

Copper and lead unite only to a certain extent; 3 lead and 8 copper is ordinary pot-metal. All the lead may be retained in this alloy, provided the object to be cast is not too thick. When the cast is.heavy, or much lead is used, it is pressed out by the copper in cooling. One lead, two copper, separates lead in cooling-it oozes out from the pores of the metal; 8 copper and 1 lead is ductile, more lead renders copper brittle. Between 8 to 1 and 2 to 1 is thc limit of copper and lead alloys. All of these alloys are brittle when hot or merely warm.

Alloys of copper are subject to the same laws as others; and as they are generally more tenacious, more use is made of them. Phosphorus renders copper very hard, brittle, fusible. and oxidizable.

Clean copper, held in the vapours of phosphorus, is successfully hardened. A very little of this substance melted together with copper, causes it to be very hard, similar to steel. Carbon combines with copper and causes it to be brittle. Silicon also combines with it, hardens it, and, if present in a small quantity only, does not impair its malleability. Arsenic has ouly a faint affinity for copper; still the last traces of it cannot be driven off by mere heat; the combination is brittle. Equal parts of copper and sılver, and 2 per cent. of arsenic, form an alloy similar to silver, a little harder, however, but of almost equal tenacity and malleability. Antimony imparts a peculiarly beautiful red colour to copper, varying from rose-red in a little copper and much antimony, to crimson or violet when equal parts of both metals are melted together.

Uses.-The application of copper, either in its pure condition or as an alloy, is so universal that but little can be said on this subject. It is used for sheathing and bolts for ships, for boilers in factories, distilleries, dyeing establishments, steam-boilers, \&c. Rollers, shaft-bearings, engravers' plates, and kitchen utensils, are manufactured of pure copper or its alloys. For cylinders, waterpumps, coins, wire, and a multitude of purposes it is also used. Its oxides form fine colours, but are deadly poisons.

Manufacture of Copper.-Smelting of copper is an extremely simple process, because it is as permanent as iron, and little affected by heat and oxygen. The metal which occurs mixed with gangue, consisting chiefly of silicious rock, is cut into small lumps that may enter the furnace; these are in some instances of a ton weight and more. Or, if the metal is disseminated through the mass of the rock, either in grains or in small veins, it is pounded and washed in a stamping mill, and the contents so far concentrated that the sand contains from 70 to 75 per cent. of copper. This is called stamp-work, and sent in barrels from the mines to the smelt-works. Copper from this kind of native metal is smelted chiefly in reverberatory furnaces. Small blast furnaces are often employed to smelt copper. For smelting it thus, from stamp-work or lumps, any reverberatory furnace may be used, either of those in which copper is refined or smelted, or a roasting furnace may be easily converted into a smelting furnace. The operation is simple, and will be described hereafter.

Smelting in Reverberatory Furnaces.-There are two distinct methods of smelting copper ores; the one is in reverberatories, and the other in blast furnaces. As the operations are similarly conducted in the various countries where they are practised, and as the smelting of copper ores in reverberatories is done with skill and much experience at Swansea, we will first describe the operation as it is there performed.

In all instances the copper ores are sorted at the mine, the lumps broken, and large pieces of rocky matter thrown away. The ore is then classified in various qualities, of which the impure ore is sent to the stamps to be crushed and washed. Clay ores are broken into small pieces and washed by hand. All the rich ore, or that ready for smelting, is broken with the beater to lumps of the size of nuts, and freed from light impurities by riddling.

The small and impure ore is washed with a sieve in water, which carries away the stony parts and leaves the metalliferous ore in the tub. Those parts of the ore which are very impure, but will pay for crushing and washing them, are sent to the stamping mill.

The stamping mill is similar to the one given, p. 273 . The ore is here converted into powder, more or less fine, and separated from gangue in the labyrinth or slime troughs; or, the ore is washed on the sweep-table, shown in Fig. 2105. In fact, the purifying of copper ore does not essentially

differ from that of other ores. But as the specific gravity of copper ore is small, much care should be taken not to crush it very fine in the stamps.

The furnaces used in this operation are five in number; they are all of similar construction, and so far all the various operations may be performed in the same furnace at different times. Still it is found to be profitable to divide the operation, and perform it in different machines. In Fig. 2106, A, is shown a plane, and in B, Fig. 2107, a vertical section of a reverberatory calcining furnace. This furnace is not essentially different from other reverberatory furnaces. The vault C is an addition; into this the ore is discharged when calcined. The furnace is constructed partly of fire and partly of common bricks, and strongly bound. The hearth is from 18 ft . to 19 ft . long, and 14 ft . to 16 ft . in width. The fire-grate is 5 ft . by 3 ft . The fire-bridge is hollow, and through it fresh air is conducted to the ore under treatment. Two hoppers $h, h$, serve for letting in the ore. The chimney is low.

The first process is the calcining. Three and a half tons of clean ore are charged into the
furnace at a time, which is, with occasional stirring at intervals of two hours, ready to be withdrawn after a heat of twelve or fifteen hours, and let into the cab-vault-beneath. Here it remains as long as possible in a close heap, at least so long as the vault is not needed for the next charge. When the ore is withdrawn it is spread evenly on a floor and damped. In this operation it loses much of its sulphur, and after being cold and wetted is ready for the next operation.


The second process is the smelting of the ore. The furnace for this purpose is much smaller, only 11 ft . long, and 7 or 8 ft . in width. The grate is as large as the one in the calcining furnace, because a higher heat is here required. The furnace has only one work-door at the flue, and in one side a similar aperture for cleaning the hearth. The hearth is formed of coarse sand, and slopes slightly towards the door in the side. Below this door there is an iron grating which covers a vault of water, into which the metal is discharged and granulated. A hopper is placed in the top of the furnace for letting in the charge.

A charge in one of these furnaces consists of 21 to 24 cwt. of roasted ore, which takes four hours for smelting, adding slags from refining, and also fluxes, if such are necessary. Two cwt. of slags are generally charged with the ore, besides lime, fluor-spar, or other fluxes, according to the quality of the ore. The time of smelting these charges is four hours, after which the slag at the top of the metal is skimmed off by means of a rabble, and drawn out at the work-door into a bed of sand. The metal is not drawn at every heat, but only once or twice each twenty-four hours. A second charge of ore is therefore thrown into the furnace, after the poor slags are removed; the furnace is then shut once more, and that charge melted. When the metal, which is matt, an alloy of all the metals in the ore, and sulphur, rises as high as the bridge at the work-door, the tap-hole below is opened, and the matt either run into the basin of water below the furnace for granulation, or into a bed of damp sand. The metallic grains which are thus formed oxidize rapidly, particularly on their surfaces. The colour of this crude metal is a steel-grey, its fracture compact, and it is of much lustre. The scoria rejected after this process contains always some metal; copper and tin are found to be present in 1 or 2 per cent. in this silicious slag. The matt produced contains about 33 per cent. of copper, or four times as much as the ore; the other 66 per cent. is chiefly sulphur and iron. If with the use of the refining slags the ore does not flux, the addition of fluorspar is resorted to. Great care must be taken not to use too much of these fluxes, for all scoria, no matter of what description, will contain copper; and the more slag there is made, the greater must be the loss in metal. The size of the smelting furnace is so regulated, that it consumes all the ore which is calcined in the first furnace.

The third operation is that of smelting the crude metal, or matt, of the second process, with the slags of the fifth process. This slag is chiefly a peroxide of iron, and the operation may be called on this account a roasting one. This calcination is performed in the large furnace, represented in Figs. 2106, 2107. The charges consist of 2 tons of matt, with nearly an equal amount of slags. The operation lasts twenty-four, and sometimes thirty or thirty-six hours, under repeated puddling of the ore. In this process much care must be taken to regulate the heat; it should be performed on the principles of roasting, by commencing with a low heat, which is gradually increased to the melting-point. The ore is tapped into the vault under the furnace, and oxidized by exposure.

The fourth process. This is again a smelting operation performed in the smelting furnace, of which Fig. 2108 shows a plan. The charges are 28 or 30 cwt., and a heat lasts from five to six hours, or when slow, eight hours. At every charge the metal is tapped, which now is a rich matt of 66 per cent. of copper. It is frequently very pure, and then it is called fine metal, and run into moulds, forming pigs; sometimes all of it is pimpled copper. In this operation there should be still so much sulphur in the metal as to cause sufficient fluidity; if there is a lack of it, some green ore is charged with the matt. When the metal from this operation is far from the reguline state, it is run into water and granulated.

The slags from this last smelting, together with some other slags, are sometimes melted in a furnace by themselves, which forms a particular operation. The matt obtained from these slags is a white and brittle alloy. The slags are also partly thrown away, but most of them are used in
the first process. The matt obtained is smelted separately, and then added to the first smelting, or the second operation.

Fifth process. The fine metal in the form of pigs of the foregoing operation, is charged to the amount of $2 \frac{1}{2}$ or 3 tons at once in the calcining furuace, and exposed for twenty-four hours to a gentle heat. It should not melt, at least not for sixteen hours, and when melted afterwards it is to be repeatedly skimmed. The metal from this calcining operation is drawn into a bed of sand, and formed into pigs, which are fine metal for the refining furnace.

The sixth process is that of refining or toughening the metal. This operation is done in the smelting furnace; a charge of metal is from 3 to 5 tons. The pigs are exposed in the furnace to a roasting heat for twelve or sixteen hours, then the charge is melted, skimmed, and worked as clean as possible. A test of the metal is, after twenty hours' heat, taken by means of an iron ladle. A small wrought-iron foundry ladle is washed and heated
 in the fluid copper until it becomes red hot, or as hot as the metal itself. A ladle full of metal is now taken from the furnace and exposed to a suow cooling in the air. If the copper is fine enough, it will settle considerably in the ladle. The surface of the metal in the furnace is now covered with fine charcoal and prepared for refining. If the copper in the ladle swells up, or shows veins or black spots, it is not fine enough. In order to accelerate the process, a pole of wood is now used for stirring the metal diligently for ten minutes, after which another ladleful is taken for trial; it is now found to be fine, it will settle in the ladle. Good fine metal is brittle, of a deep colour, coarse grain, porous, and crystalline. The surface of the melted copper is now covered with fine charcoal, and the metal repeatedly stirred by means of wooden poles. The grain of the copper becomes finer by this operation, and the metal tougher. A test of the metal is now repeatedly taken in a small iron ladle, and when considered sufficiently refined, it is tried by means of a hammer on the anvil, while still red hot. If the metal forges soft, does not crack on the edges, and the refiner considers colour and grain sufficient, it is ladled out of the furnace with large ladles and cast-iron moulds. These form either pigs or slabs, 12 in . wide, 18 in . long, and 2 or $2 \frac{1}{2} \mathrm{in}$. thick. These slabs are ready for the rolling mill.

In the progress of these different operations, the use of the slags forms a remarkable point for consideration. From the last smeltings the slags go back to the first process, to be either calcined or smelted. The refining slags are smelted with the metal in the formation of matt; and those from the smelting of matt are used in the calcining operation. The arrangement is such that the slag from the last operation is returned to a previous one. In each smelting some of the slags are thrown away, as too poor for the further work of extraction.

The fine metal of the sixth operation should be blistered or pimpled metal, containing from 94 to 96 per cent. of copper. Pimpled metal always assumes blisters, like those on converted steel, when cast into a sand-bed. The heat on the fine or blistered metal is longer or shorter according to its purity; an impure metal requires more heat than a pure metal. In some instances but a few hours' roasting are sufficient, in others a longer time is required. When the copper is melted in the refining furnace there is no harm done in stirring and cooling it, alternately, so as to chill the metal, and then melting it again. The rabbling, or puddling, must be continued until the copper is fine; in this operation the foreign metals become oxidized and vitrified. The slags of all the various operations contain more or less copper, particularly those of refractory ores. Neglect in skimming causes the slags to absorb and retain much metal. The slags of the coarse metal, or matt, take up the oxides of iron and tin, and often contain 5 per cent. of copper; they are therefore re-smelted. If the ore contains much tin, antimony, lead, and other metals, the slags of the fourth operation are smelted in a slag furnace, and the metal obtained used as pot-metal, either for brass and copper nails, or, if much tin and lead are present, pewter is formed of it.

When the point of refining is passed, in the operation of refining copper, the metal deteriorates in value, it becomes carbonized; this is prevented by exposing the hot surface to the action of the flame, and in skimming charcoal and slags off. Good metal is bright on the surface in the furnace. It is of a fine red colour when cold.

In the Blast Furnace.-The other method of smelting copper ores is in the blast furnace. The ore for this operation is sorted, washed, stamped, and in fact prepared as lead or silver ores. Poor ores, such as copper stists, are roasted in heaps, for fifteen weeks or longer. In smelting, matts are formed, as in reverberatories, which are re-smelted, and finally refined. In Figs. 2109 to 2111, two vertical sections, A and B, are shown of a blast furnace; and in C, Fig. 2111, the plane section with its two basins D D. The height of the furnace is about 14 or 15 ft .; the widest part of the boshes 39 in .; the hearth is 2 ft . square. The basins D D are 3 ft . in diameter and about 21 in . deep.

The copper ores, after having been roasted, are smelted by charcoal or coke-anthracite is perhaps preferable to either. The tuyere is generally pushed far into the furnace, so as to concentrate the heat in its centre. About 4 tons of ore are smelted in twenty-four hours with a considerably strong blast. In this operation a matt and a slag are smelted; the first contains from 30 to 40 percent. of copper, and the latter frequently 5 or 6 per cent. more or less, according to the kind of ore. The matt contains sulphurets of copper, iron, silver, zinc, arsenic, cobalt, and in fact all those metals which were originally in the ore. It is tapped alternately into the basins and the slags removed from its surface. In cooling, it forms on its surface round plates which may be lifted from the
fluid metal. These contain matt of a variety of compositions, according to the height of the metaI in the basin.


The matt thus obtained is generally roasted, either in kilns, or more generally, at present, in reverberatories, of which Fig. 2112 represents a vertical section of a German one. Fig. 2113, B, shows the same furnace in an opposite section to that of A, Fig. 2112. Above the two furnaces there is

2113.

a condensing chamber C , into which the volatile metals are conducted. These two furnaces, one above the other, are so arranged that either of them may be used separately. The flame is then conducted from the lower furnace in a separate flue into the condensing chamber, the partitions in which are so arranged that the gases are conducted from one into the other until they escape into a chimney.

The matt is roasted in these furnaces from three to six times; this is, therefore, an extremely slow operation ; subsequently it is exposed to smelting again in the blast furnace. Crude copper is now obtained of a granulated fracture, which is ready for refining. After the above-mentioned roasting is performed, the ore is lixiviated in water, in order to extract the soluble sulphate of copper, which is precipitated by means of metallic iron. The coarse or black copper forms the lowest stratum in the smelting furnace, and also the basins; above this floats a poor matt covered by a silicious slag, which is thrown off and rejected. The matt and the metal underneath are gradually lifted out as it cools, and are in the form of rosettes.

The fine copper thus obtained from the blast furnace is most generally refined in reverberatory furnaces. In all instances that copper which has been smelted in blast furnaces is subjected to refining in the reverberatory, if it is brought into market directly from the blast furnace: this kind of copper is quite impure, which renders it unfit for being rolled into sheets. The impurities are most successfully removed in the reverberatory, as they consist chiefly of carbon and oxidizable metals.

A copper-refining furnace, as it is used by the Germans, is shown in Fig. 2114 in plane. The hearth A, 7 ft . in diameter, is formed of sand, or clay and fine charcoal. B B are two receiving basins, for ladling out the copper, or forming rosettes of it. Three tons of black copper are melted at once, and as soon as the metal is fluid the bellows are set in operation, which, by means of the tuyeres C C, furnish blast on the surface of the metal, and oxidize it rapidly. A thick slag is thus formed, which is constantly drawn off, so as to expose a clean surface to the action of the blast. The refining lasts about sixteen or seventeen hours, and the loss of metal amounts to 3 per cent., which is absorbed by the slags. The latter is returned to the blast furnace.

The expenses for smelting copper ores are high, on account of the many and tedious operations which must be performed. Poor sulphureous ore, or that which contains but 8 or 10 per cent. of copper, is the most profitable in the reverberatory; rich ores should be smelted in the blast furnace. Ores of 9 per cent. consume 20 tons of mineral coal for the production of 1 ton of metal; poorer or richer ores than these cause the use of still more fuel. The labour spent in working the ore amounts to still more than the fuel consumed.

Copper is brought into market in different forms. For melting brass it is sold in a granulated form-bean-copper. This is produced by pouring it through an iron strainer, made of a ladle, into cold water. Hot metal causes round beans, cold metal oblong beans. Russian copper is sold in small square slabs: Spanish copper in the form of pigs.

The rollers used for laminating copper or
 brass are plain cylinders, as shown in Fig. 2115, not often more than 36 or 40 in . long, and 16 in . in diameter. Rollers 5 ft . long and 20 in . in diameter are used for large sheets. Slabs for rolling are gently heated on the hearth of a reverberatory furnace, Fig. 2116, to a dull red heat. At first singly, and as the sheets become thinner, they are passed in pairs, or three sheets and more at once, through the rollers. In the process of lamination the metal becomes cold, and by compression hard; it is therefore reheated, which serves in the meantime, when performed slowly, for annealing. When large sheets are to be rolled, the annealing furnace must be of a sufficient size to contain them, They are greased before passing them between the rollers.

Some kinds of copper contain large quantities of silver, for which the Lake Superior copper is particularly distinguished. We shall allude to the extraction of this metal under the head of silver.

Theory of Smelting Copper. - The copper of commerce is not pure; it is an alloy, as well as other metals. A quality of Norway copper, much esteemed by
 brass manufacturers, contains $99 \cdot 5$ copper and $\cdot 5$ lead. Hard Hungarian copper contains 99 copper, $\cdot 7$ antimony, $\cdot 1$ iron. A superior quality of Swedish copper was composed of $8 \cdot 66$ copper, $\cdot 75$ lead, $\cdot 05$ iron, $\cdot 23$ silver, $\cdot 05$ silicon, :02 aluminum, $\cdot 03$ magnesium, $\cdot 12$ potassium, and $\cdot 09$ calcium. These assays show how much impurity copper may contain, and still be considered as a good article. The purest kind of copper should be employed for sheets. A minute quantity of lead causes copper to roll badly, and iron causes it to be brittle. Other mixtures are less injurious than these metals. It has been observed that the purest copper contains protoxide of the metal, a fact which is established in most other metals. The best kinds of copper are those which have been smelted by charcoal, and contain minute quantities of potassium. Bell-founders and other workers in bronze and brass are in the habit of covering the metal with potash or soda. This causes it to be close, sonorous, and of a fine grain. The substances most injurious to copper are lead, iron, antimony, silicon, carbon, sulphur, phosphorus, arsenic, and some other. Small quantities of lead, iron, nickel, silver, aluminum, magnesium, calcium, sodium, and potassium, improve the tenacity and general qualities of the metal. In refining copper, it must be therefore of advantage to have the surface of the metal covered with charcoal which has been soaked or damped with a solution of carbonate of potash or soda. These alkalies cause the removal of lead, tin, zinc, and iron, and prevent the flying or boiling of the metal.

The fine copper of the smelter, pimpled copper, black copper, or blistered copper, is an impure copper which contains much iron. This kind of metal is so far purified copper as to show its colour and faint metallic properties. Black copper, smelted of pyrites, contained 95.7 copper, 2.9 iron, $\cdot 6$ zinc, and $\cdot 8$ sulphur. Some crude copper, smelted of carbonates and oxides in the blast furnace, was composed of $89 \cdot 3$ copper, $6 \cdot 5$ iron, $2 \cdot 4$ peroxide of iron, $\cdot 3$ sulphur, and $1 \cdot 3$ silica. We may mention that silica, combined with the protoxide of iron, exists in the form of slag in the copper. A coarse metal, which was derived from a refining cinder, contained copper $27 \cdot 6$, iron $2 \cdot 5$, cobalt
$19 \cdot 7$, nickel $35 \cdot 2$, lead $12 \cdot 4$. A metal which furnished a prime quality of copper, in refining it, consisted of $95 \cdot 5$ copper, $3 \cdot 5$ iron, $\cdot 4$ bismuth, $\cdot 6$ silver.

The composition of the crude metal depends on the composition of the ore. In metal derived from sulphurets much sulphur is found; and in that from oxides other metals form the impurities, which must be removed before the metal is saleable. Iron forms, in most instances, the bulk of the impurities, and it must be the object of the refiner to remove it entirely. The presence of silica is required to oxidize and remove iron; but, as the oxide of copper has also a strong affinity for silica, the heat should be low, and the iron slag, as soon as formed, should be removed by skimming the metal. In crude copper derived from pyrites, the iron may be supposed to be present as sulphuret; and as, in oxidizing this, the metal is oxidized to the highest degree, it is necessary that carbon should be present to reduce the peroxide of iron thus formed, and convert it into protoxide, suitable for a union with silica. Such crude copper should therefore be refined under cover of charcoal, agitated by means of wooden poles. Copper smelted of oxides contains the iron in a metallic state, in the form of grains; for the affinity between these two metals is so faint that they do not unite chemically. The proper mode of refining this kind of crude copper is to melt it at a pretty strong heat, and stir or puddle it by means of an iron rod or hook, such as shown in Figs. 2117, 2118. Other substances than those above mentioned are easily removed from copper. Lead, zinc, bismuth and arsenic are volatile, or their oxides combine readily with potash or soda, by the addition of which they will separate from the metal. A small quantity of precious metal does no harm to copper, and large quantities, such as one per cent.
2117.
 of silver or gold, may be profitably extracted from it. Cobalt is removed with the iron, and nickel does no harm, for the alloy may be used as argentan in case much of this metal is present. When iron chiefly is to be removed, a clean surface of the melted metal is required in order to facilitate its oxidation. All other metals ought to oxidize slowly, and the oxides should be supplied with some alkali to combine with.

The impurities of copper are brought into the metal either by the ore, flux, or fuel. Iron is generally used as flux; if there is not sufficient of it present in the ore, it is added in smelting. But, as this method of using iron causes the formation of balls or lumps of refractory metal, or slag, in the furnace, the poor copper ore which contains iron as a natural admixture is preferred, since it is not liable to balling. The iron is in sulphuretted slags, in the form of sulphuret of iron. In slags derived from oxidized ores, it is in the form of protoxide. In the first kind of slag, sulphur causes its fusibility; in the second slag, silica. The former is a sulphuret, the latter a silicate. Both these compounds may be present in a slag; this, however, is not often the case. Generally, the silica separates from the sulphuret, and, as the first is not so heavy as the latter, it floats on its surface. In smelting, we thus obtain a slag which is a silicate, as the highest stratum, and a slag which is a sulphuret below that; the latter is called matt. When metals are present which have only a faint affinity for sulphur, such as lead, gold, or silver, these gather below the matt and slag, as we have seen in smelting lead. So long as sulphur is present in the slags, we cannot succeed in removing all the iron from the copper, nor all the copper from a sulphureous slag. Silicate of copper is refractory. All the metal may be extracted from a silicate, provided the union of copper and silex is prevented. The metal should be separated before silex is admitted to act on its oxide. Thus we have a series of operations in the reverberatory, all calculated to remove iron by means of silex, and retain and concentrate the copper in the form of a sulphuret or matt. The addition of silica to rich ores is, therefore, a necessity; but as it is difficult to estimate the proper quantity to be used, such rich ores are not always so profitable to work as the poorer kinds. 'Too much silex causes a stiff cinder which absorbs copper; and too little silex does not absorb all the iron, and forms a stiff slag which cannot be separated from the copper, and causes it to form balls and oxidize. In smelting copper, as well as other metals, the slags are never too fusible; stiff pasty slags always retain grains of metal. It makes no difference by what means copper slags are rendered fusible, provided they melt at a lower degree of heat than the metal itself. Copper cannot be reduced from its sulphuret-it should be oxidized; therefore the smelting of copper is divided into a succession of processes, consisting of alternate calcinations and smeltings.

Slag from a smelting of copper pyrites in a reverberatory contained $48 \cdot 2$ silica, $\cdot 5$ protoxide of copper, 37 protoxide of iron, 3 oxide of tin, 4 lime, 1 magnesia, 1.8 alumina. This slag is thrown away, because it contains but little copper. Slag from roasted pyrites, smelted in a low-blast furnace, contained, -silica, $51 \cdot 8$; protoxide of copper, $1 \cdot 4$; protoxide of iron, $29 \cdot 2$; baryta, $8 \cdot 8$; alumina, 5. The same kind of ore, smelted with more iron, furnished,-silica, 35 ; protoxide of iron, 41 ; oxide of zinc, 3 ; baryta, 12 ; lime, 3 ; magnesia, 2 ; alumina, 4 . This composition furnishes a more fluid slag than the former, and is consequently free from copper. When the addition of iron is necessary, it should be made in the form of forge cinder, or puddling-furnace cinder, from the iron-works; because that form of iron fluxes well, without furnishing metal. The following is an assay of a slag which contained too much iron ; silica, $33 \cdot 6$; protoxide of copper, 3 ; protoxide of iron, $51 \cdot 5$; lime, 5 ; alumina, $5 \cdot 6$. This slag, besides containing much copper, caused the deposition of considerable iron in the smelted copper, which formed balls of refractory metal consisting of $89 \cdot 4$ iron, 2 copper, 7 cobalt, and 1.8 sulphur. We thus see that the quantity and form in which fluxes are used is of much importance in this operation. Copper may be smelted from crude ores with success, as it is performed in Sweden; but the operation requires skilful hands to manage it. The fluxes are arranged so as to form a silicate, consisting of silica $56 \cdot 5$; protuxide of iron, $14 \cdot 9$; lime, $6 \cdot 3$; magnesia, $14 \cdot 3$; alumina, 6 . This is a first-rate slag, and works well in
the now-blast furnace. The flux commonly used is limestone and forge cinder. More lime and less iron causes the copper to be very impure, and the slags contain copper; it also causes vexatious work in the furnace. Slags from copper-smelting resemble the forge cinder of the iron-works; they are, however, generally not so glassy, and often contain oxide of iron not combined with silica.

The matter obtained in the various processes is a compound of metals and sulphur, differing with the kind of ore from which it is obtained and the mode of operation by which it is formed. Roasted pyrites, smelted in a low-blast furnace, such as is used for smelting lead, furnishes a matt consisting of 27 copper, 40 iron, 25 sulphur, and 8 earthy matter. Rich matt, smelted in a blast furnace 16 ft . high, from roasted ore, contained $58 \cdot 6$ copper, $13 \cdot 2$ iron, $23 \cdot 2$ sulphur, and $\cdot 6$ earthy matter.

The slags obtained in refining furnaces are a combination of the oxides and sulphurets which are contained in the crude metal. Refining is at present exclusively performed in reverberatory furnaces, either with the assistance of blast or without it ; in either case the metals are oxidized by the oxygen of the air, and the sand of the hearth furnishes the silica for vitrification. These slags always contain a large quantity of copper, and are therefore re-smelted, either by themselves, or returned to earlier operations. The predominance of other metals than iron in the slags is indicative of a corresponding quantity of copper. Oxide of antimony is particularly apt to form and absorb oxide of copper. Lead has a similar effect, but in a far less degree. The slags obtained from the refining operations are easily reduced in a small blast furnace, and furnish an alloy. Smelted in a crucible with black flux, an assay of them is obtained in which all the other metals are present except iron. Their appearance varies greatly; when they contain much iron and sulphur, they are grey or black. Slags which contain no sulphur are brown, semi-transparent, and often blood-red, magnetic, of all shades of colour between black-brown and light red.

On whatever principle the extraction of copper from its ores is conducted, the composition of the ore and flux is so arranged that the yield does not amount to more than 7 or 8 per cent. The first smelting yields then a matt of 30 per cent. of copper ; the second smelting, one of 60 per cent.; and the third smelting, crude metal or pimpled copper of 75 to 85 per cent. In blast furnaces, ores of $2 \frac{1}{2}$ per cent., or less, in yield, may be smolted to advantage. Rich ores are smelted in a low furnace, the height of which varies from 5 ft . to 18 ft . The first smelting of a $2 \frac{1}{2}$ per cent. ore yields, in Sweden, in the first smelting, a matt of 60 per cent. of copper ; and the second smelting, after roasting the matt, yields crude metal of 85 or 90 per cent. copper. In Germany, an argentiferous copper is smelted of bituminous slate in high-blast furnaces, which yields only from 1 to 3 per cent. of copper ; the copper has not quite $\frac{3}{4}$ per cent. of silver. In all these various forms of smelting copper ore, a rapid oxidation by a high heat cannot be permitted, in order that the formation of silicates of copper may be prevented. Calcining is performed at a low heat, because if the ore was subjected to fusion in the operation, much copper would unite so closely with the silica as to become inseparable in the smelting furnace. Sulphur and silica are necessary fluxes in the reverberatory. In the blast furnace, copper ore may be smelted by fluxing it with lime or silicate of iron; and where the latter can be obtained in sufficient quantity, there is no doubt but that the smelting is cheaper when performed in the blast furnace than in the reverberatory. Refining should be invariably done in the reverberatory.

That ingenious and experienced manufacturing chemist, Peter Spence, of Manchester, has introduced some useful and important improvements in the metallurgical process of copper-smelting. Spence observes, in the Mining and Smelting Magazine (1864);-
"It is well known to those conversant with our staple chemical manufactures that, for many years back, a large proportion of the sulphuric acid required in them has not been produced from the sulphur imported from. Sicily, but has been made from iron pyrites (bisulphide of iron) which is largely found in Ireland and Cornwall, and is also largely imported from Belgium and other parts of the Continent; and that, more recently, immense deposits of pyrites, rich in sulphur and containing from 2 to 4 per cent. of copper, have been found in the Peninsula, on the borders of Spain and Portugal. These last-named pyrites have come into extensive use, and, from their richness in sulphur, are preferred to the pyrites derived from other sources; so that the importers get a higher price for them than the value of the copper they contain, and consequently seldom sell them to the copper-smelters. The chemical manufacturer, in fact, has to purchase these ores at the value of both the copper and the sulphur, and, after extracting the sulphur, has generally to sell the ore to the copper-smelter, thus incurring a heavy cost in carriage on an article of small value per ton, and sometimes also a large loss from the uncertainty of the ordinary mode of assaying.
"From these circumstances, I found that in using Spanish pyrites the sulphur was costing me rather a heavy sum; and in 1861 I began to look round for some mode of lessening this cost. By extracting the copper from the burnt ores, if it could have been done without loss, I found I could save 600l. per annum in the mere carriage of the burnt ore to the smelters.
"I first, in connection with Rumney, of Manchester, went into the wet method of extracting copper. But having operated on nearly 2000 tons by various modifications of this method, our success seemed so problematical that the experiment was abandoned, with a loss of about 2000 l . Indeed I am fully convinced by my own experience, added to that of almost all who have tried the extraction of copper by purely chemical wet processes, that such processes are not applicable in practice.
"I next turned my attention to the fact that the large copper-smelters in South Wales and elsewhere, were then, as they are at this moment, throwing out into the atmosphere, as to them an utterly useless product, the sulphurous acid gas which the chemist is so anxious to get cheaply, and that in quantities that would more than meet all the demands of the staple chemical manufactures. I asked myself whether I could not avail myself of this sulphur, which cost nothing and had no value to the smelter. The first difficulty was a mechanical one. All the pyrites then used in chemical works (whether Irish, Belgian, or Spanish) were imported in large masses, which were broken up into fragments of from 1 to 5 cub. in. in size. From these the sulphurous acid was extracted by
combustion in kilns somewhat similar in construction to small lime-kilns; but before charging the pyrites into these kilns all the dust or small had to be sifted out, as otherwise it would choke or damp the draught of the kilns and prevent combustion.
"When, however, attention was directed to the ores used by the copper-smelters, it was found that all the ores sold in Cornwall, and nearly all those imported, were crushed up before being sent to market, and that, in fact, the smelter used only small ores. Now, no method was then known by which these small ores could practically be calcined so as to economize the sulphur; for though repeated experiments had been made, at great expense, by the large copper-smelting firms to calcine their ores in such a way as to condense the sulphur, all such attempts had ended in failure-so much so that all endea vours on their part to effect this object have been abandoned.
"To prove that such is the case, I take the liberty of giving an extract from the evidence given by Dr. Percy before Lord Derby's Committee on noxious vapours.
" 'Lord Derby.-The statement of Le Play, that 46,000 tons of sulphur are annually lost, only goes to this, that if there were any known means of preventing the evolution of that sulphur, there would be a saving to that extent?
"' Dr. Percy.-Undoubtedly, if it could be collected economically, it would be an advantage to the copper-smelters.
"' Lord Derby.-You are not prepared to say that it could be.
"'Dr. Percy.-I am not prepared to say that it can, by any known method. I am acquainted with all the attempts which have been made to obviate this nuisance-for a nuisance it undoubtedly is-but I believe they have not been found effective.
"'Lord Derby.-What, in your opinion, is the reason why it is impossible to condense, or in any way to dissipate, this vapour?
"'Dr. Percy.-I do not say it is impossible at all; all I say is that I know of no method, at present in existence, whereby it can be completely and economically condensed.'
"One of the attempts probably referred to by Dr. Percy was a furnace used by a Lancashire copper-smelter, chiefly for producing arsenic from the copper ores, but partially used for the production of sulphuric acid from these ores. While my attention was directed to the subject of copper ores as a source of sulphur, a friend of mine in Manchester got the plans, and erected one of these furnaces, which were called dummy furnaces. At his request I saw it while building, and without hesitation said it would be a failure, which it turned out to be; for, after being used for a ferw months, it was pulled to pieces, its fallacious principle becoming apparent. This being then considered the best thing hitherto tried for the purpose, I at once determined to attempt the erection of a furnace on what I considered sound chemical principles. The furnace I erected was successful in calcining the small ores with a small expenditure of fuel and labour, with elimination of all the sulphur, if that was required, and enabling me to send all the sulphur so eliminated into the vitriol chamber, as sulphurous acid gas. Very soon after, I erected additional furnaces; and all the sulphuric acid made at iny works at Manchester and at Goole since the end of 1861 has been made from these small ores, treated in furnaces similar, with slight modifications, to the first one I erected. I have now (186t) eight calcining furnaces at work, and am using from 125 to 180 tons of ore a week; and, in fact, am doing exactly what Dr. Percy stated he knew no method in operation that was capable of doing-for all the ores used are precisely the same ores as those used by the copper-smelters. I buy them in Cornwall, at the same market as the copper-smelter-paying only for the copper, the sulphur not being of any value to the smelter, who would rather have his ores nearly without it, if he could so get them.
"The amount of sulphur wasted in copper-smelting, and which could be economized by the use of calcining furnaces similar to those used by myself, is something enormous. Le Play, who many years ago investigated the subject with great care, gave the ores theu used at 4000 tons a weck, and the sulphur in then as averaging 23 per cent. The quantity now used exceeds 5000 tons a week, which (by the result of many analyses made in my own laboratory) contain on an average 28 per cent. of sulphur. This gives 70,000 tons of sulphur per annum, which at the present price of brimstone ( 6 l .10 s . a ton) gives a money value of $455,000 \mathrm{l}$. This, however, is hardly a fair way of putting it, as sulphuric acid is more cheaply made from pyrites than from brimstone. But from the 250,000 tons of copper ore now used annually by the smelters, 200,000 tons of brown sulphuric acid (of specific gravity $1 \cdot 75$ ) could be produced-for I am actually making acid at that rate from these ores. This acid sells at from $3 /$. 10 s. to $4 l$. a ton ; but at the very low estimate of $2 l$, a ton, its cost price to the maker, the smelters are actually throwing away 400,000 l. a year. Now, whatever may have been the position of the copper-smelter a few years ago, he is at the present time wasting all this sulphur when there is not the slightest difficulty in economizing ncarly the who e of it as sulphuric acid, without at all interfering with the smelting processes."


In Spence's first arrangement, the calciner, Figs. 2119, 2120, had a flat bed for the ore 40 ft . long by from 6 ft . to 9 ft . in breadth. Under this bed the fire-flues traverse nearly the whole
length of the furnace, to which furnace, however, the fire itself has no access. The ore is charged on to one end of this furnace-bed every two hours, in charges of from 5 cwt . to 8 cwt .; and, after two hours, is transferred, by iron paddles or slices, to some distance from the point where it was first charged; being replaced by another charge. This transference goes on every two hours, until the ore reaches the other end of the furnace, where, being fully calcined, it is dropped into an iron truck and removed. This charging and transference goes on periodically, air being continuously drawn in at the part of the furnace where the burnt ore falls out; which air, traversing the whole surface of the ore on the bed and effecting combustion, ultimately arrives at the other end highly charged with sulphurous acid gas, and passes up a special flue or chimney into the vitriol chambers. Nitrous gas is, as usual, found mixed with it as it enters the chamber. This process goes on day and night; and the furnaces are so regularly kept at the same temperature that their wear is exceedingly small-one having been at work upwards of two years, with only three days' stoppage for repairs.

Fig. 2121 represents a modification of the furnace, Fig. 2119. Both of these arrangements were found to work well. The one shown in Fig. 2121 consists in combining the calcining furnace with the smelting furnace, so that the ore falls out of the calciner into the smelter at a red heat-thus saving the labour and nuisance of removal, and the cost of reheating it when put cold into the smelter. In addition to this, the waste heat
from the smelter, instead of passing direct to the stack, is made to pass under the bed of the calcining furnace, thus saving all the fuel now used in calcining.

Fig. 2122 represents a longitudinal section, and Fig. 2123 a cross-section, of Spence's final arrangement, now (1870) in active operation.


The bed of the furnace is at $a$, in which are mounted two rails $b$; upon these rails are wheels $c$ mounted loosely upon axles which are attached to a frame $d$; to this frame is bolted a bar e carrying
on its under-side a rack $f$ and resting upon rollers, one of which is shown at $g$; but there are several placed at intervals, carried by any suitable bearings, the length of the bar $e$ being such as not to pass off the innermost of the said rollers when the frame $d$ is at the back end of the furnace. The frame $d$ carries two rows of instruments $h h^{1}$, which project downward nearly to the bed of the furnace, and the two rows are so placed that an instrument in one row is opposite to a space in another ; at Figs. 2124, 2125, they are shown detached, the latter showing their horizontal section, and it will be seen that on one side they are pointed and on the other flat. At $\tau$ is the fireplace, and at $k$ the chamber, through which the products of combustion pass, and by means of which the furnace becomes heated. At $l$ is one of a series of doors extending at intervals along each side of the furnace. At the front of the furnace is an inclined shute $n$, through which the material passes when calcined.
 The ore having been first ground, or otherwise brought into a state of fine division, is fed across the furnace through one of the doors at the back thereof, and motion is communicated by means of the shaft and pinion $m$ to the rack $f$, whereby the frame $d$ will be caused to advance towards the back of the furnace ; the instruments $h$ moving in the direction of the arrow, Fig. 2125, the pointed parts will therefore advance through the material, and their curved bottoms, seen in Fig. 2124, acting as ploughshares will stir and turn the material over. When the frame has reached the back of the furnace the motion of the shaft $m$ is reversed, and the frame $d$ with its instruments travels towards the front of the furnace, and the flat parts of the said instruments will now therefore act against the material, a portion of which will be carried forward. The frame $d$ and its stirrers is now in a highly-heated state ; and in order to allow it to cool, the motion of the shaft $m$ is arrested, so that the apparatus may remain stationary for a period, and as it is now over the shute $n$ it receives the current of air passing through the same. When the desired cooling has been effected, say in four or five minutes, the shaft $m$ is again put in motion and the operations above described are repeated. The charge is repeatel at the back end of the furnace about every hour, and it has been found that a charge may be about $1 \frac{1}{2} \mathrm{cwt}$. for a furnace 32 ft . long and 6 ft . wide, the speed of the rakes being about two minutes from the front of the furnace to the back and back again; and as the charging continually goes on for each hour, a portion of the material is carried forward each time by the flat part of the rakes, and is ultimately delivered through the shute $n$; but as such portions are continually falling off the rakes they are only carried forward through short spaces at each raking, and they therefore ouly gradually make their way to the delivery end.

The mechanical means of causing the forward and backward motion of the rakes may be varied to a considerable extent. The means employed will be readily understood by referring to Figs. 2126,2127 , the former being a side view, and the latter a plan of a pair of engines and other parts

which may be used separately for each furnace, or for tro or more. At 1, 2, are two steam-cylinders communicating motion by their piston-rods to the crank-shaft 3 , upon which there is a pinion 4 tiking into a wheel $4^{*}$ which is on the shaft $m$, the same as that denoted by the same letter of
reference in Fig. 2122. Upon the crank-shaft 3 is a bevel 5 taking into another 6 upon a shaft mounted in a bearing 7, and this shaft is provided with a worm 8 in gear with a worm-wheel 9

mounted on a shaft 10 ; on this shaft is a tappet 11, which at intervals, as will be described, arrives in contact with a pin 12 projecting from a lever 13 turning loosely upon the shaft 10 , and the said lever 13 also carries another projecting pin 14, situate beneath the ordinary eccentric rod 15 , which at its end is connected to the valve-rod by the usual gab-motion 16. The steam-pipe for the cylinders is at 17, provided with a stop-tap 18, upon which is mounted a lever 19, 19*, the end 19* thereof being pressed in the direction of the arrow by a spring 20 , and the said end $19^{*}$ is jointed so as to be capable of turning against the force of the said spring in one direction. Adjoining the steam-cylinders 1,2 , is a supplementary small steam-cylinder 21, the piston of which is constantly going and communicates motion to a crank 22 , on the shaft of which is a worm 23 taking into a wheel 24 , which then drives the train of clockwork $25,26,27,28$, on the last of which there is a projecting pin 29 arriving at certain times in contact with the lever 19. According to the positions of the parts shown, the rakes are proceeding from the front to the back of the furnace, and have nearly reached that position, they having been driven forward by the wheel on the shaft $m$. The tappet 11 will therefore soon arrive in contact with the projecting pin 12, and when that takes place the said lever will be turned over until the weight thereon has passed the vertical line, after which it will fall over and bring the pin 14 to bear against the eccentric rod 15, which, by turning upon the rocking-lever 30, will shift the position of the gab-motion 16, and thus reverse the engines, there being a lever 13, and pins 12, 14, for each of the two driving steam-cylinders; the driving-wheels will now revolve in the contrary direction, and the rakes will travel back towards the front of the furnace, carrying with them, as before described, a portion of the material. When this has been effected, the lever 31 on the shaft 10 (and which is now moving in a direction contrary to the arrow) will arrive in contact with the lever 19*, thereby causing the stop-tap 18 to be turned and the steam to be shut off; the motive power being thus arrested, the rakes will remain at the front and colder end of the furnace a sufficient time to cool, and that period is determinec. in the following manner;-It has been stated that the small cylinder 21 is constantly supplied with steam, and the train of gearing 23 to 28 is therefore constantly revolving, and the numberis of teeth are so arranged that when the rakes have been allowed to remain stationary a certain time, 3 nin 29 upon the wheel 28 arrives in contact with the lever 19, thereby turning the ston-tap go as to open it and admit stean to the cylinders 1,2 ; when this is done, the tappet 11 will at the same time again arrive against the pins 12, but on the side thereof opposite to that above mentioned; the levers 13 will therefore be turned back into the position shown, thereby removing the pins 11 from the eccentric rods 15, and allowing them to fall, and again reverse the eugine. so that the rakes will now be moved to the back of the furnace, and thus the operation continues. When the lever 37 turns back after having shut off the steam, the spring joint of the lever 19* allows it to pass.

Copper and Copper Ores; our ascertained positive knowledge respecting.-Atomic weight $=63$; probable molecular weight $=63$.

Pure copper is found in a state of nature, but the principal ore of this metal is a double sulphuret of copper and iron. This ore is subjected to the process of roasting, which transforms the sulphuret of iron into oxide of iron and sulphurous anhydrite. The oxide of iron passes off in the scorix or slag in the form of fusible silicate. A repetition of the same process completes the

## COPPER.

expulsion of the iron. A third roasting gives the copper in its raw state; the sulphuret of copper is, indeed, transformed into sulphurous anhydrite and oxide of copper, and this latter oxide reacts on the sulphuret not yet decomposed, thus producing copper and sulphurous anhydrite.

$$
\underset{\begin{array}{c}
\text { Subsulphuret of } \\
\text { copper. }
\end{array}}{\mathcal{f u}^{2} f}+\underset{\begin{array}{c}
\text { Protoxide of } \\
\text { copper. }
\end{array}}{2 \in \mathrm{Eu}}+\underset{\text { Copper. }}{4 \cdot \mathrm{e}^{2}}+\underset{\begin{array}{c}
\text { Sulphurous } \\
\text { anhydrite. }
\end{array}}{\mathrm{f}^{2}}
$$

By roasting this raw copper in a silicious furnace, a certain quantity of oxide is formed by which the elimination of the sulphur is completed, and at the same time the oxides of the foreign metals, uniting with the silex of the furnace and forming silicates, pass away in the scoriæ.

To free the copper from all oxide, it is melted, carbon is placed on its surface, and the mass stirred with green wood. The carbonated gases evolved from the wood by the action of heat are sufficient to reduce the oxide of copper in the metallic mass. These two latter are called refining processes.

Copper may be chemically obtained by reducing the oxide of this metal with hydrogen. For this purpose, pure oxide of copper is placed in a globular chamber blown in the middle of a glass tube, Fig. 2128; one end of this tube is made to communicate with an apparatus in which hydrogen is produced, through the medium of a desiccating tube, and the other end communicates freely with the
 atmosphere.

When the hydrogen has passed through during a time sufficiently long to ensure the expulsion of all the air, a precaution necessary to avoid an explosion, the oxide of copper is heated by means of a spirit-lamp; water is thus formed and the copper is liberated.

$$
\begin{aligned}
& \left.\left.\qquad \mathrm{\epsilon u}^{\prime \prime} \Theta+\underset{\mathrm{H}}{\mathrm{H}}\right\}^{\prime}=\underset{\mathrm{H}}{\mathrm{H}}\right\}_{\text {Water. }} \boldsymbol{\theta}+\underset{\mathrm{Cu}}{ } \quad \text { Copper. } \\
& \text { Oxide of copper. Hydrogen. }
\end{aligned}
$$

The process is complete when no more steam can be evolved.
Copper is of a red colour; it is sufficiently malleable to allow of being beaten out into transparent sheets; it is also very ductile and very tenacious.

It may be obtained artificially crystallized in cubes, and it is under this form that it is found in nature. The density of this metal is 8.85 .

Copper, on being rubbed, emits a disagreeable odour. It melts at about $778^{\circ} \mathrm{C}$. It does not become oxidized in dry air at an ordinary temperature; in a high temperature it becomes oxidized without incandescence. When exposed to damp air, it becomes covered with a layer of hydrated carbonate of copper (verdigris), but this layer preserves the metal from further change.

Azotic acid affects copper when cold, and sulphuric acid when heated. In the former case are produced bioxide of azote and azotate of copper; in the latter case, sulphate of copper and sulphurous anhydrite.


In the presence of acids, copper readily absorbs the oxygen of the atmosphere; it also becomes oxidized in the presence of ammonia, and is dissolved, producing a beautiful blue liquid.

Copper, previously heated, burns in chlorine, producing bichloride of copper. It combines directly with sulphur, phosphorus, arsenic, bromine, and most of the metals.

Copper forms two series of compounds. Being diatomic, it combines directly with two monoatomic radicals, or with a diatomic radical like itself, and so becomes saturated. Thence we have a series of compounds which have received the name of maximum. These are ;-

| Bichloride of copper |  | $\mathrm{Eu}^{\prime \prime} \mathrm{Cl}^{2}$ | Protoxide of copper | - |
| :---: | :---: | :---: | :---: | :---: |
| Bibromide of copper |  | $\mathrm{Cu}^{\prime \prime} \mathrm{Br}^{2}$ | Hydrite of copper | $\epsilon u^{\prime \prime}(\theta \mathrm{H})^{2}$ |
| Bifluoride of copper |  | $\mathrm{Cu}^{\prime \prime} \mathrm{Fl}^{2}$ | Protosulphuret of coppe | $\epsilon^{6 \prime \prime}{ }^{\prime \prime}$ |

and the various oxygenated salts resulting from the substitution of radicals of acids for the hydrogen of the hydrite of copper.

By reason of the diatomic nature of copper, two atoms of this metal may unite, exchanging only a single atom between each other, and form the diatomic group $\mathrm{Cu}^{\prime \prime}$, as shown in the following

$$
\mathrm{Gu}
$$

figure ;-


The group $\epsilon_{u}{ }^{2}$ being diatomic, may combine quite as well as the atom $\epsilon_{u}$ with chlorine, bromine, iodine, and so on; and as the combinations formed by it offer sufficient stability, we have a second series of compounds of copper, known by the name of minimum, and in which, instead of the single atom $€ \mathrm{u}^{\prime \prime}$, appears the group $\mathfrak{C u}^{2}$. These are; -

and the very unstable protosalts resulting from the substitution of the diatomic group $\mathcal{C u} \mathrm{u}^{2}$ for an even number of atoms of the hydrogen typical of the acids.

Besides these two series of compounds, copper forms with oxygen a bioxide $\operatorname{\epsilon u} \theta^{2}$, and an acid the composition of which has not yet been exactly determined.

The diatomic nature of oxygen explains how sevcral atoms of this body can unite with a single atom of copper : two atoms of oxygen may exchange each one atom with the copper and one between each other, as we see by the aid of the diagram.

We will consider more particularly bichloride, protosulphuret, protoxide, hydrate, sulphate, azotite, and the carbonates.

Bichloride of Copper, $\mathrm{Cu}^{\prime \prime}\left\{\begin{array}{l}\mathrm{Cl} \\ \mathrm{Cl}\end{array}\right.$. -This compound is formed by the direct action of chlorine upon copper; it is also produced by dissolving the protoxide of this metal in chlorhydric acid.

$$
\left.\begin{array}{c}
\left.\mathrm{Cu}^{\prime \prime}+\underset{\mathrm{Cl}}{\mathrm{Cl}}\right\}
\end{array}\right\}=\mathrm{Cu}^{\prime \prime}\left\{\begin{array}{l}
\mathrm{Cl} \\
\mathrm{Cl} \\
\text { Copper. Chiorine. Bichloride of } \\
\text { copper. }
\end{array}\right.
$$

Bichloride of copper is soluble in water and in alcohol ; its watery solution, conecntrated when hat, deposits while cooling hydrated crystals, the formula of which is $\mathrm{Gu}^{\prime \prime} \mathrm{Cl}^{2}+2$ aq. These crystals are in shape like long needles of a greenish-blue colour. The alcoholic solution of this salt burns with a bright green flame.

Protosulphuret of Copper, €u" S. -This substance does not exist alone in nature ; it is obtained by sending a current of sulphurated hydrogen through the watery solution of a salt of copper, bichloride, for instance;-

$$
\underset{\substack{\text { Bichloride of } \\
\text { copper. }}}{\left.\mathrm{Cu}^{\prime \prime}\left\{\begin{array}{l}
\mathrm{Cl} \\
\mathrm{Cl} \\
\begin{array}{c}
\text { Sulphydric } \\
\text { acid. }
\end{array}
\end{array} \underset{\mathrm{H}}{\mathrm{H}}\right\}=2\left(\begin{array}{l}
\mathrm{H} \\
\mathrm{Cl}
\end{array}\right\}\right)+\underset{\substack{\text { Chlorhydric } \\
\text { acid. }}}{\mathrm{Cu} u^{\prime \prime} \mathrm{S}} \underset{\substack{\text { Suphuret of } \\
\text { copper. }}}{ }}
$$

It is precipitated under the form of a black mass, which converts itself into sulphate when expesed to the air, by attracting the oxygen.

$$
\begin{aligned}
& \left.\left.\qquad \mathrm{Cu}^{\prime \prime} \mathrm{S}+2\left(\begin{array}{l}
\Theta \\
\Theta
\end{array}\right\}\right)=\begin{array}{c}
\mathrm{S} \Theta^{2^{\prime \prime}} \\
\mathrm{Cu}
\end{array}\right\} \Theta^{2} \\
& \begin{array}{c}
\text { Protosulphuret } \\
\text { of copper. }
\end{array} \\
& \begin{array}{c}
\text { Oxygen. }
\end{array} \text { (ulphate of copper, } \\
& \text { maximum. }
\end{aligned}
$$

Protosulphuret of copper loses half of its sulphur by being heated, and is transformed into subsulphuret.

$$
\begin{aligned}
& \left.4 \subset u^{\prime \prime} \mathscr{S}=2 C u^{2} \frac{S}{S}+\begin{array}{c}
S \\
S
\end{array}\right\} \\
& \begin{array}{c}
\text { Protosulphuret } \\
\text { of copper. }
\end{array} \text { of copper. }
\end{aligned}
$$

Protoxide of Copper, Cu O. -This oxide may be obtained; 1, by heating copper exposed to the air, when a layer of oxide easily detached is formed on the surface of the metal ; 2, by calcining azotite of copper; 3, by heating hydrate of copper; to dishydrate this latter substance, it is sufticient to boil it with water.

Whatever process of preparation may be employed, with slight differences in the physical properties of the oxide, which may be more or less compact, this compound possesses the following properties;-

It is a black and amorphous powder, capable of supporting a very high temperature without being decomposed or melted. When, however, it is heated in too high a degree, the mass becomes a solid block, extremely hard, and, when pounded, of a yellowish colour. This oxide appears to be in a particular allotropic state. M. Kieben has noticed that it then possesses the faculty of agglomerating at a lower temperature than when it has not been overheated. It loses this property when it has been heated several times at a temperature insufficient to cause it to agglomerate, and then allowed to cool.

Protoxide of copper is a basic anhydrite. It is much used in laboratories, where it is employed in making organic analyses.

Hydrate of Copper, $\left.\begin{array}{|c}\stackrel{\mathrm{H}}{ } \mathrm{H}^{\prime \prime}\end{array}\right\} \ominus^{2}$. This hydrate is obtained by precipitating the solution of bichloride, sulphate, or any other maximum salt of copper by an alkaline base. The precipitate so formed must be well washed and dried in an ordinary temperature ; it is of a dirty blue colour.

If the liquid in which it is precipitated is made to boil, it loses water, and is so transformod into anhydrous oxide; a fortiori, it will be dishydrated when heated by the fire directly.

Hydrate of copper is dissolved in ammonia, when it becomes of a beautiful blue colour.
Sulphate of Copper, maximum, $\left.\mathrm{S}_{\mathrm{E} \boldsymbol{\theta}^{2 \prime \prime}}^{\mathrm{E}}\right\} \theta^{2}$. -In laboratories this substance is prepared by applying concentrated sulphuric acid to the copper when hot.


The residue of the preparation of sulphurous anhydrite is utilized for this purpose. In the arts, sulphuret of copper is heated while exposed to the air. This substance absorbs the oxygen of the air, and is transformed into sulphate, which may be st purated from the ore that has not been subjected to washing and evaporation.

The sulphate of copper of commerce nearly always contains sulphate or iron; the surest means of obtaining this salt free from iron is to dissolve it in water, and to precipitate by sulphurated hydrogen its solution previously acidulated. The copper is precipitated alone; the precipitatewell washed and exposed to the simultaneous contact of air and water-is transformed into sulphate, which is crystallized after having filtered the solution. Sulphate of copper is known in commerce under the name of blue vitriol. It is insoluble in alcohol, and soluble in water; it crystallizes in this latter liquid in blue oblique parallelopipeds. These crystals are hydrated, and are represented by the formula $\left.S \theta^{2 \prime \prime} \mathrm{Cu}^{\prime \prime}\right\} \theta^{2}+5 \mathrm{aq}$.

When hydrated sulphate of copper is heated to $110^{\circ} \mathrm{C}$., it loses 4 aq ; at $248^{\circ} \mathrm{C}$. it loses the water it contains, and becomes anhydrous. It is then a white powder like flour. As the smallest quantity of water gives it its blue tint, this substance furnishes a test of the presence of water.

The crystals of sulphate of copper are isomorphous with those of sulphate of magnesium, zinc, or cadmium, when these latter contain, as it does, five moleculæ of water. This salt forms double sulphates with alkaline sulphates. It combines with the sulphates of magnesium, zinc, iron at a minimum, \&c., producing crystals which contain five moleculæ of water, when the copper is predominant, and seven when the other metal predominates. These crystals are always isomorphous with each other when they contain the same quantity of water.

When heated to a high degree, sulphate of copper decomposes itself into oxygen, sulphurous anhydrite, and protoxide of copper.

If the solution of this salt is precipitated by a quantity of an insufficient base, an insoluble basic sulphate is produced of a green colour.

When sufficient ammonia is added to a solution of sulphate of copper to dissolve the precipitate which is first formed, and alcohol is poured into the blue liquid so produced, a precipitate of a beautiful blue colour is obtained, known as ammoniacal sulphate of copper, the composition of which is $\xlongequal[\mathcal{C u}^{2 \prime \prime}]{ }\}=\theta^{2}, 6 \mathrm{AzH} H^{5}+\mathrm{H}^{2} \theta$.

Azotite of Copper, $\mathrm{Cu}^{\prime \prime}\left\{\begin{array}{l}\theta \mathrm{Az} \Theta^{2} \\ \Theta \mathrm{Az} \theta^{2}\end{array}\right.$. -Azotite of copper is prepared by dissolving the metal in azotic acid, evaporating the liquid, and leaving to cool. The salt is deposited while cooling in large blue hydrated crystals; these crystals, when heated, first melt in their crystallized water, then this water is evaporated, and the anhydrous azotite is decomposed; at first there is formed a green basic azotite, afterwards the decomposition becomes more and more complete, and finally there remains a residue of oxide of copper.

Carbonates of Copper.-The carbonate obtained by pouring carbonate of sodium into a solution of sulphate of copper is a carbonate of copper containing two parts of this metal, and bibasic. Its formula is

$$
\left.\left.\begin{array}{l}
\epsilon_{u^{\prime \prime}}\left\{\begin{array}{l}
\theta \mathrm{H} \\
\theta \\
\epsilon u^{\prime \prime}\{
\end{array}\right\} \in \theta^{\prime \prime} \\
\theta \mathrm{H}
\end{array}\right\}=\begin{array}{c}
\left(\epsilon u^{\prime \prime}\right)^{2} \\
\Theta \theta^{\prime \prime} \\
H^{2}
\end{array}\right\} \theta^{4}
$$

This substance has the same composition as the natural carbonate known by the name of malachite. Malachite is of a beautiful green colour; in places where it abounds, it is used as copper ore, and, indeed, it is an excellent ore.

There is also found in a natural state a hydrated carbonate of copper containing three parts of this metal; it is of a beautiful blue colour, and is known under the name of mountain blue, or azurite.

The verdigris which is formed upon the surface of copper is a hydrated carbonate of copper ; this verdigris must not be confounded with the verdigris of commerce, which is a subacetate of copper.

Protochloride of Copper, $\mathrm{Cu}^{2} \mathrm{Cl}^{2}$.-The most simple means of preparing this substance is to dissolve metallic copper in aqua regalis containing very little azotic acid, and to add some water to the solution; the protochloride of copper is precipitated under the form of a white crystalline powder.

Another way of preparing this compound is to dissolve suboxide of copper in boiling hydrochloric acid, and then to allow the liquid to cool, in the midst of which will be deposited the little colourless tetrahedrons of protochloride of copper.

It may also be obtained by heating perchloride of copper, which thus loses half its chlorine.
$\underset{\substack{\text { Chloride of copper } \\ \text { maximum. }}}{2 \mathrm{Cu} \mathrm{Cl}^{2}}=\underset{\substack{\mathrm{Cl} \\ \text { Chtorine. }}}{\mathrm{Cl}}+\underset{\substack{\text { Chluride of cepper } \\ \text { minimum }}}{\mathrm{Cu}^{2} \mathrm{Cl}^{2}}$

Chloride of copper minimum is a white substance hardly soluble in water, but soluble in hydrochloric acid and ammonia; it turns green when exposed to the air by absorbing oxygen and transforming itself into oxychloride, $\mathrm{Cu}^{2} \mathrm{Cl}^{2}$. It also absorbs oxide of carbon, but it gives up this gas when its solution is made to boil. In an ammoniacal solution it gives, with the gaseous carburets of hydrogen of the series $\mathrm{G}^{\mathrm{n}} \mathrm{H}^{2 n-2}$, explosible precipitates which, heated with liydrochloric acid, give out the hydrocarburet the elements of which it contains. This property has been utilized in organic chemistry.

Subsulphuret of Copper, $\mathrm{Cu}^{2}$ S.-Subsulphuret of copper is found in a natural state under the form of beautiful crystals, belonging to the cubic system. They are of a black colour and sufficiently soft to be cut with a knife, and they melt in the flame of a candle. Their density is $5 \cdot 0$.

This substance is prepared artificially by calcining copper with an excess of sulphur ; the excess of sulphur is evaporated during the process of calcination. To ensure the whole of the copper being acted upon, the product of this first operation is crushed and calcined a second time with sulphur.

When heated in the air, it gives a sulphate of copper, if the temperature is not too high ; if not, it is transformed into oxide of copper and sulphurous anhydrite, by absorbing oxygen. Heated with oxide of copper, this sulphuret gives out sulphurous anhydrite, leaving a residue of metallic copper.

$$
\underset{\substack{\text { Subsulphurct } \\ \text { of copper. }}}{\mathrm{Eu}^{2}}+\underset{\substack{\text { Protoxide } \\ \text { of copper. }}}{2 \mathrm{Eu} \theta}=\underset{\text { Copper. }}{4 \mathrm{Cu}}+\underset{\substack{\text { Sulphurous } \\ \text { anhydrite. }}}{\mathrm{S} \Theta^{2}}
$$

Suboxide of Copper, $\mathrm{Cu}^{2} \theta$.-This substance is found in a natural state. It is sometimes found in compact masses, and sometimes in octahedric crystals of a red colour; it may be obtained artificially in the form of a red powder, and in various ways.

If acetate of copper be boiled with glucine, a red crystalline powder is prccipitated, which is su' oside of copper.

In the arts, this substance is commonly prepared by calcining a mixture of ;-


The produce of this operation must be well washed.
Suboxide of copper melts without undergoing change when heated unexposed to the arr; when heated in the air it is transformed into protoxide.

Hydrochloric acid converts it into protochloride; this oxide is, therefore, a basic anhydrite.
Azotic acid gives up its oxygen to it, and causes it to pass into the state of a maximum azotate ; strong acids decompose it into metallic copper and bioxide of copper, which, when in contact with these acids, gives a salt of copper ;-


Ammonia dissolves this oxide without being discoloured; but the solution becomes blue by absorbing oxygen when exposed to the air.

Distinctive Properties of Salts of Copper.-Salts of copper are recognized in analyses by the following properties ;-

1. A sheet of iron becomes covered with a perfectly-adhering layer of copper, when dipped into the saline solutions of this metal.
2. Hydrosulphuric acid causes in these solutions a precipitate insoluble in alkaline sulphurets; this precipitate is not produced in the presence of cyanide of potassium.

The maximum salts maxy be distinguished from the minimum;-

1. Potassium gives with the minimum salts a yellow precipitate insoluble in an excess of reactive, the maximum salts are precipitated by the same reactive, a dirty blue colour, and the precipitate becomes black on being boiled, if sufficient quantity of potassium has been added to wholly decompose the salt of copper.
2. Ammonia produces in the maximum and the minimum salts a precipitate soluble in an excess of reactive; but with the maximum salts the ammoniacal solution is of a beautiful blue colour, whilst with the minimum salts this solution is colourless and becomes blue only on contact with the air.

All the salts of copper are poisonous. The best means of counteracting this poison if accidentally swallowed is to take some white of egg, while waiting for an emetic. The albumine of the egg forms with the copper a compound almost insoluble, and by this means the absorption of the metal is prevented.

It has been proposed to substitute iron dust for albumine, which dust precipitates the copper in a metallic state, or sulphuret of iron, which will produce sulphuret of copper.

Figs. 2129 to 2131 are of the furnace, invented by Alfred Jenkin. -f Zell, for the reduction and calcination of copper and lead ores.


This invention relates to a peculiar construction and arrangement of a double reverberatory furnace, to be employed in the reduction and calcination of copper and lead ores, whereby a great economy of fuel is effected. The principal feature in this improved furnace is, that one ordinary fire answers the double purpose of reducing and calcining the ore. The fire is contained in an ordinary fire-place, situated at one end of the double furnace.

The heat and flame from this fire pass through a lateral opening or flue into the reducing or flowing furnace, and after passing over the surface of the ore contained therein, it enters by another opening or openings into the calcining furnace, which is placed on the same level, or nearly so, with the flowing furnace. From the calcining furnace the heat passes off by a suitable flue or flues to the chimney. In the passage or passages which conduct from the flowing furnace to the calcining furnace there are placed suitable doors or dampers, which are so arranged that, by opening or closing certain of these doors or dampers, the heat and flame may either be directed into the
calcining furnace, or be entirely shut off from such furnace and be directed into a waste flue, which passes downwards to an underground flue communicating with the chimney. An air or a ventilating space is left between the calcining and flowing furnaces, for the purpose of preventing the bed of the latter furnace from becoming overheated, when the flame and heat are diverted down the waste or escape flue or flues, which flue or flues are formed in that side of the ventilating air-space farthest from the flowing furnace; so that, whether the calcining furnace be in operation or not, the bed of the flowing furnace will always remain at about the same temperature.

Fig. 2129 represents an exterior elevation of Jenkin's improved arrangement of a reducing and calcining furnace; Fig. 2130 is a longitudinal transverse section; and Fig. 2131 an horizontal section. A is the ordinary fire-place, the heat and flame from which pass through the lateral opening or flue $\mathbf{B}$ into the flowing furnace C, and after traversing over the surface of the ore contained therein enter by the passages D into the calcining furnace E , which is upon the same level with the floor of the flowing furnace. $F$ are passages leading off to the main draught flue or chinney. When it is required to shut off the heat and flame from the calcining furnace, the two dampers $G$ are closed, and the pair of dampers $H$ are elevated, thereby shutting off the flow from the passages $D$, and diverting it into the descending or waste flue $I$, which conducts it underground to the main flue. An air or a ventilating space $J$ is left between the flowing and calcining furnace, in order to prevent the latter from becoming overheated. The ore to be calcined is fed into the calcining furnace by the hopper $K$, and is removed when calcined through the aperture $L$ in the bottom of the furnace. The flowing or reducing furnace is also provided with a similar hopper M, to supply the calcined ore to the flowing or reducing furnace. The small chimney N serves to carry off any dust or ashes arising from the fire when fresh fuel is supplied, or when the fire is otherwise agitated.

William Longmaid in 1842 proposed a method of treating ores. This method related to the treating of such descriptions of ores and minerals as contain sulphur; and had for its object the removal of the sulphur from such ores and minerals, in order to render the subsequent operations to which such ores are subjected more advantageous in obtaining products therefrom. Writing in 1842, Longmaid observes,-"I have discovered, after much experiment, that the use of common salt can only be generally and practically useful as a manufacture when the quantity of salt applied in respect to the ore or mineral treated considerably exceeds the quantity of sulphur contained in the ore or mineral. And I have found that when treating ores according to my plan with common salt, I obtain metallic oxides in a condition fit for metallurgical purposes; and by such means I am enabled more advantageously to obtain the metallic products of the ores treated than when operating on similar ores according to the means now practised, and with one great advantage, that in so treating ores and minerals containing sulphur, I take up the larger portion of the sulphur so contained in the ores or minerals, and convert common salt into sulphate of soda. Hence it will be understood that my invention consists of treating ores and minerals containing sulphur with such proportions of common salt that the ores are deprived of their sulphur, or nearly so; and the metallic products resulting from such process are rendered more suitable for subsequent processes for obtaining the metals therefrom, and at the same time the act of so treating the ores and the minerals will produce much larger quantities of sulphate of soda than has heretofore been obtained.
"I have found that any ore containing 15 to 20 per cent. of sulphur, and in some cases even less, may be usefully operated on according to my invention; and the treatment according to my invention may be usefully applied either before or after the ores have undergone any process of heat, provided a quantity of sulphur remains equal to the percentage above mentioned. The ores and minerals to be treated according to my invention are those containing sulphur, particularly mundics, or iron pyrites, copper ores, lead ores, tin ores, zinc ores; mundics, or iron pyrites, containing sulphur combined with copper or tin, or with both; copper ores, containing sulphur combined with iron or tin, or with both; lead ores, containing sulphur combined with copper; and tin ores containing sulphur combined with copper or iron, or with both. The process of treatment, according to my invention, is to be carried on in suitable furnaces. The one which I prefer is a reverberatory furnace, having four beds, each succeeding bed being on a somewhat higher level in proceeding from the fire towards the end of the furnace. At the same time I do not confine myselt thereto, as the furnace employed may be varied without departing from my invention; and the ore or mineral, with salt, are to be introduced on the bed of the furnace most distant from the fire, in order that the sulphur given off at comparatively low temperature may be taken up by the salt; and as the ores operated on progressively require more heat to separate the sulphur, they will progressively be brought on to a hotter bed of the furnace.
"Figs. 2132, 2133, show a longitudinal section and plan of a furnace such as I prefer to use in carrying out my invention, and which I prefer to make about 60 ft . long, and 10 ft . from back to front in the clear, haring several openings to allow of the matters under process being turned or stirred from time to time; and I find that the occasional admission of steam to the charge nearest the fire is attended with beneficial effects, promoting the oxidation of the minerals and the evolution of the muriatic acid, but this addition is not absolutely necessary."

Description of the Process.-It is better that the ores should be crushed so as to pass through a sieve of four or more holes to the inch, though this is not always necessary. The salt should be dried previous to mixing with the mundies or other sulphur ores, by placing over a flue, so as to obtain the benefit of the otherwise waste heat. The object of this drying is to prevent its caking in the furnace. The quantity of sulphur contained in the ore having been ascertained by the analysis of a carefully-prepared sample, a given quantity of salt, say 1 ton, having been weighed out, a quantity of ore containing sulphur required for the conversion of the salt into salt cake should be added and intimately mixed. The quantity of sulphur required to convert a ton of salt into salt cake is, by calculation, about 5 cwt . 1 qr . 11 lbs .; but as all the sulphur cannot be taken by the salt, it is proper to have the sulphur in excess above that quantity. And although a beneficial working may be obtained by employing a much less quantity of salt in respect of the sulphur contained, yet

Longmaid found that in all cases the emmon calt should considerably exceed the known weirht of sulphur contained in the ore or mineral under process. The mixture of about $\frac{2}{3}$ of the ore require 1 for the salt used should be put upon the upper bed of the furnace, that is, the bed farthest removed
2132.

2133.

from the fire, and left until heated throughout. It should then be turned over from time to time, so as to allow of the successive contact of the mixture with the atmospheric air passing through the furnace. At about equal intervals and at several times during the time the mixture remains on the upper bed, the remaining quantity of ore should be added. By these means the rapidity and cffectiveness of the operation is promoted, and some saving of fuel is produced. It is impossible to fix the exact quantity of ore required, as it must vary according to the quantity of sulphur contained in the ore employed, and depending, in some degree, on the nature of the substances associated with it. The less arsenic contained in the ore the better, although its presence is not an insurmountable objection, especially if associated with a small percentage of copper. A charge being drawn about every twenty-four hours from the front bed, each one of the three remaining charges will then be moved forward to the next lower bed, and a fresh charge put into the upper bed, each one of the charges being kept regularly raked in its turn. A brisk fire is to be kept up in the furnace during the whole time, and a damper is applied to the chimney to obtain regulation. As the decomposition of the salt and ore proceeds, the misture is gradually fitted to bear the increase of temperature obtained by removal from the upper to the next lower bed, and so on, approaching the fire. The operation appears to proceed best when on the bed nearest the fire it has been brought to a semi-pasty condition, or when the mass has a tendency to agglomerate, and seems to be moist on the surface. By the increase of temperature to which it is here exposed, the charge soon begins to dry up, so that it is eventually drawn in a granular condition. The sulphate ash obtained contains sulphate of soda or salt cake, the chloride of sodium, oxides of iron, a soluble salt of copper, and oxide of tin, if any tin was present in the ore employed, provided the ores be mundic ; and, if other ores are used, other products will be obtained. The ash being lixiviated with water, affords a solution containing the sulphate of soda, chloride of sodium, and salt of copper, the insoluble residue containing the oxides of iron and tin. If oxide of tin be contained in the ore employed, it may be separated from the residual matters by washing, the greater specific gravity of the oxide of tin rendering the separation comparatively easy. The copper may be separated from the solution, either with iron, as is well understood, or, as Longmaid prefers, by the addition of lime slacked in water, forming a milk of lime. Iron precipitates the copper in a metallic form; but the lime precipitates it as an oxide, associated with the slight excess of lime necessarily employed, and some small portion of sulphate of lime. This precipitate, by filtration, having been separated from the refined liquor, should be well washed, in order to the complete separation of sulphate of soda and chloride of sodium, the liquors obtained being employed in the lixiviation of fresh sulphate ash. This precipitate is bulky; but, by filtration and drying, its volume is very much diminished, and it is then obtained in a condition fit for reduction to the metallic state by the usual metallurgical process. The solution from which the copper has been separated may, if required, be concentrated by boiling, and set aside to crystallize in suitable vessels, very fine crystals of the sulphate of soda being obtainable. The mother liquor may be again concentrated and set aside to crystallize, or, if required, be employed for the manufacture of alkali by mixture with fresh lime in quantities bearing the proper proportions observed in the manufacture of black ash. If the salt cake be required only for the manufacture of alkali, the solutions obtained by lixiviation should be run off into a large tank, from which the quantity of liquor containing the desired quantity of salt cake may be run on its equivalent of newly-burnt lime, previously weighed out, by which means the water of solution is either solidified or expelled by the heat evolved in the course of the slacking. But should the solution be so weak that the heat evolved from the requisite quantity of lime would not be sufficient to expel the whole of the water, the lime, after being treated with sufficient of the liquor for slacking and converting into a very thick pasty condition, resembling a tough mortar, might be thrown either upon a flue at the end of a black ash furnace, by means of which the benefit of spare heat may be obtained, or upon the third bed of a black ash furnace, where, as evaporation proceeds, the remainder of the liquor required may be gradually added. By these means a more perfect mixture of the salt cake, with its equivalent of lime, is obtained than is the case by the usual process of mixing only the slacked lime and the dry
sulphate of soda or salt cake in powder. The mixture of sulphate of soda and lime treated with its requirements of carbonaceous matter previous to turning down to the lower bed of the black ash furnace is in the usual condition in which it is employed.

Longmaid, in 1844, stated that the great object of his method of 1842 was to obtain sulphate of soda, the metals obtained being considered a beneficial addition resulting from the process. But he afterwards observed that there were circumstances under which, and situations where, ores containing copper, tin, and zinc, with sulphur, may with advantage be treated with common salt for obtaining the metallic parts, without depending mainly on the profits derivable from the sulphate of soda. Longmaid's process of 1844 consists of an improvement in the manufacture of copper, tin, and zinc, by causing ores containing those metals to be treated with common salt below the relative quantities which he specified in 1842. In 1844 Longmaid stated that the more nearly the quantity of salt approaches sixty by weight to every forty by weight of the sulphur ascertained to be contained in the ores to be treated containing copper, tin, and zinc, the more effectually will the metallic portions of the copper and zinc become soluble in water; though the quantity of common salt may be reduced very considerably below sixty to forty of sulphur, and yet obtain very beneficial effects, particularly where the cost of common salt is comparatively great, and there is no ready sale for sulphate of soda at a price that will repay the manufacturer for using a larger proportion of common salt.

The salt to be used should be well dried, and the ores containing copper, tin, or zinc, with sulphur, to be broken to powder; and having ascertained the quantity of sulphur contained in any quantity of ore about to be treated, mix therewith a quantity of salt suitable for obtaining the metallic parts of the copper and zinc in such a state as to become readily soluble in water, using more salt when the cost thereof, coupled with the selling price and demand for sulphate of soda, do not restrict; but below the rate of sixty by weight of common salt to forty by weight of the ascertained quantity of sulphur, the object of this process being to obtain the metallic parts of the ore separate, without materially, and in some cases not at all, depending on the value of the sulphate of soda resulting from the process, this invention being useful only in those cases where the manufacturer, from local or other causes, does not desire to produce a quantity of sulphate of soda so large as would result from the sulphur contained in an ore if common salt were used in the manner described by Longmaid in 1842. The ore containing copper, tin, or zinc, mixed with the salt, is to be placed into a suitable furnace, similar to that to which Figs. 2132, 2133, appertain. The ore and salt mixed together are to be treated in the same manner as that described by Longmaid in 1842, except that, there being a reduced quantity of salt employed, the whole may be at once mixed before being introduced into the furnace. Each charge of ore and salt is to remain from twenty to twenty-four hours on each bed of the furnace, and to be drawn in about eighty to ninety-six hours, which the workman will judge of by the muriatic acid being driven off. And it should be stated that Longmaid found that some ores, when so treated with salt, are liable to flux. In such cases he applied about $\frac{1}{3}$ cwt. of small anthracite coal or other carbon mixed with a charge of a ton of mixed ore and salt, either when the same indicates fluxing in the furnace, or with all future charges of the same ore. The charge being drawn from the furnace, is then to be lixiviated with water in suitable vessels. The liquor obtained will contain metallic matters in solution, according to the nature of the ores operated on, together with sulphate of soda, muriate of soda, or chloride of sodium. The copper contained in any liquor obtained as above explained may be precipitated, as is well understood, by means of iron; and the milk of lime may be subsequently employed for separating the zinc, associated with an excess of lime and with some oxide of iron. Longmaid, in 1844, stated that the oxide of tin separates from the liquor by gravity, with residuary matters; and if they be not broken fine enough for the washing process to separate the oxide of tin, they are to be broken before washing to separate the tin in the ordinary manner (?). If the whole of the copper and zinc be not converted into the soluble form by the first operation, the insoluble residue may be treated with weak muriatic acid obtained by condensing (?) that product as it is evolved from the furnace where the ores are being treated with common salt, or weak muriatic acid otherwise obtained may be employed to dissolve the copper and zinc not before rendered soluble in water; and these metals may be separated from the solutions thus obtained, as above explained.

Longmaid, in 1851, says;-"In my processes, of 1842 and 1844, for treating ores and minerals, I have found considerable difficulty in condensing the gas and vapours arising therefrom, in consequence of the volatile carbonaceous matters contained in bituminous coal, which, when volatilized together with free carbon, have a tendency to choke the condenser, and thus retard the operations in the furnace, as well as hinder the condensation of the other volatile products of the process of calcination.
"In using coke in the decomposition of common salt and ores and minerals containing sulphur, I prefer having the fire-place of the furnace closed with a donr, and the ash-pit supplied with water, which, being gradually converted into vapour, ascends through the fire, and promotes the combustion of the fuel.
"In using anthracite coal in the decomposition of ores and minerals and salt, I mix a small portion of (coking) bituminous coal, about $\frac{1}{6}$ part by weight of the anthracite coal, which has the effect of causing the mass to cohere on the application of heat. The opening to the fire-place $I$ fit with an iron plate, about 2 ft . by 18 in ., over which I turn an arch 15 in . high-that is, leaving the opening for charging the furnace with fuel 18 in . wide by 15 in . high in the clear. When the fire is well raised, I fill the opening with the mixed coal, and whilst in this position it becomes partially coked; when it is necessary to replenish the fire with more fuel, I move the partially coked coal on to the burning mass in the fire-place, and place a further quantity of coal on the plate, as before; I also apply water to the ash-pit, in a similar manner to that described when coke is the fuel used.
"The volatile products of the sulphating process possess great affinity for water, and their condensation is facilitated by the introduction of steam into the flue leading from the sulphatiug
furnace to the condenser at a convenient distance from the furnace; the object of the application of stcam is, to obtain an intimate mixture of aqueous vapour with the volatilized matters, and thereby to promote their more perfect condensation.
"I have described processes for treating ores and minerals, and the manufacture of alkali, copper, and other products, by mixing such ores and minerals with common salt, and subjecting the mixture to heat in a furnace. In carrying these processes into practical operation, when treating ores and mincrals containing silver and copper, these metals have been converted into a condition soluble in a solution of alkaline and metallic salts, being the product of some of these processes, and I have hitherto precipitated them by means of metallic iron, and then proceeded to oxidize the copper of such precipitate, and dissolve the oxide of copper so obtained with sulphuric acid, producing sulphate of copper in the well-known manner; the residual product, containing the silver, has hitherto been separated by the ordinary process of smelting silvery lead ores.
"I will now describe my improved processes for treating ores and minerals containing silver and copper, and the manner in which the same are performed. I treat ores and minerals containing sulphur in the manner described by me in 1842. I grind the ore and minerals containing sulphur, and mix with common salt, and calcine the mixed material, and obtain solutions therefrom, which I treat as hereinafter described; but if the mincrals contain a considerable quantity of arsenic, antimony, or zinc, or any of them, I prefer to calcine such mineral, in order to drive off some or all of these volatile metals; I then mix the calcined material with ore or mineral rich in sulphur, so as to make the average quality about twenty to thirty of sulphur-that is, when one of my principal objects is to obtain sulphate of soda.
"If the ores or minerals be rich in sulphur, and I have none of the calcined ore, I prefer to reduce the percentage of sulphur by the addition of oxide of iron in a finely-divided condition; in all cases the process is best conducted when the sulphur in the mixed material is about 30 parts by weight to 100 parts of salt employed.
"If the primary object be to scparate silver and copper from ores and minerals, it is sometimes convenient to calcine the matcrial until the sulphur is nearly driven off', and then to add common salt to the charge, and subject it to further calcination; from eight to twelve hours will generally be found sufficient to convert the silver into the state of chloride.
"I usually grind the ores and minerals sufficiently fine to pass through a sieve of at least six holes to the inch; the richer the ores and minerals are in silver and copper, the finer I prefer to grind them, in order to ensure the whole or nearly the whole of the silver and copper being converted into a condition soluble in the sulphate liquor.
" I place the calcined mass, which I call sulphate ash, in a suitable vessel or vessels, and dissolve the soluble portions in water; the solution thus cbtained consists of sulphate of soda, and soluble salts oî silver and copper, and other matters; this solution I call sulphate liquor; I cause it to flow through $\overbrace{}^{2}$ vessel or series of vessels furnished with metallic copper, the liquor coming into contact with the copper, the silver is precipitated, and an equivalent of the metallic copper is diss?lved; when the silver is precipitated, I run the liquor into another vessel or set of vessels, in which the copper is precipitated by iron. If, however, the liquor be so rich in these metallic salts, that the whole of the silver or copper is not readily precipitated, I increase the number or size of the vats, or repeat the operation. Having carefully examined the liquor, to ascertain that the precipitation of the silver and copper is complete, I run the liquor into the alkali department of the works, to convert the sulphate of soda into carbonate. In order to dissolve the whole of the soluble portions of the sulphate ash, I repeat the washing until no more sulphate of soda or metallic salts are obtained; I preserve the weak liquor, and apply it to further portions of the sulphate ash; I convert the precipitate obtained by means of iron, above mentioned, and in fact any copper containing silver into regulus, by the well-known means. I prefer to granulate the regulus, in order to facilitate the further operations whereby the copper and silver are separated by cominon salt.
" In the manufacture of sulphate of copper, I find it convenient to convert the sulphide of copper into sulphate by calcining the regulus or other sulphide, being the product of my processes for precipitating silver and copper from thin solutions, by means of the compounds of sulphur, which I use for precipitating the sulphides of silver and copper, or the sulphides, carbonates, or oxides of these metals from their solutions at a low temperature with access of atmospheric air, and thus produce sulphate of copper and a soluble salt of silver; these I dissolve with water. precipitate the silver from this solution by means of metallic copper, and draw off and crystallize the liquor containing the sulphate of copper, the silver being obtained as a residual product. If the sulphate of copper be not wholly converted into sulphate, or if it be the mixed precipitate containing carbonate, or oxide, or both, I add sulphuric acid, and thereby dissolve the copper.
"I precipitate silver and copper from thin solutions by means of sulphide of calcium, such as alkali waste, which compound contains sulphide of calcium. I also precipitate silver and copper from thin solutions, by means of compounds containing alkaline and metallic sulphides and other alkaline salts, such as the third product (which I call green ash), which product contains sulphides of iron and sodium, carbonate of soda, caustic soda, and other matters, or one or more of these alkaline compounds; or I use solutions thereof. I also precipitate silver and copper from thin solutions by means of black ash or crude alkali, which contains sulphides of calcium and sodium, carbonate of soda, and caustic soda, or one or more of these alkaline compounds; or I use solutions thereof. When I use the carthy sulphides, or the alkali waste containing sulphide of calcium, I find it convenient to sift these substances through a sieve of six or more holes to the inch, in order to separate the coal and cinder and the larger aggregated masses of the alkaline earthy matters; having thus prepared a sufficient quantity, I place a layer in the bottom of the precipitating vessel; having previously provided means for filtering the liquor, I prefer to put a layer of straw, coke broken into fragments, or cinders, and on this I place the precipitant, and proceed to fill the vessel or vessels with the liquor containing the metals to be precipitated. Into a vessel or series of vessels about

15 to 20 ft . square and about 6 ft . deep, I put a layer of alkali waste or other earthy sulphide, about 18 in . deep, through which I cause the liquors to permeate slowly; the silver and copper during this operation combine with the sulphur of the sulphides and the chlorine or acid, and the oxygen in the latter case; and passing over from the metallic salts to the alkaline earthy matters, the metallic sulphides are precipitated, being insoluble in the sulphate liquor; or I allow the solution containing the metallic salts to remain in contact with the precipitant until the metals are precipitated. This operation will usually occupy only an hour or two, but it is desirable that sufficient time be allowed in order to obtain deposition of the precipitated matters; this will be ascertained on examining the liquors from time to time. When the precipitation and deposition are complete, the liquor is to be drawn off and the residual product fluxed to convert the silver and copper into regulus. The black or green ash, herein described, may be used in the solid form in a similar manner to that described when using alkaline earthy sulphide; but I prefer to make a strong and hot solution, and having ascertained the quantity of silver and copper in the sulphate solution, I run a quantity of the sulphide solution into a vessel or set of vessels, and then add the sulphate solution thereto. If the whole of the silver and copper are not precipitated, I add a further quantity of the precipitant. When the precipitate is fully subsided, I carefully draw off or filter the sulphate liquor which is to be used in the manufacture of sulphate of soda, and for the purposes to which sulphate of soda is usually applied. When the precipitate is sufficiently accumulated, I wash and dry it, and proceed to heat the precipitate, as herein described, to manufacture silver and sulphate of copper, or silver and copper, as the case may be.
"From such ores and minerals as are rich in silver, but containing little sulphur, with or without copper, I obtain solutions containing silver or silver and copper, as the case may be. I dilute the strong solutions with water; I heat the diluted solution by preference to boiling, and the silver is precipitated in the state of the chloride. I usually find this accomplished when Twaddle's hydrometer stands at 30 when applied to the diluted liquors. When the chloride of silver is fully subsided, I run the liquor, if it contain copper, into another vessel or set of vessels, and precipitate the copper, and I collect the chloride of silver and smelt it. I can treat such ores and minerals as are rich in silver, but with little sulphur, when containing copper, with common salt, and obtain solutions containing silver and copper. I produce a weak or diluted solution, and thereby precipitate the silver in the state of chloride on the residual matters of the ores or minerals. I mix the precipitate in this case with lead or lead ores, and smelt in the ordinary manner of smelting silvery lead ores; but if the silver has been precipitated separately by dilution, it may at once be smelted and metallic silver obtained. The copper solution I treat by any of the well-known methods for separating copper.
"In treating regulus containing silver and copper, I mix it with 5 or 10 per cent. of common salt by weight of the regulus, and grind it so as to pass through a sieve of 10 or more holes to the inch, and calcine the ground material in a furnace. As it is better to conduct this operation gradually, I prefer a furnace of three or more beds, each bed of about 12 ft . square, but a furnace with one or more beds may be used. I place the first charge on the bed farthest from the fire, and when it has remained about eight hours in the back bed I move it on to the next bed, and so on in rotation, occasionally stirring it and drawing the finished charge at the bed nearest the fire ; the calcined material is put into vessels. When the soluble portions are dissolved, if the calcined material does not contain alkaline and metallic salts, I add strong and hot sulphate liquor, this solution having the property of dissolving chloride of silver; if, however, the whole of the chloride of silver is not dissolved by the first operation, I repeat it. In treating such ores, if the sulphate of soda be not an object of importance to the manufacturer, as will sometimes happen, I precipitate the chloride of silver by means of diluting the solution with water; having obtained the dilute solution, I heat it as before mentioned; this operation of heating causes the particles of chloride of silver to aggregate, and facilitates its deposition. I allow the liquor to rest until the chloride of silver has subsided, and then draw off the clear liquor ; about forty-eight hours will generally be sufficient for its subsidence. I collect this precipitate and smelt it for silver.
"The ordinary regulus of the copper-smelter frequently contains a notable quantity of antimony; in this case having obtained solutions as before, I proceed to precipitate the silver and antimony together by the process of dilution with water; I collect and smelt the precipitate with lead, or a compound of lead, in the well-known manner of smelting silvery lead ores, and thereby obtain the antimony and silver. But if the ore or mineral treated be a sulphide of copper containing silver mixed with any compound of iron, I mix such material with common salt, calcine and dissolve the copper with hot water, taking care that the solution be so weak as not to dissolve the chloride of silver, which is obtained with the oxide of iron as a residual product, which I smelt with lead or compound of lead, as above described. If the ore or mineral I treat be a mixed mineral, such as sulphides of silver, lead, and copper, I proceed to mix such ore with common salt and calcine the mass, and dissolve the copper with water, and I prefer the use of a solution so dilute that the chlorides of silver and lead are not retained in the cold solution, but are deposited with the mass of the lead contained in the ore ; the copper of the weak solution I precipitate. But if it be desired to produce chloride salts of lead, I dilute it, and thereby precipitate the silver; I draw off the hot solution and allow the chloride of lead to subside on cooling, after which I precipitate the copper as before. If the mineral I treat be regulus of copper, or the sulphide precipitate herein described, a portion of the copper only will be rendered soluble; this portion may be precipitated, but I prefer to use solution of black or green ash, and smelt the precipitate separately with great care; by this improvement I produce copper of a superior quality; the portion remaining undissolved I smelt. It is sometimes convenient to precipitate the silver and copper by means of other compounds of sulphur. To produce sulphides of silver and copper in all cases, I treat such sulphides as before explained, when describing my processes for treating such sulphides.
"Lastly, I select such product of the calcined ores and minerals as consist of oxide of iron ; I separate the oxide of iron from the earthy matters by washing, and mix it with sufficient carbon to deoxidize the oxide of iron, and a small quantity of clay to cause it to cohere; I then mould the
mixture into balls, or other convenient shape, and subject it to a smelting process in a reverberatory furnace; the product is iron of fine quality. The carbon I use by preference to anthracite coal or charcoal, and in all cases the clay and carbon should be as free as possible from sulphur."
G. Hähner, in 1856, proposed to decompose certain metallic oxides at a high temperature in contact with alkaline chlorides, or other chlorides forming oxychlorides, or chlorides soluble in water, in avoiding the formation of free soda by the addition of a mineral acid, and lastly, separating the metals contained in the solution, and utilizing the residuts.

For this purpose, the metallic ore is reduced to pieces and roasted, then pulverized, and again roasted with the admixture of coke, coal, or charcoal reduced to minute particles. After perfect oxidation of these matters, Hähner proposed to introduce into the furnace, to be mixed with the ore, a mixture of about two parts or more of chloride of sodium (common salt) or other alkaline chlorides, and three parts of ore already roasted to each part of metal to be extracted. When there was no longer any trace or smell of muriatic acid vapours, he introduced the roasted ore into vessels provided with filters, into which vessels water slightly acidulated was poured to wash the ore. According to this process, if the ore contains copper or silver, these metals will be found in the solution. The oxides of iron, tin, zinc, and so on, remain in the vessel; the oxide of tin is separated by washing, and the oxide of zinc by reducing it to metallic zinc. Gold remains also in the vessel, and is converted into chloride of gold by means of a stream of chlorine which is introduced into the vessel, and the chloride of gold dissolved in water. In certain cases, Hähner prefers to precipitate the copper by means of a stream of sulphuretted hydrogen, or by a solution of common ash, potash, or soda alone or mixed with lime.

Hähner further observes;-"To form the oxides I submit the ore to roasting either in the open air, or in kilns or furnaces for the purpose of expelling sulphur, arsenic, and other volatile substances, and render the ore more friable. If the metallic rock-gang contains calcareous substances, it must be burnt in a similar manner to lime, and dissolved in water; the oxides, and so on, of this ore will deposit at the bottom of the vessel in which the lime has been dissolved and driven off. Oxidized and other ores which do not contain sulphur or other mineralizing substances only require to be brought to a red heat. The ores treated as before described are then reduced to powder by the ordinary means, and again roasted in a reverberatory furnace, a small quantity of coke, charcoal, coal dust, or other combustible being added to facilitate the operation. To decompose metallic oxides obtained, and also other oxides, the red-hot ore remaining in the furnace after being completely roasted is mixed with an alkaline chloride (chloride of sodium being preferred on account of its low price) in the preportion of about two parts by weight (more or less, according to the nature of the ore) of chloride for each part by weight of metal to be extracted from the ore. To obtain a more perfect mixture, I add to the shloride, before its introduction, about an equal weight of ore already roasted, mixing them intimately, and moistening them if dry. The moistened chloride or mixture of chloride and roasted ore ought then to be incorporated as intimately as possible with the red-hot ore in the furnace, and kept in a continual movement, and at a red heat until the smell of muriatic acid becomes less perceptible, and the ore commences to adhere to the workman's tools; the ore is thin withdrawn from the furnace, and a fresh charge added. It is advantageous to leave the red-hot ore thus withdrawn for some time in heaps, which renders the process still more perfect. If the ore contains no silica, it is requisite to add about 10 per cent. of this substance. The ores treated as before described are then submitted in a hot state, if possible, to lixiviation. I add to the water employed for the lixiviation of the roasted ore, about 5 parts by weight, more or less according to the nature of the ore, of sulphuric, muriatic, or other acid to 1000 parts by weight of ore, to render more soluble the oxychlorides or chlorides, and to decompose the free soda, silicates of soda, and so on, which may have been formed during the roasting, and which would cause a great loss of metal. The vessels in which the lixiviation is performed, may be of wood or masonry work, and may be of any form and dimension, according to circumstances, they should be furnished with an ordinary filter to allow the water to run off freely. The precipitation and purification of the metals contained in the solution, or the formation of other commercial products, can be effected by the usual processes. The copper, however, may be precipitated also by common ashes, lime water, and caustic water, and the products obtained may be used in the nanufacture of different colours, salts, and so on, or reduced to the metallic state in ordinary furnaces, or by other known processes. The copper may also be precipitated in the state of arsenite or arseniate of copper for the formation of Schieles or Vienna green, by means of a solution of arsenite or arseniate of potash. The refuse wash waters which have served for the separation of the metals may be employed for moistening the roasted and pulverized ores, or for other purposes. After the lixiviation, there remains in the vessel the powdered metallic rock gold, oxides of iron, tin, zinc, and so on, if the ore contained such metals, which can be utilized by known means."

Other methods of treating ores containing copper, as well as those of Napier and Henderson, will be treated of hereafter. See Allors. Arsenic. Atomic Weights. Chimeey, p. 951. Firinaces. Gold. Iron. Kilns. Lead. Nickel. Ores, Machinery and processes employed to dress. Reagents, employed in smelting ores. Silver. Sulphur. Tin.

COPING. Fr., Larmier; Ger., Mauerabdeckung; Ital., Coronamento; Span., Caballete.
The Coping is the highest or covering course of masonry in a wall, often with sloping edges to carry off water; sometimes called capping.

COP-SPINNER. Fr., Bobinoir ; Ger., Spulmaschine; Ital., Filatoio; Span., Hilandera.
See Cotton Machinery.
CORK-CUTTING MACHINE. Fr., Bouchonnier ; Ger., Pfropfenschneidmaschine; lral., Macerhina da turaccioli; Span., Máquina para cortar corchos.

Figs. 2134 to 2139 relate to a peculiar construction and arrangement of a cork-cutting machine, invented by Hammer and Butz, of Philadelphia, U.S., whereby all the successive operations for transforming the bark of the cork-tree into any of the various descriptions of corks, may be conducted in such a manner, that the several operations can be simultaneously employed at different
manipulations upon the same machine, without hindrance to each other. This machine consists of an arrangement and combination of mechanism for cutting tapered corks from square or cylindrical blocks; of an improved arrangement for effecting the preliminary operation of cutting the raw bark into strips, and for subdividing these strips into parallel or tapered blocks of any required size, to be subsequently reduced to cylindrical or tapered corks in the first-mentioned part of the machine; and of mechanism combined with these arrangements for cutting flat and cylindrical corks by means of revolving crown cutters; the whole of these various subordinate combinations forming one complete machine, driven by one main driving belt. The main shaft of the machine carries a circular disc-cutter at each end, the edges of which are kept sharp by oil-stones fitted to the framing, and held in contact with the cutting edges as they revolve. One of these cutters, which is of larger diameter than the other, is for cutting tapered corks from square or cylindrical blocks; these blocks are contained in a tray or receptacle supported by a fixed bracket, and at the delivery mouth of this receptacle there is an adjustable gauge, against which the block is held by a blade spring until it is grasped between the adjoining ends of two rotating spindles. These spindles are contained in a sliding frame, and one of them, which may be termed the live spindle, reccives a positive rotatory motion from an endless chain and chain pulley, the other being merely carried round by the frictional contact with the cork. A self-acting clutch and clutch-lever throw the chain-actuating pulley out of gear when the sliding frame is receding from the cutter, so that the spindles may be at rest when in the act of gripping the block, and, consequently, they take hold of it with greater certainty and accuracy. The grip is obtained by the aid of a spiral spring on one of the spindles. As the sliding frame approaches the cutter with the block of cork to be cut, the clutch is thrown into gear again, and the rotation of the spindles recommences. The cork being cut, and the sliding frame receding from the cutter, the spindles are separated, in order to allow the cut cork to fall into an inclined shoot below, which delivers it into a receptacle for the purpose. The clutch above referred to is thrown in or out of gear by a projection in the sliding frame coming in contact with the tail of the clutch-lever, as the slide moves to and fro. The separation of the spindles, in order to release the cut cork, is effected by a movable or spring incline, which acts upon the lower end of a lever, the upper end of which grasps a collar on the spindle to be slid back. This incline enables the lever to pass freely in one direction, namely, when approaching the cutter; but acts upon the lever and releases the cork when moving in the opposite direction, or receding from the cutter. The reciprocating motion of the sliding frame is obtained by means of a chain or cord attached to opposite ends of the slide, and passing under a drum or pulley to the periphery of which it is secured. By turning this drum or pulley in one direction or the other, by the aid of a lever handle, a reciprocating motion is imparted to the sliding frame. The apparatus for cutting the bark into strips consists of the smaller one of the two circular disc-cutters, in combination with a table and adjustable gauge. The bark is laid upon the table with its edge against the gauge, and the cutter, by revolving in the direction of the feed, draws in the bark as fast as it is cut, without any force being required to pass it through the machine. When subdividing these strips into blocks, a sliding saddle is fitted on to the table, and upon this saddle is placed a head-piece having a squaring strip thereon, against which one side of the strip of cork is held whilst being cut. This head-piece is pivoted to the saddle by a vertical centre pin, upon which it is free to turn slightly in an horizontal plane', so as to present itself, and the strip of cork upon it, at an horizontal angle with the plane of the cutting edge of the circular cutter. This angular movement is controlled by two screw pins passing through slots in the headpiece, which screws, when tightened, serve to hold the head-piece fixed in a position perfectly parallel to the cutter, when parallel blocks are to be cut. When tapered blocks are required, the head-piece is moved alternately from one strip to the other between the successive cuts, and hence in lieu of the strip of cork being cut across at right angles, which would produce parallel blocks, it is cut across alternately at two opposite angles, thereby producing tapered blocks. The apparatus for cutting flat and cylindrical ends consists of a vertical spindle, to the lower end of which are fitted different sized crown cutters, according to the diameter of the corks required. Beneath this spindle there is provided a table for supporting the cork to be operated upon. The vertical spindle is driven by a twisted strap, from a driving pulley on the main or disc-cutter shaft of the machine, such strap passing around a small fixed pulley on the vertical spindle. By having the usual fast and loose driving pulleys on the main-cutter shaft of the machine, and driving the same by a belt, the whole of the different operating parts receive motion simultaneouly and in concert.

Fig. 2134 is a side view of this cork-cutting machine, combining the above-mentioned devices for effecting the several successive operations; Fig. 2135 is a front view of the machine; Fig. 2136 is a plan of that part of the machine in which the bark is first sliced and then cut into blocks; Fig. 2137 is a detached side view of the feed arrangement of the tapering machine for cylindrical blocks; Fig. 2138 is a plan thereof; Fig. 2139 is a detached plan view of part of the mechanism for cutting tapered corks; and Fig. 2140 a detached view of the feed-table of the tapering machine for square or bevelled blocks. A is the main shaft of the nachine, provided with fast and loose driving pulleys $a a^{1}$, and carrying at its extreme ends, outside of the adjustable bearing $b b$, the circular cutting dises or knives $\mathbf{B}$ and $\mathbf{C}$, the larger one of which (B) serves to cut conical or cylindrical corks from blanks fed to it by means of the sliding spindle frame D. This frame-hereafter more fully described-is fastened in an adjustable manner to the slide E , reciprocating in a bed F . The feeding device for supplying cylindrical blocks to the spindles for transmittal to the knife $\mathbf{B}$, is supported upon a stand $c$, and is constructed as follows;-Attached to the stand $c$, and vertically adjustable upon the same by means of a screw $d$, there is a rectangular piece $d^{1}$; on the top of the horizontal part of $d^{1}$ is placed the gauge $e$, which is adjustable lengthwise along $d^{1}$, and secured by a screw and nut. A receptacle $G$ for the corks to be fed to the spindles is provided on the top of $e$. The position in which the cork is placed for being grasped by the spindles $J$ and $J^{1}$ is best understood from Fig. 2137; resting upon the inwardly projecting end of $d^{1}$, the cork is lightly pressed against the gauge $e$ by a spring pad $f$, swinging upon a small rock-shaft $g$, which pad is
sufficiently yielding to allow the cork to be removed horizontally towards the cutting disc. A sliding movement of the spindle frame D , for alternately carrying the corks to the knife and returning to the feedtable for a next one, is given by means of a cord or band $h$, so attached to the opposite ends of the slide E , and to the periphery of a pulley H, that by means of a hand-lever $\mathrm{H}^{1}$ on the pulley-shaft $i$, motion is transmitted to E by the cord $h$ in either direction. The outward movement of the slide $\mathbf{E}$ is arrested by a fixed stop I on the plate $\mathbf{F}$, while a screw $\mathbf{I}^{1}$ serves as an adjustable stop for the movement of E in the opposite direction; by varying the position of this screw, the finished diameter of the screw is regulated with accuracy. The spindle J, for rotating the cork as it is presented to the cutting edge of the circular knife B , is driven by means of a chain $j$ passing over the pulleys $j^{1}$ and $j^{2}$, the latter being the driver, and receiving motion through the bevel-wheels $j^{3}$ and belt $j^{4}$ from the main shaft A. $j^{2}$ is provided with a clutch and

so actuated by a clutch-lever $k$ and arm $k^{1}$ projecting from the slide $\mathbf{E}$, that the spindle $\mathbf{J}$ ceases to rotate as it recedes towards the feed-table, and is in turn thrown into action when approach-
ing the cutting disc. The spindles being thus at rest when grasping the blank to be conducted to the knife, will be much more certain to take accurate hold of the blank than if they were revolving at the time. A rocking arm K, actuated during the sliding motion of $\mathbf{E}$ by an inclined plane $i^{1}$, and by a spiral spring $i^{2}$, on the spindle $J^{1}$, serves to give the requisite sliding movements to that spindle for alternately grasping and releasing the corks as they pass through the machine. Although this sliding motion of one spindle only is ordinarily sufficient, it may in some cases be advantageously given to both spindles. The arrangement of the inclined plane $i^{1}$, and the manner in which it actuates the double-armed lever K through the spindle $J^{1}$, will be best understood upon reference to Fig. 2139, where it will be seen that $i^{1}$ is attached to a plate L in the following peculiar manner;-It has a limited vibrating movement upon a central axis $l$, between two small stops 2 and 3 , the axis $l$ being confined in an oblong opening in the plate $\mathbf{L}$; a light coiled spring 4 bears against one end of $i^{1}$, and brings its other end in contact with the pin 3.

The operation of the whole is as follows;-The lever K, while holding the cork to the knife, occupies the position represented in Fig. 2135, and as it recedes towards the feed-table, its small friction-roller 5 brings up against $i^{1}$, as shown in dotted lines at Fig. 2139; by yielding to the inclined plane the lower arm of K is drawn inward, and by a consequent movement of its upper end in an outward direction, the cork just finished is released from between the two spindles, and falls down; in this separated position the spindles remain during the whole outward movement, until at the instant of the slide E, bringing up against the stop I, the friction-roller 5 is liberated from the inclined plane ; the spiral spring $i^{2}$, being thus freed from its previous compression, suddenly pushes the spindle $J^{1}$ inward, to make it grasp the blank on the feed-table. The roller 5 on the $\operatorname{arm} \mathrm{K}$ is now in the position shown in dotted lines, Fig. 2139; and as upon the advance of $\mathbf{E}$ toward the knife, this roller comes in contact with the inclined plane $i^{1}$ on the opposite side, the latter will yield to it in the oblong bearing of its axis $l$, so as not to disturb the spindles in their hold upon the cork to be cut; in returning from the knife the described routine of movements is repeated. By an arrangement of hoppers or inclined planes, shown in Fig. 2134, the chips are separated from the finished corks; the former, curling up on the inner side of the knife, fall into the large hopper M, and are heaped up under the frame of the machine; while the corks as they leave the spindles are carried over two inclined planes $m$ and $n$ into a separate receptacle. $N$ and $\mathrm{N}^{1}$ are small oil-stones, so attached to their supports as to press lightly against opposite sides of the cutting edge of the knife, thus keeping it uniformly sharp. The mechanism for cutting the bark into strips consists, in addition to the small circular knife C , of a table O and gauge $\mathrm{O}^{1}$; the end of the piece of bark to be cut into slices is held against the gauge $O^{1}$, and then laterally advanced toward the edge of the revolving circular knife C , the direction of motion of which is such as to draw the bark through without the least application of force on the part of the workman. For the subsequent operation of reducing these strips into blocks, a sliding saddle $P$ is employed in addition to the gauge $O^{1}$, which saddle has a vibrating head-piece $Q$. When required for cutting blocks with parallel sides, this head-piece is permanently fastened upon the sliding saddle P , in the position shown in the engraving, Fig. 2136; but the same parts are in a very simple and efficient manner adapted to cutting blocks with tapering sides, from which tapered corks can be eut most economically. To this end, the head-piece Q is made to vibrate upon an axis $p$, this movement being limited between adjustable stops $q q$; the strip of cork is laid upon the front part of the head $\mathbf{Q}$ against the squaring strip $Q^{1}$, and between successive cuts of the knife the head is moved alternately from one stop to the other, so that the cork blocks become tapered by thus reversing the angle for each successive cut. In addition to the above mechanism, the machine is provided with a frame R, carrying in bearings SS the hollow cutter-spindle T, for cutting cylindrical and flat corks of any required diameter and thickness, by means of changeable cutters set into the lower end of the spindle. U is the table upon which rests the slice of cork to be operated on by the revolving cutter, and V is the lever for actuating the cutter-spindle; the latter is driven from a pulley on the main shaft A by means of a half-twist belt running over a small pulley $W$.

CORNICE. Fr., Corniche; Ger., Karniess ; Ital., Cornice; Span., Cornisa.
Any moulded projection which crowns or finishes the part to which it is affixed is termed a cornice; as the cornice of an order, of a pedestal, of a door, window, or house.

CORNISH ENGINE. Fr., Machine à vapeur pour élever l'eau; GEr., Wasserhebungsmaschine; Ital., Macchina di Cornovaglia.

See Pumps and Pumping Machinery.
CORN MILL. Fr., Moulin à blé; Ger., Mahlmühle; Ital., Mulino; Span., Molino harinero.
See Mills. Barn Machinery, Fig. 546 .
CORROSION. Fr., Corrosion; Ger., Zerfressung-Corrosion; Ital., Corrosione; Span., Corrosion.

With the exception of Robert Mallet's extensive experiments on the action of air and water upon iron and steel, corrosion and anti-corrosion have not received that amount of attention on the part of scientific men-chemists in particular-that their industrial importance demands.

Malleable iron undergoes no change in dry air, or in water free from air; but in moist air, or water containing air, it gradually becomes oxidized or rusted from the surface inwards, until eventually the entire mass is converted into oxide. The carbonic acid present in atmospheric air appears to contribute largely to the production of this change. The presence of saline substances in water also facilitates the oxidation of iron, while alkalies and oily or resinous substances retard it. Contact with more highly electro-positive metals, such as zinc, also hinders the oxidation of iron within a certain distance around the point of contact. Caustic alkalies or alkaline carbonates act as preventatives. There is a theory that, so soon as the first thin coating of oxide has formed upon the surface of the metal, a galvanic action sets in, whereby the process of oxidation is greatly accelerated, the iron acting as a positive agent. the oxide as a negative one. Brass and copper are attacked by ammonia, for which no efficient preventive is known.

Corrosion of Steam Boilers.-The process of corrosion is very similar to that of the combustion of
fuel, the only difference being that in corrosion the metal unites with the corrosive agent slowly, while in combustion the fuel unites rapidly with the supporter of combustion.

The external corrosion of a boiler is due to simple oxidation, caused principally by atmospheric exposure.

In the boilers of sea-going vessels it is also caused by the contact of the bottom of the boiler with bilge-water, and by the exposure of the top to leakage from the deck. The best means of preventing this is to cover the top with felt and sheet lead sollered at the joints, and to keep the bottom thoroughly painted. The internal corrosion is due to simple oxidation, and to the galvanic action taking place whenever two different metals or a metal under different conditions are either wholly or partially immersed in a fluid in which either of them would be oxidized-that is, united with the oxygen of the corrosive agent, and which has the effect of confining the corrosion principally and sometimes wholly to one of the two metals in contact. The sheets are eaten away around the rivets before the rivet is injured, on account of the iron in the rivet being in a different condition from that in the sheet, owing to its being more dense from being hammered until cold, and, consequently, producing a galvanic action by which the sheet is corroded. Tube-shects are ait to leak, when the sheets and tubes are composed of different metals, from the effect of the galvanic action produced by them.

The hot brine or sea-water contained in marine boilers is a most powerful corrosive agent of wrought iron. Hence the stays are corroded, and the pins or bolts which hold the stays are eaten and loosened. A very thin film of scale is the best protection against this kind of corrosion. The corrosion of the steam drum is caused by the high temperature of the uptake, about $600^{\circ}$ Fahr. for natural draft, thereby superheating the steam, and oxidizing the iron in a similar manner to the making of hydrogen gas, by sending steam over red-hot iron.

Boilers not in use are liable to corrosion on the fire side of the heating surface as well as on the water and steam side. To prevent this, the smoke-stack should be covered over to keep out rain and moisture: the man-hole plates taken off, so as to allow a free circulation of air inside; and a light fire of shavings should be built occasionally to dispel all moisture

Corrosion of Hiyh-pressure Boilers.-Speaking on the explosion of locomotive and other highpressure boilers, William Kirtley, of Derby, in a paper printed in the Proceedings of the I. M. E. (1866), observed that in the large majority of exploded locomotive boilers it has been found that the explosion has arisen from the plates of the boiler having become weakened by corrosion at particular places. In the paper referred to it was the writer's object to describe the nature and extent of this corrosion, and to endeavour to show the causes of its occurrence, together with the means of prevention.

In the present ordinary construction of locomotive boilers with lap-joints, as shown in Figs. 2141,2142 , the wear by corrosion of the plates is found principally round the smoke-box end of the boiler barrel, in the interior, opposite to the edge of the outside angle-iron as shown at A A in Figs. 2143 to 2145 , where an annular groove is found to be eaten out of the plates by corrosion. This grooving extends sometimes so deep into the plate that only a thin shell of metal is left on the outside at the bottom of the groore, as shown in Fig. 2145, which is a full-size section of an actual case of the grooving; and the corrosion takes place so rapidly in many cases that the plates require renewal after only a few years' work. A similar grooving also takes place along the edge of the inside lap at the longitudinal joints, as at D and E in Fig. 2146, and also at the transverse circular joints, as at B B in Figs. 2147, 2148; but in the latter case the grooving does not occur so frequently, nor is the extent of corrosion so great, as at the smoke-box end and at the longitudinal joints.

It may be remarked first that this grooving is only found below the water-line, showing that it must be due to the chemical action of the water on the plates; and the special point to be inquircd into is the cause of this action being so remarkably concentrated at the particular lines where the grooving takes place. It was evident from the specimens shown Kirtley, which were taken from locomotive boilers that had been at work for various periods of from three years to as much as nineteen years, that some corrosion also takes place over the general surface of the plates; but this is very limited in extent compared to the grooving at the seams, and it occurs very irregularly, being apparently influenced by some irregularities in the structure of the plates, causing them to be pitted irregularly by the corrosion.

In the ordinary construction of locomotive boilers with lap-joints, as shown in Figs. 2141, 2142, the barrel of the boiler is constructed of three rings, each ring formed by two plates of $\frac{7}{16}$ in. thickness, riveted with lap-joints FF and H H. The general amount of lap is $2 \frac{1}{4} \mathrm{in}$. for single-riveted and $3 \frac{1}{2} \mathrm{in}$. for double-riveted joints. The smoke-box and fire-box are each united to the barrel of the boiler by an angle-iron K K, Fig. $2142,3 \mathrm{in}$. or $3 \frac{1}{2} \mathrm{in}$. wide, welded into a ring. General experience has shown that after five or six years' wear of these boilers the grooving action that has been described is developed at the joints and at the edge of the angle-iron rings.

Now the longitudinal strain upon the joints of boilers constructed in this manner tends to spring and bend the plates at the joints, when under pressure, into the form shown exaggerated in Fig. 2149, in consequence of the plates not being originally in the line of strain, as shown by the dotted line S S in Fig. 2142, which, it will be seen, runs along the outer face of one plate and the inner face of the next. Also, in the longitudinal joints of the barrel, shown at F F in Fig. 2141, a similar mechanical action takes place, the strain acting in the true circle shown by the dotted line $\mathbf{S}$ S, springing and bending the plates at the edge of the joints, as shown at G G, each time that the boiler is under pressure. The continued alternation of expansion and contraction in the boiler causes the scale that is deposited upon the plates from the water to be continually broken off at the edge of the joints by the mechanical action of this springing and bending of the plates at the lines of the joints; and the plates are thereby laid bare at those parts, and kept continually exposed to the corroding action of the water, instead of being protected from the action of the water by the deposited scale remaining attached to them.

Though the corrosion produced by the water is slow in action and but slight in effect on the rest of the boiler-plates, which are protected by some deposit of incrustation remaining almost con-


Before pressure.

2149.


214 .
Grooving at B.


Grooving at C.

stantly upon them, it becomes very serious on an exposed raw surface of iron; and this action is particularly severe in the case of locomotive boilers, in consequence of the total quantity of water evaporated in a locomotive boiler being much greater in proportion to the surface of the plates than in stationary boilers. The particulars are given in the accompanying Table of the total work done and water evaporated during the time they were running, by the several locomotives specified; from these particulars $10,000,000$ gallons of water may be taken as
having been evaporated in a locomotive boiler during the five to eight years in which the plates have become corroded through. As this is nearly double the quantity evaporated in the same time by an ordinary stationary boiler having three times the surface of boiler-plate exposed to the action of the water, the total work of the locomotive boilers may be considered as amounting to six times the evaporation the square inch of plate in the same time, and the total length of working as equivalent, consequently, to from thirty to fifty years' working of a stationary boiler.

Partioulars of Locomotives from which Specimens of Corroded Plates were taken.

| Number of Engine. | Years of Working. | Miles run. | Water consumed. Gallons. | Nuinber of Engine. | Years of Working. | Miles run. | Water consumed. Gallons. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 99 | 3 | 83,349 | 1,462,774 | 274 | 131 | 303, 249 | 10,644,039 |
| 121 | $11 \frac{3}{4}$ | 334,711 | 5,874,178 | 306 | 111 $\frac{1}{4}$ | 229, 162 | 8, 043,587 |
| 123 | 12 | 290,380 | 5,096,169 | $306 \dagger$ | $6 \frac{1}{4}$ | 142,808 | 5,012,560 |
| 141 | $8 \frac{1}{4}$ | 268,679 | 4,715,316 | 369 | $10 \frac{1}{2}$ | 246,956 | 8,668,155 |
| 162 | $8 \frac{3}{4}$ | 255,042 | 4,475,987 | 375 | $3 \frac{1}{4}$ | 67,072 | 2,354, 227 |
| 187 | $8 \frac{1}{2}$ | 229,099 | 8,041,374 | 388 | $8 \frac{1}{4}$ | 180,985 | 6,352,573 |
| $235^{*}$ | 14 | 315,227 | 11,064,467 | 410 | $6 \frac{3}{4}$ | 158,801 | 5,573,915 |
| 250 | $14 \frac{3}{4}$ | 316,391 | 11,105,324 | 422 | $8 \frac{1}{2}$ | 231,035 | 8,109,328 |
| 255 | 14 | 293,559 | 10,293,920 | 658 | $18 \frac{3}{4}$ | 249,672 | 4,381,743 |

* Flanged tube-plate.
$\dagger$ After renewal mith thick-edge plates.

It must be noticed that the pressure under which the locomotive boilers are worked is much higher than in the case of stationary boilers, and the injurious action caused by the springing of the plates at the joints is therefore proportionately increased; and taking the pressures at $3 \overline{5} \mathrm{lbs}$. the inch for the stationary boiler and 140 lbs . for the locomotive, this makes the action four times greater in the locomotive boiler from this cause, taking the increase to be only at the same rate as the increase in pressure. Hence as the action is six times greater from the previous cause, a total is given of twenty-four times as great an extent of injurious action in the locomotive builer as in the stationary boiler in the same length of time. As an illustration of the effects of increased pressure in increasing the corroding action, it may be mentioned that this grooving of the plates has been found to be materially increased in amount since the working pressure of locomotives has been increased from 100 lbs . up to the present 140 lbs . the inch.

In some of the older classes of locomotive engines Kirtley found that there is an increased local action of serious amount caused in the boilers by the rigid points of attachment to the boiler barrel, such as frame-stays, brackets, and so on, which offer special points of resistance to the expansion of the boiler when under pressure. A specimen of this grooving, taken from No. 187 engine in the preceding Table, is shown at C C in Figs. 2147, 2148, caused by the rigid attachment of the spectacle bracket M to the boiler barrel. The result is made worse when the fire-box is rigidly fixed to the frames, or not allowed full freedom for expansion by the provision of a sliding bracket; as a great additional strain is thereby thrown on the tube-plate, springing the angle-irons round the ends of the boiler. The expansion of a 10 ft . 6 in . or 11 ft . boiler barrel being about $\frac{3}{16} \mathrm{in}$. at a pressure of 140 lbs . the inch, an attachment to the frame at any other place besides the fixing of the cylinders and tube-plate at the front end must subject the boiler to a bending strain at the points of attachment, causing a risk of corrosion at these points. In the Midland Railway engines all the other attachments except the smoke-box angle-iron are now removed, including that of the motion-plate which carries the inner ends of the slide bars; and the boiler is allowed in expanding to slide freely throughout upon the frames.

In the longitudinal joints of the boiler the grooving from corrosion is generally found to be more marked when the inside edge of the lap faces upwards, as at E in Fig. 2146, than when it is turned downwards, as at D. In the former case it may be considered that the deposit will collect upon the projecting ledge in larger quantities, forming a thickness of deposit sufficient to be detached bodily by the springing of the plate under pressure; and it will consequently leave the bare plate more frequently and extensively exposed to the direct action of the water, than when the edge of the plate faces downwards, as at D , because in the latter case the thinner film of deposit will not be so readily and frequently detached from the plate by the same action. It must be borne in mind that the earthy deposit itself, being chemically neutral, cannot have any injurious action upon the plate; except in the case of a stationary boiler heated from an external flue, where undue heating and expansion of the plate are caused wherever its inner surface is separated from the water by any considerable thickness of non-conducting deposit.

In the preceding Table are given the particulars of seventeen locomotives on the Midland Railway, from which the specimens of corroded plates were taken; showing the length of time of working, and the mileage and consumption of water before the plates had become so defective as to require removal. The average result is, $10 \frac{1}{4}$ years' working, 255,645 miles run, and $7,618,778$ gallons of water consumed by each engine.

In the case of the boilers constructed in the ordinary manner, as already described, the plates cut out show the grooving action of the corrosion below the water-line, while they are comparatively clean above. In No. 235 engine the tube-plate was flanged and riveted inside the boiler barrel; and the result of working shows the advantage of this mode of construction over the ordinary angle-iron joint, since the plates at the smoke-box end are not grooved along the edge of the flange, as they would have been with an external angle-iron.

From the foregoing consideration of the subject, observes Kirtley, it therefore appears that the special corrosion of the plates at the joints is to be attributed to the combined operation of chemical and mechanical causes, the chemical action of the water in the boiler being concentrated upon those particular parts in consequence of the mechanical action produced at those parts by the strain upon the plates. That the combination of these two causes is requisite for producing this effect is shown by the middle of the plates being free from it, where they are exposed to the chemical action alone, without the mechanical action; and further by the joints in the upper part of the boiler above the water-line being also free from it, where exposed to the mechanical action alone, without the chemical action. The removal of one of these causes will therefore be sufficsent; and in the locomotive boilers now to be described this object has been aimed at by removing the mechanical cause which produced the springing of the plates at the joints.

From the particulars already given of the corrosion which takes place in locomotive boilers, it appears that the greatest mjury takes place at the smoke-box end of the barrel, where there is not only a great and sudden change in the thickness and rigidity of the plates at the edge of the angleiron, as at J in Fig. 2142, but also a leverage for the springing of the plate from the outer line of rivets, as at L in Fig. 2149. The consequence is the bending of the plate at the point J, as in Fig. 2149, each time of being under pressure of steam, owing to the outer line of rivets L being entirely outside of the line of strain $S$ of the boiler-plates. There is also a great tendency to injury of the angle-iron, by this action tending to split it between the rivet-holes at the outer line of rivets L .

The present plan adopted on the Midland Railway is found to obviate the injury previously experienced from corrosion; and this is accomplished by the use of plates rolled with thickened edges, as shown in section in Fig. 2150, and shown exaggerated in thickness in Figs. 2151, 2152. The ordinary thickness of $\frac{7}{16} \mathrm{in}$. is preserved in the borly of the plate, and the edges are thickened to $\frac{5}{8}$ in., with a long gradual taper in the thickness from I to I, Fig. 2150, about 4 in. long. The effect of this long taper is that, when the plate is flanged, as at N , in order to do away with the angle-iron,
the taper ensures a gradual springing of the plate, distributed over all that length, instead of the sudden bending concentrated at one point, as at J in Fig. 2149.


A similar section of thick-edge plate is also used at the transverse circular joints, as shown in Figs. 2151, 2152, causing a gradual springing of the plates over a considerable length when under pressure, as in Fig. 2152, instead of thê former sudden bending at one point, as in Fig. 2149. There is also an increased strength gained by this mode of construction, as the increased thickness of the plates between the rivet-holes compensates for the loss of section by the holes.

The practical working of the thick-edge plates is shown by the specimen No. 306 engine in the preceding Table. The original boiler of this engine, constructed in the ordinary manner, was removed after $11 \frac{1}{4}$ years' working, as the plates were much grooved and pitted; and a new boiler, constructed with the thick-edge plates, was substituted, which has continued at work $6 \frac{1}{4}$ years. It was then found that the plates were free from grooving, although they were badly pitted.

A consideration of the ordinary construction of locomotive boilers and their defects shows that their construction admits of important improvement in the barrel, by removing the injurious strains resulting from the employment of lap-joints, which throw the plates out of the line of strain, and by making the barrel truly cylindrical and circular throughout. These objects are effected by welding the longitudinal joints of the three rings forming the boiler barrel, and making these rings all exactly the same diameter, uniting them to one another with flush butt-joints. This plan is now carried out upon the Midland Railway, as shown in Figs. 2153 to 2155, and exaggerated in thickness in Fig. 2156. The meeting ends of each ring are turned in a lathe, and united by covering strips O O, formed of welded flush rings, shrunk on over the joints and double-riveted. Strengthening hoops P P are also shrunk on the centre of each of the plates, crossing the longitudinal welded joints, and are secured by a few rivets. These hoops and covering strips for the joints are carefully blocked before being shrunk on, and the whole of the rivet-holes are drilled after the hoops are in their places.

These boilers are consequently truly cylindrical at all parts, and no strain to which they are subjected has any tendency to change their circular form. The effect of the longitudinal strain upon the transverse circular joints, as in Fig. 2157, is found to be altogether inappreciable in practice, because the covering rings 00 could not yield to it without contracting in circumference in the form of a double cone, Fig. 2157; and on this account, to-
gether with their greater thickness, they

2155. offer a greatly increased resistance as compared with simple lap-joints. All possible effect of the longitudinal strain might indeed be entirely got rid of, if desired, by the further addition

of inside covering strips at the butt-joints, as shown in Figs. 2158, 2159. At present the circular plates of these welded boilers are in two semicircular segments for the circumference of the boiler, and therefore require two welds; but Kirtley thinks the barrel of the boiler would be improved if
each length were made of one plate only, whereby only one longitudinal weld would be necessary.


A remarkable corroboration of the correctness of this mode of construction is given by the samples, No. 653 engine, the boiler of which was constructed with butt-joints all flush throughout, the transverse joints being covere 1 by external hoops and the longitudinal joints by internal strips. This boiler has been at work nearly nineteen years, having been started in 1847; but the engiue being of smaller size than those now used with trains, has only been employed as a spare engine for some years past. The plates of the boiler, which are the original ones and have never been repaired at any part, are all good, and the grooving has not taken place at the butt-joints, a little irregular pitting alone being visible on the inside of the plates. The boiler has now been cut up only on account of the engine being abandoned from the great length of time it has been worked. The remarkable contrast shown by the freedom of the butt-joints in this boiler from the grooving so universal with the lap-joints in the ordinary boilers appears only to admit of being accounted for by the difference of construction of the joints in the two cases. In another engine of the same class, which was last broken up, after attaining the maximum mileage of 343,000 miles, the boilerplates were very badly grooved at the angle-iron joint at the smoke-box end, showing that this part of the boiler remained as defective as in the ordinary boilers, the construction being the same as regarded this joint; while the rest of the joints being butt-joints were free from the grooving.

The flanging, bending, and welding of the thick-edge plates for forming the boiler barrel are performed by the aid of machines specially designed for the purpose, which are shown in Figs. 2160 to 2172.

The Flanging Machine is shown in Figs. 2160 to 2162. It consists of a horizontal table A, on which the thick-edge plate, shown black, having been previously heated, is laid and secured by

clamps, being pushed forwards against the adjustable stop B, Figs. 2161, 2162, so that the thick edge projects leyond the edge of the table to the required extent for forming the flange. The
roller C is then brought down with a slow motion by the eccentrics D, as shown in Fig. 2161, being firmly held by guides E at each end in the frame of the machine; and the edge of the plate is thus gradually bent down to form the flange. The table $\mathbf{A}$ is made to slide upon the bed F of the machine, and is set up by adjusting screws $G$ to the required amount of clearance from the bending roller B , according to the thickness of the plate to be flanged. The front edge of the table is faced with a separate wrought-iron or cast-iron edge-piece I, which can be removed and changed for another having a different curve for the edge, according to the curve that is desired in the neck of the flange. The holding-down bar H is screwed down tight upon the plate, immediately behind the edge of the table, so as to hold the plate down flat on the table while the flange is being bent by the roller. The working speed of this machine is seven double strokes per minute.

The Bending Machine, for bending the thick-edge plates into the semicircle to form the boiler barrel, is shown in Figs. 2163, 2164. It consists of three horizontal rollers, of which the two lower ones
2163.



A A are carried in fixed bearings at each end in the frame of the machine; while the third roller $\mathbf{B}$ slides vertically in the frame, and is lowered by the screws C C at each time of passing the plate through the rolls, to give the required degree of curvature to the plate. The screws C C were at first worked by hand, but are now driven by gearing from the main shaft. As the thickness of the body of the plate is only $\frac{7}{16} \mathrm{in}$., while the thickness of the edges is $\frac{5}{8} \mathrm{in}$., a liner-plate, $\frac{3}{16} \mathrm{in}$. thick, is laid over the body of the boiler-plate in the bending process, in order to make up the same thickness of $\frac{5}{8} \mathrm{in}$. throughout for passing through the rolls; and the liner-plate is afterwards flattened again ready for subsequent use. At one end of each of the lower rollers A A is a groove D to receive the flange of the plate; this groove is shown enlarged in Fig. 2165, and is formed by a glut-piece or ring E, screwed upon the roller-spindle $\mathbf{F}$ and tightened by a set screw $G$, by means of which the width of the groove can be increased or diminished according to the thickness of the flange of the plate. A corresponding groove is provided at the opposite end of the upper roller B, to allow of bending plates with the flange inside instead of outside. In order to obtain a sufficient hold upon the plate to pass it through the rolls, the surfaces of all the rollers are fluted longitudinally with shallow flutes at $1 \frac{1}{4}$ in. pitch, as shown in Fig. 2165, and enlarged to half full size
2165.

in the section, Fig. 2166. The lower rollers only are driven by gearing, the upper roller being merely a pressing roller for giving the required curvature to the plates, and weighing about 25 cwt . The working speed of the rollers is three revolutions a minute, or about 12 ft . a minute speed of surface.

The two semicircular plates are then welded together into a single ring, to form one length of the boiler barrel. The edges to be welded are first heated in the fire at A, Fig. 2167, and upset sufficiently to give the required thickness of metal for forming the scarf weld. A welding heat is then taken on a short length of the joint of the plates, and the plates $B$ are welded together along the joint upon the welding anvil, shown in Figs. 2167, 2168. The anvil-face $C$ is shaped to the internal diameter of the boiler barrel, and is separate from the pedestal D of the anvil, so that it can he exchanged for other sizes of face, according to the diameter of the boiler. During the heating and welding, the plates B are held in the circular frame E, in which they are securely clamped; and the frame E being slung from a crane, the plates are readily handled. The two ends of the joint are first tacked together by welding, to secure the correct diameter of barrel, and the joint is then welded in short lengths from the centre towards each end.

The rings of the boiler barrel, when welded up, are blocked, to test them by stretching and bring them to the diameter of the boiler. For this purpose the rings are first heated in the

blocking furnace shown in Figs. 2169, 2170, having the fire-grate A under the centre, with six chimney-flues C C round the circumference. The ring B to be heated is put in from the top, and placed on end, with the heat from the fire passing up through the inside of the ring and then down all round the outside to the flues, so as to give a uniform heat to the ring.


The ring is then put on the blocking press, shown in Figs. 2171, 2172; which is an ordinary hydraulic wheel-tire blocking press, worked by a centre cone $\mathbf{D}$ forcing out the blocking segments


E E. These blocking segments are carried up for the purpose and strengthened by brackets, as shown in the drawing. One half of the height of the ring B is blocked at once; and the ring is then turned over for blocking the other half.

The welded joints of these boilers have been tested by a series of experiments upon the tensile strength of strips of plate cut out across the weld, which were taken from several boilers from the
opening cut out for the steam dome R, Fig. 2153. Three sets of strips were tested, of $1,1 \frac{3}{8}$, and $1 \frac{1}{2} \mathrm{in}$. width respectively, and each $7 \frac{1}{2} \mathrm{in}$. length, cut out of the plate transversely to the weld, which was in the middle of each piece. The following was found to be the average breaking streugth a sq. in. of these strips;

Experiments to test Strength of Welded Joints.

| Wijdth of strips. | No. of Strips tested. | Broke in Weld. | Broke in Sulid. | Breaking Strength per square inch. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Least. | Greatest. | Average. |
| inch. | 15 | 8 | 7 | tons. | tons. | tons. |
| 1 | 15 |  |  | $16 \cdot 5$ | $23 \cdot 8$ | $20 \cdot 2$ |
| $13 \frac{3}{8}$ | 4 4 | 2 1 | 2 3 | $19 \cdot 6$ $18 \cdot 1$ | $22 \cdot 2$ $23 \cdot 5$ | $\begin{aligned} & 21 \cdot 0 \\ & 21 \cdot 7 \end{aligned}$ |
| Total | .. 23 | 11 | 12 | $16 \cdot 5$ | $23 \cdot 8$ | $20 \cdot 6$ |
| Also | .. 11 Str | of the <br> elded.. | e plate | $20 \cdot 7$ | $25 \cdot 8$ | $23 \cdot 6$ |

From these results it appears that more than half of the strips broke in the solid and not at the weld, and the average breaking strength of the twenty-three welded plates was within one-eighth of the full strength of the eleven unwelded plates; while the worst pieces, including some cases of as extremely defective weld as are at all likely to occur in practice, had more than two-thirds of the full strength of the unwelded plates.

In reference to the cost of construction of the welded boilers in comparison with the ordinary class of lap-jointed single-riveted boilers with angle-iron ends, it has to be noted that there is an increase of weight of $1 \frac{1}{4}$ ton in the new boilers, the weight of the $11-\mathrm{ft}$. boilers, 3 ft .11 in . diameter, being $7 \frac{3}{4}$ tons as compared with $6 \frac{1}{2}$ tons in the old class of boilers of the same dimensions. This increase arises from the thick-edge plates, and from the hoops and joint strips, which weigh about $2 \frac{1}{4}$ cwt. each; and the joints, instead of being single-riveted, are double-riveted on each side of the joint, making four rows of rivets.

There have, says Kirtley, been nineteen of these welded boilers in constant use upon the Midland Railway for the last $6 \frac{1}{2}$ years, and the result has proved so thoroughly satisfactory that this construction has now been permanently adopted for the engines of this line. Up to the present time (1866) all these nineteen boilers have been examined once, and have been found in good condition; the mileage of each during the $6 \frac{1}{2}$ years that they have been running has been equal to about 175,000 miles, and each boiler has had one set of tubes worn out.

Anti-corrosive Paints.-Tarr and Wenson's anti-corrosive paint for ships' bottoms consists of a composition made by reducing an alloy of zinc, tin, iron, and quicksilver to powder, and adding to the mass 20 per cent. of white arsenic. This is mixed with a composition of 40 gallons of wood tar, 30 gallons of coal naphtha, and $\frac{3}{4} \mathrm{lb}$. oxide of iron.
'G. W. Morse's paint for ships' bottoms is composed of antimony, 80 parts ; lead, 15 ; cementcopper, 5 ; naphtha, 1 ; benzine, 1 ; and tar, 2.

See Bollers. Construction, p.1053. Galvanized Iron. Increstation of Bollers. Kyanizing. Locomotives. Stone, artificial.

Cotter. Fr., Clavette ; Ger., Keil ; Ital., Bietta, zeppa; Span., Costilla.
A cotter is a wedge-shaped piece of wood, iron, or other material, used for fastening the parts of a structure; a key.

COTTON GIN. Fr., Machine à éplucher le coton; Ger., Fieissuolf; Ital., Macchina da nettar cotone; Span., Desgranadora.

See Cotton Machinery.
COT'ON MACHINERY. Fr., Machines à flature de coton; Ger., Spinnerei Maschinen ; Ital., Macchine da lavorare il cotone; Span., Maquinaria para fabricar algodon.

John Platt, of Oldham, in a paper printed in the Proceedings of the I. M. E., observes that the process of spinning involves three essential and distinct operations;-

1st. Drawing, in which the fibres of the raw material are drawn out longitudinally, so as to lay them all parallel with one another, and overlapping at the ends; as is done by the fingers of the hand spinner for forming a continuous sliver out of the short fibres lying irregularly in the bundle that is tied upon the distaff.

2nd. Twisting, in which the sliver previously formed is twisted into a roving or thread, for giving it longitudinal tenacity by increasing the lateral friction between the fibres as is done by the hand spinner by twirling the bobbin on which the portion of thread already twisted has been wound.

3rd. Winding, in which each portion of the thread, after it has been sufficiently twisted, is wound upon the bobbin.

In the application of machinery to the performance of these operations, the great difficulties experienced have arisen from the irregular character of the cotton fibre on the one hand, and on the other from the unyielding action of machinery, which has to take the place of the delicate feeling of the fingers in hand spinning, whereby the spinner is enabled to accommodate the action continually to the variations in the material. It is a point of special mechanical interest, however, to note at how early a period in the application of machinery correct ideas were developed as to the principles of action in the important successive steps; so correct indeed that they have
remained unaltered in principle to the present time, although many highly ingenious improvements in detail have subsequently been effected.

In Paul's first machine the raw cotton was passed through a succession of pairs of rollers, each pair running faster than the precedins, so as to draw out the sliver of cotton longitudinally to any degree of fineness required. The machine thus accomplished only the drawing process, leaving the sliver so formed to be twisted aud wound afterwards by hand. The great feature of this invention was that the important principle of drawing by rollers running at different speeds was thus established at the outset, to supersede drawing by the fingers in hand spinning; and this mode of drawing has been adhered to ever since as the fundamental principle in the preparation of fibrous materials for spinning.

Paul invented, also, a carding machine, for carding or combing the raw cotton in preparation for the drawing rollers. It consisted of a number of flat parallel cards fixed upon a table with spaces between them; and the teeth of the cards being all bent in the same direction, the cotton was carded by being drawn over them by hand by means of an upper flat carding board set with teeth bent in the opposite direction. In another arrangement this flat upper card was replaced by a horizontal carding cylinder, made to revolve by hand; and the lower carding table was made concave to fit the under-side of the cylinder. When the cotton was sufficiently carded, it was taken off each card separately by hand by a needle-stick, and then comnected into one entire roll or lap.

Paul improved his original machine, in 1758, by rendering it capable of performing the two other processes of twisting and winding requisite to complete the operation of spinning by machinery; and he constructed a spinnin. machine having a circular frame containing fifty spindles. The cotton was drawn by rollers, as in his previous machine, and the sliver was delivered from the rollers to a bobbin upon each spindle, by means of an arm or flier fixed upon the spindle; and the spindle being so contrived as to go faster than the bobbin, the sliver was thus twisted into thread by the flier, before being wound upon the bobbin.

Although the two mechanical principles which have formed the basis of all subsequent spinning machinery-namely, the drawing rollers, running at different speeds, and the differential motion of the flier and bobbin - were thus originated by Paul, it does not appear that his machines were ever practically successful; and Arkwright's spinning machine, in 1769, appears to have the merit of being the first that was brought into successful operation. This machine cannot be called more than an improvement in detail upon Paul's, as the principles of the two were the same; and it is difficult to imagine that Arkwright had not seen Paul's machine. The success of the later machine may be attributed to its superiority both in workmanship and in the material employed, the earlier machine having been com posed almost entirely of wood.

In 1770 Hargreaves invented the Spinning Jenny, shown in Fig. 2173, the principle of which is identical with that of the present spinning machinery. It thus presents a remarkable instance

of a correct perception respecting the best mode of working having been attained at so carly a stage in the application of machinery to a new purpose. The operation of spinning into threads the rovings produced by the machines of Paul or Arkwright, or by the modern improved machines similar in principle, comprises the two processes of twisting and elongating the roving to form it into a thread, and then winding the spun thread into the form of a cop upon the same spindle by which the spinning or twisting has been performed. These two processes still continue to be effected in essentially the same manner as in Hargrea ves' spinning jenny.

The twisting of the thread is effected by causing the spindles $\mathbf{C}$, Fig. 2173 , to revolve, as though
for winding up the thread, but allowing the thread to slip off the free end of the spindle once in each revolution. For this purpose the thread is led off from the top end of the spindle at an angle so much greater than a right angle that its tendency to wind in a spiral brings it to the top extremity of the spindle in each revolution, causing it to slip off the end of the spindle at each successive revolution; and the top of the spindle is shaped conical to facilitate the slipping of the thread off the end. The result is that the thread is twisted one turn by each revolutinn of the spindle, witiout disturbing or interfering with the portion of spun thread already wound up into a cop on the lower part of the spindle. The cross-bar J, carrying the guiding eyes through which the several threads pass, rests at each end on a carriage that runs along the side framing of the machine; and before the commencement of the spiming by the spindles $G$, the cross-bar is first drawn backwards from the spindles by hand through about one-third the length of the machine, drawing off a continuous supply of roving from the bobbins $K$ below, which are free to turn oil their bearings. The clasp $H$ is then pressed down tight upon the cross-bar $J$, holding the rovings fast, and the spindles $G$ are set in motion, twisting the lengths of thread between the spiudles and the cross-bar; and during the twisting the cross-bar is gradually drawn backwards by hand to the end of the machine, thus producing the required elongation of the threads by tension during the spinning, as is done in the case of hand spinning by the weight of the bobbin or spindle hanging from the twisting thread. The spindles G receive tieir motion from the drum M driven by the driving pulley L, which is turned with the right hand by the handle N, while the left hand draws back the cross-bar $\mathbf{J}$ by means of the handle upon the clasp $H$.

When the cross-bar $J$ has been drawn back to the extreme end of the machine and the spinning of the threads has been completed, they are then wound up on the spindles by depressing them all simultaneously to the lower portion of the spindles by means of the faller wire 0 , which is brought down upon the tireads by the rotation of the dises P P in the direction of the arrow. The rotation of the discs is effected by tightening the cord $R$ which runs along the side of the machine, and they are turned back again by a counterbalance weight for raising the faller wire when the cord is released; the cord passes round three horizontal pulleys on the top of the carriage $\mathbf{S}$, and is tightened for depressing the faller wire by a transverse sliding movement being given to the middle pulley by means of a hand lever, which is worked by the left hand whilst holding the clasp on the cross-bar J. The threads being depressed by the faller wire, the further rotation of the spindle now cuses the threads to be woind up in cops upon the spindles, the sliding cross-bar $J$ being pushed forwards gradually by hand as the winding proceeds, until it again reaches the spindles, when it is ready for beginaing the spinning of a fresh length of rovings. During the winding of the threads already spun between the spindles and the cross-bar J, this length of the threads is secured and separated from the untwistel rovings beyond the cross-bar by the pressure of the clasp H, which is kept pressed down tight upon the threads.

On the completion of the spinning, however, of each length of the threads, and before the change can take place from spinning to winding, it is necessary first to unwind the short spiral of thread extending up from the top of the cop previously wound to the top extremity of the spindle. This spiral is unavoidably formed at the commencement of the twisting, before the thread can reach the point where it ceases to wind and begins slipping off the end of the spindle at each revolution; but if this portion of thread were not entirely removed before the faller wire $O$ is lowered at each time of changing from twisting to winding, an irregular and loose accumulation of thread would take place upon the upper end of the spindle, spoiling the form of the cop and interfering with the proper slipping-off action in twisting. The motion of the spindles has therefore to be stopped and reversed for a few turns when the twisting is finished, to unwind these few spiral coils; and this was done in the spinning jenny by the spinner stopping the driving wheel L, and then giving it a partial turn backwards by ha'l, for backing off the thread before driving forwards again for winding the thread on the spindles.

Tuis backing-off motion and the faller wire are identical with those now in use in the modern spinning machines, the only difference being that they are now made self-acting. In winding the cop each successive layer of thread is so regulated that a conical form is given to each end of the finished cop, in order to prevent the thread from ge tting loosened upon it at the ends in subsequent handling; while at the same time the crossing of the thread in the alternate spiral layers gives firmness to the cop, and still allows the thread to be afterwards drawn off it, when required for use, either by slipping off the end as in a shuttle, or by unwinding the cop on a spindle.. In the spinning jenny this shape of cop was obtained by regulating the winding of the thread by means of the cord $R$ acting upon the discs P P, raising and lowering the faller wire $O$ during the winding so as to guide the thread upon the spindle as requir ed for producing the desired shape of cop. The same shape of cop and mode of guiding the thread on are still adhered to in the present syinning machines; but the whole of the movements are now effected entirely by self-acting machinery.

Further improvements in the preparatory processes of carding and roving were introduced by Arkwright in 1775, which may be said to include the principal features contained in the carding and roving machines now used. The cotton delivered from the preliminary machine was formed into a roll or lap, for supplying a continuous fleece of cotton to the carding cylinders, the carding operation being repeated until the irregular mass of fibres in the raw material had been combed straight and laid parallel in the fleece of cotton with a sufficient degree of uniformity to allow of proceeding to the subsequent operations. Comb-plates worked backwards and forwards by cranks were also added for combing off the cotton in a continuous fleece from the dofier or taking-off cylinder of each of the carding machines. The sliver delivered from the last carding process was passed between a pair of rollers for the purpose of consolidating it by the pressure of the rollers after the loosening action of the doffing comb-plate; and it was then coiled down into a can.

The doubling and drawing process employed at this stage of the manufacture was also introduced by Arkwright at the same time, the object being to intermingle the fibres more completely in the siver, and thereby render it more uniform in quality, ready for twisting into a roving. For
this purpose two or generally more of the slivers from the carding engine are passed side by side through a series of pairs of drawing rollers, each pair in succession being made to run faster than the preceding; and the last pair of rollers runs as many times faster than the first pair as there are slivers doubled together, so that the single combined sliverdelivered from the rollers is drawn down to the same size or weight a foot as one of the original slivers. The doubling and drawing operation is usually repeated three times, and the ultimate sliver so prepared is then ready for the roving frame, in which it is again drawn and twisted and wound upon a bobbin.

In the roving frame Arkwright now effected an important advance upon his previous machine, by introducing the new principle, which has since been adhered to, of driving the bobbin and the spindle inde-
 pendently by separate motions, instead of letting the bobbin be simply dragged round by the thread. At the same time he also introduced the conical regulating drum, for reducing the speed of the bobbin in proportion as its diameter was increased by the thickness of the coils of roving wound upon it, so as to avoid increasing the tension upon the roving, as would be the case if the speed of the bobbin were uniform, since the spindle and flier are driven always at a constant speed. This mode of regulating the speed of the bobbin by a conical drum is the same that is still employed in the modern machinery, with modifications only in the mode of application.

Opening.-The first process in the preparation of the raw cotton was known as willowing, and took its name from the willow switches formerly used for beating or opening the tufts of cotton by hand. A number of small parallel cords were stretched tight and close together on a horizontal frame, so as to form a sort of table, upon which the cotton to be opened was laid; and the cotton being then beaten with switches made of willow rods kept smooth for the purpose, the fibres of the cotton remained on the cords,

principle, having revolving cylinders to act as beaters. The most primitive of these is known as the Oldham Willow, and has a revolving cylinder about 36 in . diameter, set with spikes placed in parallel rows, and revolving against a grid of bars set with similar spikes; the cotton is fed upon the grid, and beaten for a longer or shorter time, according to its condition, and an exhausting fan is employed for taking away the sand and bits of dried leaves beaten out. This machine is now used only for separating hard lumps of cotton in bales packed too tightly, and for cleaning cotton waste and the refuse cast out by other cleaning machines.

Fig. 2174 shows a section of the Cotton Opener in use at the present time for the purpose of opening out the fibres of the cotton after it has been pressed in the bales, and for extracting the sand, dried leaves, and other impurities imported with it, the object being to do this without entangling or injuring the fibre. The crude cotton from the bales is spread by hand upon the endless travelling lattice A, which conveys it underneath the iron guide-roller B with longitudinal ribs on its surface to the pair of fluted feed-rollers C. These are pressed together by the weighted lever D, and deliver the cotton to the picker-cylinder E, set with twelve rows of teeth, which are spaced so that the teeth follow one another spirally round the cylinder, as shown in the plan, Fig. 2175. The tufts of cotton being gripped tight between the rollers $\mathbf{C}$ are caught by the tips of the teeth on the cylinder revolving in the direction of the arrow, and are thus torn open and dashed by the teeth against the circular grid F, formed of angular bars set with spaces between them, which allow the dirt disengaged by the beating action to fall through. A perforated plate G forms the remainder of the casing of the cylinder on the under-side, allowing the dust to drop through while the cotton passes over it. The picker-cylinder delivers the cotton against the teeth of the beater H, which has four rows of teeth, similar to those on the picker E. Here the cotton is further beaten and passed over a circular grid and perforated plate; and the beater-cylinders being covered in at the top by a sheet-iron casing, the current of air produced by their revolution wafts the light fleece of cotton forwards over the straight grid J. It is whilst the cotton is thus floating in the air that the heavier impurities, loosened by the beaters, drop out and fall through the grid $J$ into the dust-box. The cotton then passes between the two wire-gauze cylinders K, which serve as fine sieves, the interior of the cylinders being exhausted by the fan $L$; by this means the more minute particles of dust remaining in the cotton are sifted out, and discharged by the fan through the aperture M, thereby keeping the rooms where the machines are at work perfectly free from dust. There is thus a continual deposit of impurities taking place throughout the whole passage of the cotton through the machine, from the feed-rollers C to the wire cylinders K. In the drawing, only two beater-cylinders E and H are shown; but the machines are more generally made with four cylinders, for cleansing the cotton more effectually, the two additional beaters being provided with four rows of teeth, the same as the second cylinder H. From the wire cylinders the loose cotton is collected again and consolidated into a fleece by the fluted stripping rollers I, running close to the cylinders K, but not touching; and these deliver it to the travelling lattice N , which discharges it ready to be taken to the next process of scutching, a lap machine being sometimes attached, so as to form the fleece into a lap or roll for supplying the scutcher. The beater-cylinders $E$ and $H$ run at about. 1000 revolutions a minute; the feeding lattice $A$ travels at 6 ft . a minute, which is also the surface speed of the feed-rollers C ; and the surface speed of the wire cylinders K , the stripping rollers I , and the delivery lattice N , is 60 ft . a minute.

Scutching:-The Scutching Machine now in use for further beating and cleansing the cotton delivered from the opener is shown in Fig. 2176, having also combined with it the Lap Machine for forming the fleece of cotton delivered from the scutcher into a roll or lap. The cotton from the opener is supplied to the scutcher upon the travelling feeding lattice A, and in order to produce a uniform fleece for the further processes, it is necessary at this stage to regulate the quantity fed into the scutcher, which is effected in two ways. In feeding by hand the tedious method of weighing the cotton supplied has to be resorted to, so as to distribute a uniform weight of cotton over each foot of length of the feeding lattice, which together with the feed-rollers C is driven at a uniform speed. But in the improved mode of feeding by laps B, supplied from a lap machine in connection with the opener, the top feed-roller is allowed to rise and fall, according to the variations in the thickness of cotton fed in, and the amount of its vertical movement multiplied by means of levers is employed to regulate by a self-acting arrangement the speed at which the feeding lattice and rollers are driven. By this means an almost uniform supply of cotton is fed to the beater H, which is composed of three plain bars, as shown in the plan, Fig. 2177, and is driven at about 1250 revolutions a minute. The cotton is beaten, as before, against the circular grid $\mathbf{F}$ and perforated plate G, and the current of air from the beater wafts it forwards over the straight grid $\mathbf{J}$ to the wire cylinders $\mathbf{K}$, exhausted by the fan $\mathbf{L}$, the action being exactly the same as in the opening machine. The rollers I, which strip the dust-cylinders, deliver the fleece to a set of four callender rollers 0 , placed over one another, so that the cotton in passing through them receives three compressions, which consolidate it into a kind of felt; the surfaces of the callender rollers are kept clean by rubbers of iron covered with flannel, which are pressed in contact with them.

The Lap Machine, for coiling the fleece into a roll or lap, has two fluted driving rollers P P running in the same direction, as shown by the arrows; and the lap, resting in the channel between them, is driven by contact, and wound upon the iron rod $R$, guided in a vertical groove at each end. For tightening and closing the coils of the lap, the rod R was weighted in the former machines by a heavy weight suspended from the ends of the rod, and it was then necessary to lift the whole of this weight at each time of changing the lap; but this is now effected by the friction brake $S$ pressing against a friction wheel on the shaft $T$, on which are pinions gearing into the vertical racks $U$, and these racks carry rollers at the top, bearing down upon the ends of the rod $R$ in the lap. By this means, as each successive coil is wound upon the lap, the brake S slips and allows the lap to rise; and when the lap is completed, the brake is released by a treadle, and the
racks are lifted clear by the hand-wheel on the shaft $T$, so as to allow the finished lap to be removed. The driving pinion $V$, from which the callender rollers $O$ receive their motion, is held up in gear by the lever W supported by a catch; and when the lap is finished this catch is released by a tappet upon the pinion X , which is driven from the bottom callender roller, the speed of the pinion being so reduced that one revolution of it corresponds to the size of lap required to be made. The catch being released allows the driving pinion V to fall out of gear, whereby the callender rollers 0 are stopped; and the laprollers P continuing to revolve, break off the fleece, ready for removing the finished lap from the machine. By changing the pinion $\mathbf{X}$, carrying the tappet, the size of lap made by the machine can be varied as desired.

The scutcher shown in Fig. 2176 is a single machine, and it is usual to pass the cotton first through a double scutcher of similar construction, but having a second set of feed-
 rollers with beater and wire cylinders, running at a higher speed; the second pair of wire cylinders and stripping rollers are driven three times as fast as the first pair, so that they deliver a fleece of one-third the thickness first supplied to the machine. Three of the laps from this double scutcher are then fed into the single scutcher shown in the drawing, as at BBB, Fig. 2176, being spread upon the surface of the feeding lattice $A$ in three layers on the top of one another, so as to present to the feed-rollers a uniform fleece equal in thickness to that fed into the first scutcher. By thus doubling the laps the fibres are more thoroughly mixed, and the fleece is thereby made more uniform in thickness.

Scutching machines having a revolving beater, composed of two plain bars describing a circle of about 14 in. diameter, were introducel about 1810; and these machines contained also a travelling feed-lattice, two pairs of feedrollers, and a second travelling lattice for conveying the beaten cotton underneath a perforated revolving cylinder, the interior of which was exhausted by a fan. The cotton passing through this machine was delivered in a loose fleece, and a few years later a lap machine was ad led for coiling the fleece into a lap. Other improvements have gradually been introduced up to the present time, as regards both design and workmanship; the cylinders and beaters have been put in perfect balance so as to revolve steadily at the high speed required, and the forms of teeth on the cylinders have been arranged for greater strength and greater facility of construction; stronger and simpler gearing has been employed, improvements have been made in the form and construction of the bearings of the beaters and other quick revolving shafts, so as to ensure more efficient lubrication,
and air-tight dust boxes have been added with movable doors for facility of cleaning ; and the self-acting arrangements have been introduced for stopping the machine when a given length of fleece has been delivered, and for regulating the rate of feed according to the thickness of the cotton supplied, so as to dispense with the previous plan of weighing the cotton in feeding. Thus by successive improvements through a long series of years the difticulties which originally presented themselves in the successful adaptation of machinery to cotton cleaning have been overcome.

Carding.-In the carding process the felted fleece delivered by the lap machine of the scutcher, with its fibres crossed in all directions, is combed out a great number of times so as to straighten the fibres; and the light impurities still adhering to it are taken out, such as short fibres and bits of the moss-like covering of the seeds, which if allowed to remain in the sliver produced by this operation would give a roughness to the yarn. For making coarse yarns one carding process only is employed; but for finer yarns the fleece is first passed through a breaker carding engine, which performs the first rough carding, and the slivers delivered by this are then doubled by laying together a large number of slivers, side by side and overlapping one another, into a new fleece, so as to obtain sufficient thickness and breadth of material to allow of a further carding; and the lap formed of this new fleece is then fed into a second or finisher carding engine. As many as ninety-six slivers from the breaker card, each drawn out of a separate can, are laid together by the doubling machine into a single fleece for the supply of the finisher, in order that the mixing of the cotton may be more thoroughly effected, and more perfect uniformity ensured in the sliver delivered by the finisher. For the finest qualities of yarn the finisher card is itself used as a breaker, and the sliver delivered by it is afterwards combed by a combing machine.

The Roller and Clearer Carding Machine employed at the present time as a breaker card for performing the first carding of the fleece is shown in section in Fig. 2178, and consists of a main carding cylinder A, round which are arranged a series of pairs of carding rollers or workers B B and clearing rollers or strippers C C. The surfaces of all these are covered with cards, and they are made to revolve so close together as to allow the tips of the card teeth just to clear one another. The cards are a kind of wire brush with inclined teeth, as shown full size in Fig. 2179, and are made of staples $D$ of fine steel or iron wire, each about $\frac{3}{8} \mathrm{in}$. long and $\frac{3}{16} \mathrm{in}$. wide, with a side bend in the middle of their length. These are fixed close together into a strip of webbing about $1 \frac{1}{2}$ in. wide, which is wound tight round the cylindrical rollers in a continuous spiral, keeping the staples pressed home in the cloth by the surface of the cylinder, so that they have an elastic firmness which keeps their points up to the work.

The working width of the machine is about 40 in . on the card teeth, corresponding with the breadth of the fleece in the lap $\mathbf{E}$ by which the carding engine is fed. The unlapping of the fleece is performed by the rollers $F$ on which the lap rests, and the fleece is then drawn forwards under the feed-roller $G$, and delivered to the taker-in roller $H$ revolving in the direction of the arrow. At this point the carding or combing action commences, the fleece being held by the feed-roller G travelling at the slow speed of only about $\frac{3}{4} \mathrm{ft}$. of surface a minute, while the taker-in H runs much faster, at about 800 ft . a minute surface speed; and the carding teeth on the taker-in being bent forwards in the direction of motion, the points of the teeth strike down into the fleece held by the feed-roller, and comb out the fibres, while the impurities separated fall to the ground. The fibous tufts of cotton are carried round on the under-side of the taker-in to the main carding cylinder A, which revolves in the same surface direction with a speed of about 1600 ft a minute. The teeth of the carding cylinder being bent forwards in the direction of motion sweep off the cotton from the taker-in teeth inclined in the same direction but running at only half the speed, and carry it forwards to the dirt-roller $J$, the teeth of which face those of the carding cylinder, and travel with a very slow motion of only about 16 ft a minute. The dirt-roller thus assists in combing out the fibres, and holds in the interstices of its wires any impurities that it receives from the cotton, which are carried forwards and stripped from it by a vibrating comb, so that they accumulate in a roll on the upper surface of the dirt-roller, to be taken away by hand at intervals.

The carding cylinder then carries the cotton forwards to the several pairs of workers and strippers, one of which is shown to a larger scale in Fig. 2180; and at each pair in succession the fibres undergo a further combing out and straightening. The n otion of the teeth of all these pairs of rollers is in the same direction as that of the adjacent teeth on the main carding cylinder, as shown by the arrows in Fig. 2180, but at a much slower speed; and the teeth of the strippers C are inclined forwards in the direction of motion, while those of the workers B are set the opposite way so as to present the points of the teeth facing those on the carding cylinder A. The cotton on the carding cylinder is therefore carried past the stripper $C$ without being caught by its teeth, and is caught upon the teeth of the worker B running at only about 20 ft . a minute, so that a combing action for straightening the fibres and dividing the tufts of cotton is obtained by the excess of speed in the carding cylinder running at the high velocity of 1600 ft . a minute. All fibres failing to pass the worker B are carried round upon its teeth to the stripper C , which runs at a surface speed of about 400 ft a minute, being thus intermediate in speed between the slow worker B and the quick carding cylinder A; the teeth of the stripper therefore sweep off the cotton from the worker, and are themselves stripped in the same way by the carding cylinder running at the higher speed. After passing the six pairs of workers and strippers, the fleece of straightened fibres is taken off in a continuous sheet from the carding cylinder A by the doffer K, the teeth of which face those of the cylinder and move in the same direction but at a much slower speed of only about 65 ft . a minute; the fleece thus receives a further straightening and stretching in quitting the carding cylinder, and is carried round on the under-side of the doffer to the vibrating comb I, which describes a slort are of $1 \frac{1}{4} \mathrm{in}$. vertical motion and is driven by balanced cranks at about 800 double vibrations a minute. This comb strips the fleece from the face of the doffer in its down-stroke and clears itself in rising; and the thin fleece, of the full width of the machine, 40 in ., is then gathered in by lateral guides to a width of 6 in., and finally into a smooth bell-mouthed round funnel L , having a hole only $\frac{1}{2} \mathrm{in}$.
diameter, through which the contracted ribbon or sliver is drawn by the two pairs of drawing rollers M, the second pair running one-half faster than the first, whence it passes to the coiler $\hat{N}$ and can 0 .

The coiler consists of a revolving plate N having an eccentric aperture, through which the

sliver is passed from the pair of rollers P above the plate, so that it is delivered into the can in circular coils. The can O, however, is also made to revolve with a slow motion in the opposite direction to the coiler, and the centre line of the coiler $\mathbf{N}$ is eccentric to the axis of the can, whereby the sliver delivered from the coiler describes a succession of hypocycloid curves in the can, the circles of sliver being laid into the can so that the outsides of the coils touch the inside of the can.

The sliver thus forms coils continually crossing one another, so that the can is filled up solid throughout, and when taken to the doubling frame the coils of sliver come out again without adhering to one another.

The Flat Carding Machine employed at the present time as a finisher card for performing the second carding operation is shown in Fig. 2181; and consists of a main carding cylinder A, as in the breaker card, but the pairs of workers and strippers employed in the first carding are here ro-

placed by a series of flat cards D D, connected together by links so as to form an endless travelling lattice. The lap E, formed of a number of slivers from the breaker card laid together into a fleece by the doubling machine, is supplied to the carding cylinder A by the feed-roller G and taker-in H , in the same way as in the breaker card; and the carding cylinder A is driven at the same speed of about 1600 ft . of surface a minute. A single worker B, called the fancy roller, with a stripper C, is placed immediately beyond the taker-in H, running in the same direction as the adjacent surface of the carding cylinder $\mathbf{A}$; but in this case the teeth of the fancy roller $\mathbf{B}$ are bent forwards in the
direction of motion, and it therefore requires to be driven at a higher velocity than the carding cylinder, and has accordingly a surface speed of 2000 ft . a minute. It thus seizes the cotton from off the teeth of the main carding cylinder A, and throws it against the teeth of the stripper C facing those of the fancy roller - and the fibres having thus been subjected to a preliminary carding are again swept off the teeth of the stripper, moving at only 400 ft . a minute, by the higher speed of the main carding cylinder A.


The cotton is then carried forwards by the carding cylinder to the series of flat cards $\mathrm{D} D$, Fig. 2182, which are made of cast-iron bars faced with card teeth, as shown to a larger scale in Figs. 2183, 2184; these extend the entire width of the machine, and rest at each end upon a cir-
cular guide on the top of the side frames of the machine, which is concentric with the main carding cylinder A, so that the teeth of the flats are kept in close proximity to the teeth of the carding cy!inder during the whole of their forward traversc. The teeth of the flats D are set to face those of the carding cylinder A, and travel forwards in the same direction as the surface of the cylinder, but at a very slow rate of only 1 in . a minute: and the cotton thus undergoes a very thorough carding and straightening in passing the twenty-one cards that are always in contact with the top of the carding cylinder. The flats are arranged to work at a slight inclination to the surface of the carding cylinder, so that the delivering side of each flat is closer to the cylinder, and a wider space is left at the entering side between the flat and the cylinder for the cotton to enter, as shown half full size in Fig. 2184. The angle thus formed is called the bevel of the flat, and the correct adjustment of this inclination is a point of great importance and delicacy; the bevel is obtained by cutting a bevel groove $Q$ in each end of the that at the part where it is to rest upon the circular guide on each side of the machine, as shown enlarged in Figs. 2183, 2184, where the dotted circle $\mathbf{X} \mathbf{X}$ represents the edge of the guide on which the flat travels, and the dotted circle Y Y indicates the surface of the main carding cylinder A .

The endless lattice of flats D D is carried over the three shafts P P P , and on quitting the carding cylinder A, each flat in turn is stripped of any fibres or impurities adhering to it by the vibrating comb $R$, which describes an arc of 1 in ., and is driven at the rate of forty double strokes a minute by the cam S, Fig. 2185. The flats are further cleaned by the brush T, shown in plan in Fig. 2186, running at a surface speed of 50 ft . a minute; and they are then passed over a guide U , which holds them up against an emery wheel V , running at the high speed of 550 ft a minute, and traversing across the machine along the length of the flats, whereby the faces of all the cards are successively ground to a true surface whilst at work, and the points of the wires sharpened. The same mode of grinding is also employed for keeping true the surfaces of the carding cylinder and doffier. The fleece of straightened fibres is taken off in a continuous sheet from the carding cylinder by the doffer K and vibrating comb I, and is contracted into a sliver and coiled down into the can $O$ in the same manner as previously described.

The stripping by hand labour, however, was an unhealthy and a disagreeable process; and bad work and spoiled cotton were the consequences whenever it was not done regularly and thoroughly. Many arrangements have been introduced from time to time for stripping the flats mechanically. Within the last few years a simple mechanism for this work has been introduced by Mr. Wellman, an American, the application of which has received a great stimulus frem the difficulty of obtaining men to perform the stripping by hand; and it is now extensively used. Metal flats were introduced by Smith, of Deanston, and these were linked together in the form of an endless lattice, which was made to travel slowly furwards over the carding cylinder; and each flat was adjusted to the proper bevel by two screws at each end, which travelled over two circular guides concentric with the carding cylinder. Another short flat guide at the back of the fats brought them into contact with a stripping brush, which was cleaned by a comb, and the comb teeth were scraped clear by a tin knife. These flats were used in several of the Scotch mills, but few of them were introduced into Lancashire. The flat calling machine was further improved by Mr. Evan Leigh, by cutting the bevel on the ends of the flats, as shown at Q Q, Fig. 2183, and making the circular guides over which they travel adjustable for wear; at the same time a second face was formed on the back of the flats, to work over the guides U, Fig. 2182, which hold them up in the right position for being ground by the emery wheel V ; and the vibrating comb R was also added for stripping the flats before they are finally cleaned by the brush $T$, instead of the brush alone being used for the purpose.

The finisher carding machine until recently was censtructed without the taker-in roller H , Fig. 2181, the main cylinder taking the fleece direct from the feeding roller G. This caused the fibres to clog the cards, and any impurities passing the feed-roller damaged the teeth of the main cylinder, which was of serious importance on the large extent of surface of the cylinder. By using the taker-in, however, these evils are prevented, the fibres being delivered to the carding cylinder in a more divided state, and more equally distributed over its surface. Carding machines are also sometimes made, which are a combination of the breaker and finisher card, having rollers and clearers on the side of the cylinder next the feeder, and flats on the side next the doffer.

The practical difficulty originally experienced with the carding engine consisted in getting the cards to work sufficiently near to one another without occasionally coming in contact, which destroyed the carding points. The surfaces on which the cards were fixed were generally constructed of wood, and therefore varied with every change of the atmosphere from the shrinking or swelling of the wood, so that the faces of the cards had to be made true each time by grinding down the points of the wires at the full parts. Moreover, the cylinders and rollers were not carefully constructed so as to run with a steady motion; and the fixings for carrying the different journals were not capable of a fine adjustment, nor were they steady after being set. These defects are now overcome by using iron instead of wood, and by the aid of machinery and tools adapted for making all the parts accurately; fine adjustments are provided, and the adjustable portions are made as firm when set as if fixed. These improvements cause less grinding and stripping to be required, as the finer and truer the points of the wires can be maintained, the clearer the cards continue in working.

Drawing.-The slivers from the finisher card are next taken to the Drawing Frame, shown in section in Fig. 2187, which contains generally four pairs of drawing rollers A, each pair running faster than the preceding, and the front pair running at six times the surface speed of the back pair. Six slivers B in separate cans from the carding engine are fed up together to the back pair of drawing rollers, being combined together by passing between two guide-pins C ; and after being laid together and drawn out to six times the original length, the single sliver so produced is passed through a funnel to the pair of callender rollers D, by which it is delivered to the coiler E, and coilcd down into the can F in the same manner as in the carding machines. This combined sliver, having
been doubled six times and drawn six times, is the same weight a foot as each of the original slivers fed up to the back pair of rollers; and the object sought in the doubling and drawing process is to equalize the distribution of the cotton fibres and produce slivers of more uniform strength and texture by the combination. The process is repeated three times in this machine, and the extent of combination or intermixture obtained in the ultimate slivers is therefore represented by the cube of six, or 216 times, in comparison with each of the original slivers first supplied to the machine.

In order to ensure the drawing rollers being always supplied with the full number of six slivers, each of the slivers fed up to the rollers is passed over a guide $G$, turning on a centre pin and nearly balanced, so as to turn with a slight pressure; and during the working of the machine this guide is depressed by the weight of the sliver into the position shown by the full lines. But in the event of the sliver breaking or running out, the tail of the guide, being overweighted, drops into the position shown by the dotted lines, and catches the vibrating finger $J$, which is worked by an eccentric on the shaft H running at 70 revolutions a minute. This shaft is driven at the end

by a small crown ratchet-clutch, held in gear by a spiral spring behind and driving by the inclined faces; so that whenever the shaft $H$ is stopped by the tail of a guide G catching one of the vibrating fingers $\mathbf{J}$ and the ratchet is consequently held stationary, the clutch is thrown out of the ratchet by the inclination of the teeth, and the end-motion thus produced releases a catch which holds the strap-fork of the machine; the fork is then reversed by a spring always acting upon it to throw off the strap from the fast to the loose pulley, thus stopping the machine. After the sliver run out has been renewed or the broken sliver joined up by the attendant, the machine is set in motion again by moving the starting rod I, which extends the whole length of the drawing frame.

The improvements made in the drawing frame since its introduction include, amongst other details, the construction of the roller-supports so as to be easily set and adjusted to the different distances required to suit the different lengths of cotton fibre worked; the addition of the self-acting stop-motion, for stopping the machine when any one of the slivers breaks or runs out; a simpler mode of suspending the weights from the top rollers; and the use of an endless travelling cloth over the surfaces of the top rollers, as shown in Fig. 2187, for cleaning off the waste or fly from the
rollers. The top rollers are also made with dead spindles and loose bosses, the bosses being driven by independent motions. In drawing frames used to prepare slivers for the finer counts of yarn, an additional stop-apparatus is provided for stopping the machine whenever a breakage of the sliver occurs between the front pair of drawing rollers and the callender rollers D, Fig. 2187, or when tho can $\mathbf{F}$ is full or a given length of sliver has been delivered; this stop-action is worked from tho same shaft H as that employed for breakages of the supply slivers.

Slubbing, Intermediate, and Roving.-The slivers delivered from the drawing frame are conveyed to the Slubbing Frame, into which they are fed either single or double, passing over a guide, like that in the drawing frame, to a set of three pairs of drawing rollers; the last pair runs at five times the speed of the first, so that the sliver is again increased in length five times in passing through the rollers. From the drawing rollers the sliver, now called slubbing, passes to a revolving flier carried upon a vertical spindle, by which it is twisted and then wound upon a bobbin revolving loose upon the same spindle. The flier runs at a constant speed, while the bobbin is driven by means of a differential motion at a speed varying according to its increasing diameter as the slubbing is wound on; and the lifting movement, for raising and lowering the bobbin upon the spindle so as to wind on the slubbing in regular coils, is also worked by the same differential motion. The speed of the flier is 500 revolutions a minute, and the slubbing is delivered from the drawing rollers at the rate of 50 ft . a minute, so that the number of twists put into it in this machine is $\frac{5}{6}$ twist an inch of length.

The bobbins from the slubbing frame are next supplied in pairs to the Intermediate Frame, in which the slubbings are doubled by passing two of them together through a set of three pairs of drawing rollers, and then twisting them into one by means of a flier, and wrinding on a bobbin in the same manner as in the slubbing machine. In this process, intermediate between the slubbing and roving, the amount of drawing produced by the rollers is five times, and $1 \frac{1}{4}$ twists are put into the doubled slubbing by the flier an inch of length delivered from the rollers.

In the Roving Frame, shown in Fig. 2188, the contents of two bobbins A A from the intermediate frame are again doubled, drawn, and twisted as before; and the cotton, now called roving, is wound upon bobbins ready for being finally spun into thread. The extent of drawing in
 the three pairs of draw-
 and the front pair runs at a surface speed of 29 ft . a minute; the flier $\mathbf{C}$ is driven at 900 revolutions a minute, and thus puts $2 \frac{1}{2}$ twists an inch into the roving. The bobbins $A \mathrm{~A}$ supplying the cotton for the rovings are fitted upon wood spindles called skewers, pointed at the lower end where they rest upon their bearings, which are shallow cups made of glazed earthen ware; these
are found very durable, lasting for twenty years before requiring renewal; but when brass or cast-iron bearings were previously tried they were found to be worn through in as many months, whilst the skewers made of lance-wood were but little worn. The spindles $D$ carrying the fliers are driven by skew-bevel wheels on the shafts $E$ at the bottom of the machine. The bobbins $\mathbf{F}$ are loose upon the spindles, and are driven by skew-bevel wheels on the shafts G, carried in the bobbin-lifter $H$, which supports the bobbins and gives them the vertical movement on the spindles D for winding the roving in uniform coils from top to bottom of the bobbins.

As the winding is effected by the difference of speed between the bobbin and flier, both of which revolve in the same direction, the speed of the bobbin may either exceed that of the flier, or the converse; and both plans are in use in the present machines. When the bobbin runs in advance of the flier, the speed of revolution of the bobbin has to be gradually diminished as its diameter increases by each successive layer of roving wound on; otherwise the delicate roving would be irregularly stretched or broken by the relatively increasing surface speed of the bobbin, as the speed of the drawing rollers $B$ and the flier $C$ is required to be constant in order that an equal amount of twist may be put into the roving throughout its entire length. On the other hand, when the bobbin follows the flier, its speed of revolution has to be gradually increased as its diameter increases by winding. In either case the vertical reciprocating movement of the bobbin-lifter $H$ has to be gradually retarded, to allow a longer time for winding each successive layer of roving upon the increasing circumference of the bobbin : and the length of the vertical motion is also diminished at each reciprocation, so as to give the required conical form to the ends of the bobbin, which is effected by means of a separate shortening motion.

The arrangement employed at the present time for obtaining the differential speed of the bobbin and bobbin-lifter in the slubbing, intermediate, and roving frames, is Houldsworth's Differential Motion, shown in Fig. 2189. It consists of three portions, the first of which is driven at a constant speed, and drives the spindles and fliers; the second is driven from the first at a speed varying in proportion to the increase of diameter of the bobbins; and the third portion, from which the bobbins and bobbin-lifter are driven, receives a differential motion compounded of the other two, and therefore also varying with the increasing diameter in winding. The shaft $K$ being driven at a constant speed by the driving strap over the pulley L imparts a uniform speed to the spindles and fliers by the pinion M, which is fast upon the shaft. The pinion N driving the bobbins and bobbin-lifter runs loosely upon the shaft, and is driven through the differential bevel gearing $O$ by the bevel wheel $P$ keyed upon the shaft $K$. The two bevel wheels $O O$, through which the differential motion is obtained, are centred in the dise wheel $Q$ running loose upon the shaft $K$. If the disc wheel were held stationary, the pinion $N$ would be driven through the wheels $O O$ at the same speed as the wheel $P$, but in the contrary direction, and would therefore drive the bobbins at the same speed as the spindles; but if the disc wheel $Q$ were made to revolve upon the shaft $K$ at half the speed of the wheel $P$, but in the contrary direction, the pinion $N$ driving the bobbins would run at double the speed of the wheel $P$. If therefore the dise wheel $Q$ be driven at an intermediate speed, and this speed be also made to vary in proportion to the increasing diameter of the bobbins, the pinion N will receive and impart to the bobbins and bobbin-lifter a differential speed, which also will vary in the ratio of the diameter of the bobbins. This object is obtained by driving the dise wheel $Q$ through the pair of regulating cones $R$ and $S$, which are parallel but reversed end for end in respect to each other; the first cone $R$ is driven at a constant speed direct from the shaft $K$, and drives the second cone $S$ through the strap $T$, which is made to travel gradually from one end of the cones to the other. Hence the disc wheel Q , which is driven by the second cone S , runs at a varying speed depending upon the position of the strap upon the cones; and by making the strap travel along the cones at a rate corresponding with the increasing diameter of the bobbins, the speed of revolution of the bobbins is accurately proportioned to their diameter so as to give the required uniformity in surface speed throughout the winding. The travel of the strap $T$ is effected by a rack-motion I and ratchet-wheel J, Fig. 2188, each vertical reciprocation of the bobbin-lifter releasing the ratchet-wheel $J$ one tooth, and allowing the strap to be drawn forwards the correspending distance along the cones $R$ and $S$ by the weight $W$ constantly acting upon the strap-fork.

In the case shown in Fig, 2189, it will be seen that the speed of the first cone $R$ is $\frac{1}{2}$ that of the driving shaft $K$; and the speed of the disc wheel $Q$ is $\frac{1}{5}$ that of the second cone $S$, and its rotation is in the contrary direction to the driving shaft $K$. The diameter of the cones being 6 in. at the large ends and $3 \frac{1}{2}$ in. at the small ends, when the strap is at the end U, as shown by the dotted lines, the ratio of speed of the disc wheel $Q$ to the driving shaft $K$ is $\frac{1}{2} \times \frac{6 \cdot 0}{3 \cdot 5}$ $\times \frac{1}{5}=\frac{1}{6}$ nearly ; that is, the disc wheel makes one revolution for every six of the
 driving shaft, and in the contrary direction; and therefore the bobbin-pinion $\mathbf{N}$ makes eight revolutions for every six of the flier-pinion M. Similarly when the strap is at the end $V$ of the cones, the ratio of speed of the disc wheel to the driving shaft $K$ is $\frac{1}{2} \times \frac{3 \cdot 5}{6 \cdot 0} \times \frac{1}{5}=\frac{1}{16}$ nearly; that is, the disc wheel makes one revolution for every sixteen of the driving shaft, and in the contrary direction; and therefore the
bobbin-pinion N makes eighteen revolutions for every sixteen of the flier-pinion MI. Hence the ratio of speed of the bobbin-pinion to the flier-pinion is 32 to 24 in the first case and 27 to 24 in the second; and the total reduction of speed of the bobbins, whilst the strap travels along the entire 30 in . length of the cones, is $\frac{5}{32}$, or 16 per cent. of their original speed, which is the range of variation required to allow for the increasing diameter of the bobbins in winding.

The differential motion affords the means of obtaining this delicacy of adjustment with a compact and easily-worked apparatus; and by virtually magnifying the range of variation required avoids the use of cones with too small a taper for good working. The arrangement shown in Fig. 2189 is for the case of the bobbin running in advance of the flier, when the speed of the bobbin has to be reduced as its diameter increases in winding; and the action of the differential motion is exactly similar in the converse case of the bobbin following the flier, the only difference being that the disc wheel Q must then be made to rotate in the same direction as the driving shaft K , instead of in the contrary direction. As the advance of the driving strap T along the cones is a uniform amount at each reciprocation of the bobbin-lifter, the driving cone $I$ requires to be shaped with a concave outline and the driven cone $\mathcal{S}$ with a corresponding convex outline; since the absolute increase made in the diameter of the bobbin by each successive laycr of roving bears a continually diminishing ratio to the increasing diameter of the bobbin, requiring the variation of speed therefore to be effected also in a continually diminishing ratio.

Houldsworth's differential motion requires only a single rack and pinion of uniform pitch, with ratchet-wheels of varied pitches for giving motion to the rack-pinion; so that for a change in the fineness of the roving it is only necessary to change the ratchet-wheel, which is readily effected and is much more convenient than having to change the rack. When this form of the differential motion was first applied, one cone drum only was used, with counter-pulleys and a weighted pulley for keeping the strap tight; but latterly the two cone drums shown in Fig. 2189 have been used instead, made with corresponding concave and convex surfaces, so that the strap continues equally tight in all positions.

Spinning. - In the practical working of Arkwright's spinning machine and Hargreares' spinning jenny, it was found that the rovings and threads produced were both coarse and uneven, only fit for the manufacture of quiltings, and poorly adapted even for that purpose. A great improvement in this respect was effected in 1779 by Samuel Crompton's spinning machine or mule, which was a combination of Paul's or Arkwright's spinning machine and Hargreaves' jenny, combining the drawing-roller arrangement in the former with a modification of the sliding cross-bar and spinning spindles in the latter. In this machine the spindles were placed in a movable carriage, which had a stretch or run of about 54 in .; and the rovings delivered from the drawing rollers in a soft state were further drawn by the spinner in pulling the carriage backwards from the rollers, and completely twisted by the receding spindles, ready for being wound upon the spindles during the run-in or return traverse of the carriage and spindles. In the spinning jenny each successive length of the rovings was held by the clasp on the sliding cross-bar, and the stretching of the rovings was done entirely by drawing back the cross-bar by hand from the spindles; and in Arkwright's machine the stretching was performed entirely by the rollers; but in Crompton's mule the stretching was accomplished partially by the drawing rollers, when the carriage and spindles began to recede from the roller-beam, and partially by the continued run-out of the carriage after the rollers had been stopped. The rollers were stopped when the carriage had receded nearly the length of its run, and they then acted as a clasp to hold the threads during the completion of the stretching and twisting.

Crompton's first mule contained about thirty spindles; and the threads spun by it were far superior in regularity, strength, and fineness to any ever spun before. They realized about double the prices obtained in 1743 for the same counts of yarn spun by other machines, and must therefore have been very superior in quality, having been produced much more cheaply; and in order to show what could be done with the mule, small quantities were spun as fine as No. 80, which is such a quality of thread that 80 hanks of 840 yards each weigh together 1 lb . The adoption of these mules extended so rapidly that in 1811, thirty-two years after the first was made, there were 600 mills containing $4,209,000$ spindles working on this plan, and only 310,500 spindles on Arkwright's plan, and 155,900 spindles on the spinning-jenny plan.

Many of the principal movements, however, in the working of Crompton's mule still required to be performed by hand by the spinner, the same as in the previous machines. This was the case with the backing-off motion, and with the working of the two faller wires, a second or counterfaller having now been added underneath the threads, which was lifted for the purpose of taking up the slack in the threads after the backing off, the first faller being depressed for guiding the threads upon the cops during the winding. The speed of the spindles also required regulating by hand during winding, so as to correspond with the increasing diameter of the cops formed on the spindles, and to suit the conical-shaped ends of the cops. Great skill was therefore necessary on the part of the spinner, in order to make the cops regular in shape, size, and hardness, suitable for transport and for being uncoiled without waste. To supersede this skilled labour and render the mule self-acting was therefore the great aim in the subsequent improvements.

In 1818 the entire operation of winding up the spun threads into cops on the spindles was rendered altogether self-acting by William Eaton. This involved both a self-acting method of performing the backing off, which has to be done at the conclusion of the twisting of each stretch, before the winding begins; and also a self-acting arrangement in connection with the faller wire, for guiding the threads regularly upon the cops during the winding, and a self-acting contrivance for regulating the speed of the spindles according to the increasing size of the cops.

The arrangement of Eaton's Backing-Off Motion is shown in Figs. 2190, 2191. The main shaft or rim shaft A, from which the driving motion of the spindles in the travelling carriage is derived, is itself driven in the forward direction during the twisting, and again during the winding, by the driving strap running on the fast pulley B, as shown by the dotted lines in Fig. 2191. The loose
pulley C communicates a slow motion through intermediate pinions to the wheel D revolving loose upon the shaft A, but in the contrary direction; and at the other end of the shaft A is a corresponding wheel E fast upon the shaft. The two toothed sectors F F are keyed upon a shaft G, which is carried in the rocking frame H ; and the weight K on the rocking frame is constantly acting to draw the sectors back, out of gear with the wheels $D$ and $\mathbf{E}$; while the sectors themselves are only partly counterbalanced by the second weight $L$, and are ready to fall down into gear with the wheels as soon as the catch I, by which they are held up out of gear, is released. When the twisting of the threads is completed, the driving strap is shifted to the loose pulley C , and the forward motion remaining in the shaft A is arrested by friction brake carrying a ratchet-wheel, which is caught by a hook falling into gear at the moment of reversing the strap. The pull upon this hook extends a spiral spring, the recoil of which is made to release the catch I; and the sectors $\mathbf{F}$ falling into gear with the wheels D and E , a backward motion is then communicated to the shaft A from the loose pulley C running for wards, whereby the spindles are made to turn backwards through the few revolutions necessary for backing off the spiral coils of thread at the top of the spindles, preparatory to winding. As the form of cop employed was a simple cone, increasing in height at the same time as in diameter, as shown in Fig. 2192, the length of the spiral coils that require backing off at the top of the spindles becomes less with the increasing height of the cops on
 the spindles, and the number of backward turns in the backing off has therefore to be gradually diminished as the cops approach completion; this is effected by an adjustable stop underneath the sectors $\mathbf{F}$, which is gradually elevated in proportion to the increasing height of the cops. This stop is connected with a lever catching against a stud at the lower extremity of the arm H of the rocking frame; and the downward movement of the sectors F , while in gear with the wheels D and E , depresses the stop until at length the arm H is liberated: the weight K then withdraws the sectors out of gear, whereby the backward motion of the shaft A is stopped. By then shifting the driving strap to the fast pulley B, the shaft A is again driven in the forward direction, and the threads previously spun are wound up on the spindles as the carriage runs inwards. The pin J fixed upon the carriage, travelling inwards in the direction of the arrow, now comes in contact with the tail of the lever M, and lifts the sectors up again into their highest position, in which they are retained as before by the catch I at the other end of the lever M and when the run-in of the carriage is nearly completed, the same pin $J$ comes in contact with the tail of a second lever $N$, bearing against the extremity of the arm H of the rocking frame, whereby the sectors are thrown forwards again in readiness for the next time of backing off.

Eaton's Faller Motion is shown in Fig. 2192, and was almost identical with that in use at the present time, the difference being that the faller wire $A$ was depressed by a weight $B$, instead of, as in the present mules, by a chain passing round a pulley upon the faller shaft C. The direction of the run-in of the carriage D carrying the spindles and cops E is shown by the arrows; and during the run-out in the opposite direction the weight $B$ is held up in the position shown, by the catch $\mathbf{F}$ holding the tail of the lever $G$. This catch is withdrawn by the downward movement of the sectors in the backing-off motion, and the weight $B$ then brings the front end of the lever $\mathrm{C}_{\pi}$ down upon an arm on the front side of the faller shaft $C$, depressing the faller wire $A$ upon the threads H. The roller I, carried upon an arm on the back of the faller shaft, is thus brought up against the pin J fixed in the parallel-motion bar K, and is locked by the latch L; so that by the vertical movement of the bar K the faller wire A is raised and lowered during the winding of the threads, for guiding them upon the cops from end to end. The reciprocation of the bar K is obtained by its bottom end resting upon the shaper fusee or long tapered cam M, which is driven by the pinion $N$ from the toothed wheel $O$ travelling along a rack P fixed upon the floor. As soon as the carriage has begun to run in, the weight B is lifted off the faller and raised again to its original position by the tail R of the lever coming in contact with a fixed stop S . When the carriage arrives at the end of its run-in, the sliding bolt $T$ coming against a fixed stop pushes
back the latch L , and unlocks the roller I; and a balance weight upon the back of the faller shaft C raises the faller wire A clear off the threads into the extreme position shown by the dotted lines. For regulating the shape of the cop as its size increases, the shaper fusee $M$ is gradually traversed endways along its shaft N by the rack and pinion U driven by a worm wheel from the ratchet V , which $i$ s turned round one tonth at a time by the lever $W$ coming against a stop $X$ fixed on the floor at each end of the run of the carriage.

Further imp rovements were introduced by Maurice de Jongh, the backing - off motion leing driven by a rack instead of by sectors; and with the lacking off was combined the process of putting down the faller wire to the required part of the cops fur the commencement of the winding. The working of the faller for guiding the threads during winding was effected by an arm on the back of the faller shaft, carrying a roller, which travelled along a template or copping rail exten ling the whole length of the stretch. The upper edge of this copping rail was shaped acoording to the form of cop required, and the entire rail was gradually lowered by a regulating screw at each end as the cop was built up. The wiuding of the threads on the cops was done by employing a slack strap or friction strap for driving the main shaft or rim shaft of the mule, during the run-in of the carriage; and this strap was tightened by a weight and two friction pulleys pressing against it, the weight being adjusted so as to make the strap drive or slip as required for keeping the threads in proper tension.

Richard Roberts' Self-Acting Mule is the form of self-acting mule almost universally employed at the present time for spinning cotton. In this mule the faller wire was for the first time put
 down by the agency of the rim slaft, or main drivixg shaft of the machine, during the time that the shaft is turning the reverse way for backing off.

The arrangement of Faller-Wire Motion, as employed in the present spinning mules, is shown in Figs. 2193 to 2196. A is the top-faller arm, which is made of the sickle shape shown in the drawing for the purpose of elabling it to put down the faller wire to the bottom of the cops $J$, without the arm itself being required to pass down between the cops, so as to save room in the length of the mule. On the front of the faller shaft I is keyed the sector C, and a chain D attached to the sector passes round the pulley $E$ to a snail $\mathbf{F}$ upon the shaft of the tin roller, which is a long hollow cylinder made of tin, and driving by separate cords the whole row of spindles T. The snail $\mathbf{F}$ is geared to the tin roller by a ratchet-clutch, with the teeth set so as to engage only when the tin roller is driven the reverse way for backing off, as shown by the arrow in Fig. 2194. Whilst the tin roller is running forwards during the spinning, and again during the winding, in the direction shown by the arrow in Fig. 2193, the snail F is not in action; but as soon as the carriage $G$ of the mule has run out to the end of the stretch, as slown in Fig. 2194, the tin roller is turned through part of a revolution in the reverse direction, as indicated by the arrow, sufficiently for unwinding the coils in backing off; and the snail $F$ then comes into action and winds up the chain D , thereby bringing the top-faller wire A down upon the threads W and depressing them towards the bottom of the cops. On the back of the faller shaft I is fixed the curved arm B, against which bears the vertical locking bar $H$; and when the arm B is lifted by the depression of the faller $\mathbf{A}$, its extremity is caught by the recess in the bar H , which is thrown forwards by the bell-crank lever K, as shown by Fig. 2195; the tail of this lever having been brought, by the runout of the carriage $G$, under the corresponding bell-crank $L$ fixed in the end frame of the mule, has previously extended the spiral spring attached to the bell-crank L, Fig. 2194, the recoil of which throws the locking bar $H$ forwards as soon as the arm B is sufficiently raised, Fig. 2195. The pulley $E$ is carried on a rocking lever $R$, the tail of which presses against the stop $S$ in the end frame of the mule during the time that the chain D is depressing the faller, Fig. 2194; but at the moment when the locking bar $H$ is thrown forwards to lock the faller arm $\mathbf{B}$, the stop $\mathbf{S}$ is lowered, as shown in Fig. 2195, clear of the tail of the lever R, allowing the pulley E to yield to the further
pull of the chain D until the reverse motion of the tin roller in backing off is stopped; by this. means the snail F, Fig. 2194, is prevented from depressing the faller wire A beyond the required distance down the height of the cop.


The faller being thus locked, the carriage $G$ begins to run in, in the opposite direction to that indicated by the arrows in Fig. 2193; and while the spindles wind up the threads on the cops, the
faller wire is gradually anowed to rise by the locking bar H running down the inclined copping rail M, the curved arm B being kept constantly pressed home in the notch of the locking bar, by a counter-balance weight or spring acting on the back of the faller shaft I to raise the faller A. The length of the stretch or runin of the carriage $G$ is 63 in ., which is therefore the length of thread to be wound upon the cop $\mathbf{J}$ at eacli time of winding; and this whole length of 63 in . of spun thread in each stretch is wound upon the cop during each stroke of the faller wire. The mode of building up the cop in successive stages is shown half full size in Fig. 2197; and in order to allow for the increasing diameter of the cop, the successive layers of thread are wound upon it in more open coils as the size increases, as in dicated by the dotted lines, which is effected by gradually increasing the range of the faller wire; at the same time the ends of the cop are made of the conical form shown in the drawing. The length of range or chase of the faller wire at the commencement of the cop upon the bare spindles is only from A to B ; but this is gradually increased until the cop has attained its full diameter C C, when the length of range is from $C$ to $\mathbf{D}$; after which the range is

2194.

$\qquad$ 2196.

slightly diminished again to the length EF in finishing the cop. For the purpose of obtaining the requisite motion of the faller wire for giving these successive shapes to the cop during the winding, the extremities of the copping rail M, Fig. 2193, are supported on the two sliding wedges N and O , which are kept at an invariable distance apart by a connecting rod. In
commencing the winding of a set of cops upon the bare spindles, as shown at $\Lambda \mathrm{B}$ in Fig. 2197, the copping rail is set at the top of the wedges and is at its smallest inclination; and after each successive layer has been wound on, the two wedges are slided from under the rail by a traversing
screw worked by a ratchet-wheel, which is advanced one or more teeth during each run-out of the carriage G, Fig. 2193. By this means the copping rail M is gradually lowered at both ends, and at the same time its inclination is increased by the outer wedge N being made with a rather smaller angle at the top than the inner wedge $O$, for the purpose of forming the cop with a more gradual taper at the top than at the bottom, as shown in Fig. 2197. This increase of inclination continues until the cop has attained its full diameter CC and has assumed the shape ACD; after which the inclination slightly decreases again until the cop is completed to the finished shape A CEF, by the latter part of the outer wedge N being made slightly steeper than the corresponding portion of the inner wedge 0 , as shown in Fig. 2193. The inner end of the copping rail being the lowest, the winding of each stretch leaves off at the top; and at the commencement of winding each stretch the faller wire puts down the thread to the point at which the winding of the new layer is to be started, about three coils being wound on during the descent of the faller, as indicated by the spiral dotted line from $\mathbf{F}$ to $\mathbf{E}$ in Fig. 2198, and the remainder during the rise of the faller. When the spindles arrive at the rollers P, as shown in Fig. 2196, having wound up the 63 -in. stretch of threads, the stop $\mathbf{U}$ pushes back the locking bar H, thereby releasing the faller A, which immediately rises clear

2199. of the threads W .

The counter-faller wire is carried by the arm V from a second shaft behind the topfaller shaft I, and during the winding it bears up constantly against the underside of the threads W, as shown in Figs. 2194 and 2195 , with a slight pressure from a counter-balance weight or spring acting on the shaft, so as to ensure keeping the threads in proper tension; during the spinning the counterfaller is held up just beneath the threads, but without touching them, as shown in Fig. 2193. The arm V of the counterfaller is curved as shown in the draw-
 ing, so as to reach over the shaft I of the top faller, and also to avoid passing down between the cops; and the curved arm Bon the back of the top-faller shaft I is shaped so as to clear the shaft of the counter-faller. The height of the counter-faller wire is employed as a means of regulating the speed of the spindles in
winding, in the manner afterwards explained, so as to avoid the occurrence of any slack in the threads.

Roberts' Backing-Off Motion as employed in the present mules is shown in Fig. 2199, which is a plan of the main driving shaft or rim shaft A of the machine, carrying the large rim-wheel Z or double-grooved pulley driving the whole of the mule-spindles by the endless cords $\mathrm{X} \mathbf{X}$, Fig. 2193, passing round the pulleys Y Y. On the boss of the loose pulley B is a pinion C, which, through a train of intermediate wheels D D, drives in the reverse direction and at the required slower speed the spur wheel and friction cone E, also running loose upon the shaft A and sliding longitudinally upon it. This friction cone engages in a corresponding hollow cone inside the fast pulley $F$; and when the driving strap is shifted from the fast pulley $F$ to the loose pulley $B$ for the purpose of backing off, the friction cone is also brought up against the fast pulley, thereby first arresting by friction the forward motion of the driving shaft A, and then giving it the reverse motion for backing off.

Roberts' Winding Quadrant, for regulating the winding of the threads by diminishing the speed of the spindles in proportion as the diameter of the cops increases, is shown in Fig. 2200. This very ingenious contrivance has never been superseded, and is employed in almost every self-acting mule at the present day. The quadrant A turns upon a fixed centre C in the frame of the mule, and a pinion B gears into it, which is driven by a band and pulley receiving motion from the traverse of the carriage $G$, the arrows indicating the direction of motion during the runin of the carriage. The grooved arm D of the quadrant contains a doublethreaded screw, by which the sliding nut E is traversed outwards from the centre of motion C towards the extremity of the arm D. When the carriage is at the outer end of its stretch, the arm D stands inclined $12^{\circ}$ outwards from the vertical, as shown by the dotted lines; and during the run-in of the carriage it turns inwards through an arc of $90^{\circ}$. A chain F , attached to the nut E , is coiled round a drum H inside the carriage $G$, and as the carriage recedes from the quadrant arm during the run-in the chain thus causes the drum to rotate, and thereby drives the spindles T through the intervention of the tin roller I geared to the drum H. At the commencement of a set of cops, the nut E is at the bottom of the quadrant arm $D$, nearest to the centre of motion C, as shown dotted; and the number of revolutions then given to the drum H by the uncoiling of the chain during the run-in of the carriage is nearly as many as if the end of the chain at the nut were held stationary, and is sufficient to wind up on the bare spindles the length of threads spun in one stretch.

As the cops increase in diameter from their original size AB to their full diameter C C, Fig. 2197, the nut E is gradually advanced outwards along the quadrant arm D, Fig. 2200, so as to increase its arc of motion and thereby diminish the number of revolutions of the drum $H$ and the speed of the
 spindles T. This advance of the nut is obtained from the counter-faller V bearing against the under-side of the threads W during the winding. The depression of the counter-faller towards the lower part of the cop $J$ brings down the end of a governing lever upon a horizontal strap, which passes round a pulley on the headstock of the mule and round another on the centre shaft C of the quadrant; and on this shaft is a bevel pinion gearing into a second bevel pinion on the end of the double-threaded traversing screw in the arm D ; so that when the governing lever is depressed upon the strap by the counter-faller, the forward motion of the lever as the carriage runs in drags the strap along with it by friction and turns the shaft C forwards, sliding the nut E outwards towards the circumference of the quadrant. At the moment when the backing-off motion has ceased and the carriage begins to run in for winding up the stretch of thread spun, as shown in Fig. 2195, the counter-faller wire V is at its highest woriking position, compensating for the
additional length of thread that has been uncoiled from the top of the spindle in backing off after the spinning of the stretch was completed. The nut E, however, Fig. 2200, is still at the same distance from the centre $\mathbf{C}$ of the quadrant as it was at the conclusion of winding the previous stretch; and, therefore, as the diameter of the cop is now greater by winding the new layer of thread outside the previous one, the winding of the new stretch commences rather too fast, and begins at once to take up the length of thread given out in the backing off. The counter-faller V is thus depressed, and by means of the governing lever slides the nut E farther out from the centre C, until the speed of winding is sufficiently diminished to allow the counter-faller to rise again high enough for lifting the governing lever off the strap. It will be seen that, in consequence of the arm D describing the quadrant of a circle, the horizontal motion of the nut E in the winding of each stretch is greatest at the commencement of the winding, and gradually diminishes as the carriage runs in; and the effect of this is that the speed of winding is gradually increased towards the end of each stretch. By this means the threads are wound uniformly upon the cops, with an equal degree of tightness throughout.

The whole mule is driven by a strap $3 \frac{3}{4} \mathrm{in}$. broad, running over the fast pulley F, Fig. 2199, on the rim shaft A, and travelling at about 1670 ft . a minute, or about 19 miles an hour. The driving power required is about 1 indicated horse-power the 230 spindles, or $4 \frac{1}{2}$ horse-power for each mule containing 1000 spindles. The speed of the endless cord X passing round the rim wheel Z, Fig. 2193 , is 2640 ft . a minute, or about 33 miles an hour. The carriage of the mule makes 3 to $3 \frac{1}{2}$ double journeys out and home a minute, the length of stretch being 63 in. ; but the velocity varies at different parts of the traverse, the carriage being taken in by a pair of scrolls in the centre of the machine, and drawn out by three spiral grooved pulleys keyed upon a shaft running the entire length of the mule, one pulley being in the middle of the shaft and one at each end. The length of the carriage being upwards of 100 feet, a parallel motion is required for keeping the carriage straight; and this is obtained by a horizontal traversing pulley at each end of the carriage, traversing along fixed cords and thereby made to revolve; and these two pulleys being also coupled together by a crossed cord, are compelled to revolve at the same rate, and consequently cause each end of the carriage to travel at the same rate. There are three pairs of drawing rollers P, Fig. 2193, by which the rovings are drawn about eight times before being delivered for spinning. The last pair of rollers delivers the rovings at a speed of 26 ft . a minute, until the carriage G has run out the $63-\mathrm{in}$. length of stretch, when the rollers are stopped, and hold the threads fast during the winding up as the carriage runs in again. The actual length of roving delivered by the last pair of rollers for each stretch of 63 in . is about 61 in .

The spindles make about 1260 revolutions in twisting each stretch of 63 in ., thus putting about 20 twists an in. into the threads; and the total time occupied is about $12 \frac{1}{2}$ seconds. In winding up the threads, as the total length to be wound up remains constant, namely 63 in., the number of revolutions is diminished in each successive stretch according to the increasing size of the cop, from about 70 revolutions at the commencement to 23 revolutions at the full diameter of the cop, with cops of an average size winding an average number of yarn, such as No. 32 , which is such a quality that 32 hanks of 810 yards each weigh together 1 lb .; the time of winding each stretch is about $3 \frac{1}{2}$ seconds. The velocity of the spindles is about 390 revolutions a minute in winding; and in twisting the speed ranges from 8000 down to 3000 for coarse work, a common average being about 6500 to 7000 revolutions a minute. In backing off, the velocity of revolution is about $\frac{1}{20}$ of that in twisting. The direction of rotation of the spindles is the same in twisting and in winding, and the thread is wound on in a right-handed spiral when spinning twist, and left-handed when spinning weft. Fig. 2198 shows full size the conical form of the top of the spindles for the purpose of letting the thread slip off freely at each revolution in twisting; and the two lines G and $H$ show the extreme inclinations of the thread to the spindles during the twisting. The larger angle shown by the dotted line G, when the spindles are nearest to the drawing rollers, is about $145^{\circ}$; and the smaller angle shown by the full line H is about $105^{\circ}$, when the spindles are at th乌 outer extremity of the stretch. The spindles themselves are inclined inwards towards the drawing rollers at an angle of about $12^{\circ}$ from the vertical, as shown in the drawing.

Cotton Gin.-For the purpose of ginning cotton, or separating the cotton fibres from the seed, the roller gins, as they are called, are of the most primitive construction, simply and ingeniously made, and are of Indian origin. Fig. 2201 shows a section of a roller gin of modern construction. It is formed of two small rollers about $\frac{3}{4} \mathrm{in}$. diameter and from 6 to 9 in . long, made to revolve in opposite directions, as shown by the arrows, by means of toothed wheels. The bottom roller turns on fixed bearings, and the upper roller is kept in close contact with it by means of a lever and screw. The cotton seeds are fed forwards between the rollers from the table in front, a space being left between the edge of the table and the bottom roller, to allow the seeds to drop down as they become cleaned of the cotton fibres by the revolving action of the rollers. A brush is fixed underneath the bottom roller to brush off any cotton fibres adhering to its surface. It is necessary that gins acting in this way should have rollers of small diameter, say from $\frac{5}{8}$ to $\frac{3}{4} \mathrm{in}$., because
 the smaller the rollers the more obtuse is the angle they present to the seed which is being cleaned, and the seeds are thereby better prevented from being drawn in between the rollers and crushed and mixed up with the cotton fibre. It is evident from Fig. 2202 that large rollers present an acuter angle to the seed, and with such rollers the seed would unavoidably be drawn in and crushed
and mixed with the fibre. The object to be attained in ginning cotton is to get it free from all impurities; and it is found that the smaller the rollers and the slower their motion, the cleaner is the cotton fibre separated from the seeds; for if the rollers are above an inch in diameter, and if they revolve very rapidly, they draw in soft, small, and false seeds, crushing them in their passage, and straining and otherwise injuring the cotton fibre.

The most improved gin of the roller class is given Fig. 2203, and is constructed with one large roller covered with leather and having small spiral grooves formed round it both right and left. A guard-plate $\mathbf{A}$ is fixed near the surface of the roller, having a grating along its bottom edge just wide enough for the seeds to pass through to the roller; and between it and the roller a thin steel striker blade B vibrates a short distance up and down with a rapid motion. By this means the seeds are shaken and turned round in contact with the surface of the roller revolving in the direction of the arrow, and the cotton fibres are drawn between the blade B and the roller, while the stripped seeds are rejected and drop down through the space between the blade B and the edge of the feed-table, having been thus cleaned of their fibres in a most thorough manner.

In 1793 Whitney invented a machine, Fig. 2204, known by the name of Whitney's saw gin. It consists of a wooden cylinder with a series of circular saws C, about 8 in . diameter, fixed upon it at regular distances. The edges of the saws project a short distance through a grid, the divisions of which are too narrow to permit the seeds to pass through. Care is taken that the saws revolve in the middle of the grid spaces, for if they rubbed against the bars they would tear the cotton filaments to pieces. A cylinder with brushes D, the tips of which touch the saw teeth, sweeps off the adhering cotton wool from the teeth of the saws by revolving in the opposite direction to the saw roller. The


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2204 .
$$

 cotton seed as picked from the pods is thrown into the hopper E , and the saws in turning round snatch the filaments from the seed which remains against the grid, and drag them inwards and upwards. The stripped seeds, being too large to pass through the grid, accumulate at the bottom of the hopper $E$, and are let out at intervals.

The Carding Engine of Curtis, Parr, and Madeley, of Manchester, Fig. 2205, is constructed on the roller-and-clearer principle. Its dimensions are as follows, namely;-

1 large or main cylinder, 50 in . diameter, and 40 in . on the wire;

5 knives placed horizontally across the main cylinder" between the clearer and roller ;
6 boxes or receivers, placed across the main cylinder, behind the clearer;
1 revolving shaft, placed inside the boxes or receivers, working horizontally.
It is obvious that the main operations of carding, on the roller-and-clearer carding engine, take place between the roller and main cylinder. The novelty of this carding engine consists in the application of a knife between each pair of rollers and clearers, combined with a box, behind the clearer, to receive all the short-fibre cotton, fly, and shell-dirt liberated through the action of the knife and thrown into the box by the centrifugal force of the clearer. The action of the knife is almost equal to that of a combing machine; for the cotton, being held on the roller, cannot pass in fleeces to the clearer, as is the case when no knife is used, but is drawn by the roller under the edge of the knife, and by this means the fibre is more evenly laid and combed. All extraneous matter is thus liberated and immediately thrown into the box or receiver, and there retained by means of the small revolving shaft. By this principle a better quality of carding can be produced than by the ordinary common roller-and-clearer principle, without the knife and box.


COTTON PRESS. Fr., Pressoir à coton ; Ger., Baumwollenpresse ; Ital., Macchinada comprimer cotone; Span., Prensa para algodon.

See Presses.
COUNTER. Fr., Compteur ; Ger., Zählapparat ; Ital., Contatore; Span., Contador.
See Details of Engines.
COUNTER-BALANCE. Fr., Contre-poids; Ger., Gegengewicht; Ital., Contrapeso; Span., Contrapeso.

An equal opposing weight, power, or agency, acting in opposition to anything, is a counter-balance ; as the mass of iron cast on the side of a locomotive wheel, opposite the crank-pin, to counterbalance the weight of the latter and its connected parts.

COUNTERFORT. Fr., Contre-fort; Ger., Strebepfeiler ; Ital., Contrafforte; Span., Contrafuerte.

See Fortification.
COUNTER MINE. Fr., Contre-mine; Ger., Gegenmine; Ital., Contromina; Span., Contramina.

See Fortification.
COUNTERSUNK. Fr., Fraise; Ger., Versenkt; Ital., Acceccato; Span., Con cabeza embutida.

The head of a screw or bolt sunk below a surface, by drilling or turning, is said to be countersunk.

COUPLING. Fr., Accouplement, Attelage; Ger., Kuppelunj; Ital., Organi d'accoppiamento ; Span., Riueda de embrague, de conexion.

A coupling is that which serves to couple or connect, as a hook, chain, or other contrivance ; as the coupling of railway carriages; any contrivance for connecting shafts end to end, either permanently or so as to admit of their being joined or disjoined at pleasure, as by a box, clutch, heads with interlocking teeth, and so on. See Gearing.

COUSINET, or CUSHEON. Fr., Coussinets; Ger., Kampfer ; Ital., Mossa dell’arco ; Span., Salmer.

A cousinet is the stone placed on the impost of a pier for receiving the first stone of an arch.
COVERED WAY. Fr., Chemin couvert de l'enceinte; Ger., Gedeckter Weg; Ital., Via coperta; Span., Camino cubierto

See Fortification.
CRAB. Fr., Vindas, Chèvre ; Ger., Krüppeispill, Erdspill; Ital., Verricello; Span., Torno.
A crab is a form of crane used for raising or moving heavy weights. A contrivance for launching ships or raising them into dock is also termed a crab. See Jack. Winch.

CRADLE. Fr., Ber, Berceau; Ger., Stapel ; Ital., Invasatura; Span., Grada.
See Gold.
CRAMP. Fr., Crampon; Ger., İlammer ; Ital., Lega; Span., Laña.
A cramp or clamp is a piece of iron bent at the ends, serving to hold or compress together pieces of timber, stones, and so on; a cramp-iron.

CRANE. Fr., Grue; Ger., Krahn; Ital., Gru; Span., Grua.
See Lifts, Elevators, and Cranes.
CRANK, Fr., Manivelle; Ger., Kurbel ; Ital., Zanca; Span., Cigüeña.
See Mechanical Movements.
CROSS-HEAD. Fr., Tête croisseé ; Ger., Kreuzhopf ; Ital., Crociera; Span., Cabeza en T.
See Engines, Varieties of. Marine Engines. Stationary Engines.
CROWBAR. Fr., Levier. Pince; Ger, Brechstange; Ital., Piè di capra; Span., Palanqueta.
A crowbar is a bar of iron sharpened at one end, and used as a lever for raising heavy bodies.
Hydraulic Crow, Figs. 2206, 2207.-In this tool hydraulic power is employed, it being a hydraulic crow, for straightening or setting rails, designed by $J$. M. Budge. The construction of this tool consists of a small hydraulic press cylinder. $2 \frac{3}{-1}-\mathrm{in}$. bore, having hinged to it a pair of arms by which the rail which is to be straightened or set is held. On the top of the press cylinder is screwed a casting, which forms an oil tank, and which contains a small brass pump of the kind used in Tangye's hydraulic lifting jacks ; this pump, which has a $\frac{5}{8}-\mathrm{in}$. ram, being worked by means of a rocking shaft, which projects through the casting and is furnished
 with a lever handle outside. The whole arrangement is very compact and convenient, and it dispenses with the necessity of using the long awkward levers required with the ordinary screw crows.

CROWN WHEEL. Fr., Roue de rencontre; Ger., Kronrad; Ital., Ruota a corona; Spax., Rueda de escape.

See Mechanical Movements.

CRUCIBLES. Fr., Creusets ; Ger., Schmelztiegel; 1tal., Crogiuoli; Span., Crisoles.
Vessels used for the fusion of metals, and generally for all other chemical purposes in which intense heat is employed.

The use of the crucible appears to have originated with the old alchemists, who were in the habit of marking them with the sign of the cross before commencing their operations; whence the derivation of the name. The principal requisites of a good crucible are, that it should be capable of enduring the strongest heat without becoming soft or losing much of its substance; that it should not crack on being exposed to sudden alternations of temperature; that it should withstand the corrosive effect of the substance fused in it; and lastly, that it should be sufficiently strong to support the weight of the molten metal when lifted from the furnace. In the present day the consumption of crucibles is very large ; they are extensively employed by the brass-founder, the gold and silver refiner, the manufacturers of cast steel and gun-metal, as well as in the melting of zinc and copper, in the various operations of the analytical chemist, assayer, and in the production of the coinage of different countries. The crucibles in most common use in Birmingham and its neighbourhood, as well as in Sheffield, are made of a fire-clay found near Stourbridge, which is generally mixed with some other substance, such as powdered coke, in order to lessen its tendency to contract when strongly heated. These Stourbridge clay crucibles, or casting pots, are not burnt until required for use, when they are put into the furnace first with the mouth downward, and when red hot are taken out, and put in again with the mouth upward.

The material from which the most refractory crucibles are now made is plumbago, or, as it used to be called, black-lead. This is one of the various forms assumed by carbon, and in its pure state is nearly identical in composition with the diamond, although so very different in its structure and physical character. Until a few years ago the use of black-lead, Plumbago, pots was exclusively confined to the melters of precious metals, but they are now employed for melting all descriptions of metal; and large numbers of Morgan's far-famed crucibles, which are composed of plumbago, are used by the brass-founders and others in Birmingham. Immense quantities of these crucibles are annually manufactured by the Plumbago Crucible Company, whose works cover a large space of ground at Battersea, near London; and the manufacture as at present carried on at the Battersea works, presents a striking illustration of the rise and progress of a branch of industry comparatively unknown a quarter of a century ago.

In making the crucibles, the materials are first ground to powder and sifted, after which they are mixed with certain proportions of various constituents, so as to give a sufficient degree of coherence and plasticity Formerly, crucibles were made by hand, common wooden moulds being employed, but those of Morgan are made by machinery, thus securing an exactness of form and uniformity of weight which could never be attained under the old processes. Morgan's Plumbago Crucibles, shown in the Exhibition of 1862, had been used for respectively 80, 90, and 100 pourings -a vast improvement on the ordinary clay pot, wnich is considered to have done a fair average of work if it lasts throughout the day.

The advantage, indeed, of using plumbago crucibles wherever durability is required is now so apparent, that although comparatively expensive at first, they are ultimately found to be the cheapest that can be used. They effect a great saving of time, labour, and fuel (as much as $1 \frac{1}{2}$ ton of the latter being saved to every ton of steel fused), and are consequently gradually superseding all others in steel foundries.

Crucibles are made of various forms and sizes, according to the kind of work for which they are intended; those used for assaying are scarcely larger than a lady's thimble, whilst others made for zincing shot will hold as much as 800 lbs . of molten zinc. Some are nearly cylindrical, others triangular, and others skittle-shaped. They are generally numbered according to their capacity; thus, No. 30 will contain 30 kilogrammes, or something more than 60 English pounds, and so on, the adoption of French weights being found useful to Continental consumers.

Cornish crucibles are principally used for assaying copper; they are made of a clay found in some parts of Cornwall, and the smaller sizes are capable of resisting sudden alternations of temperature (a quality which is probably due to the large proportion of silica mixed with the clay), but they are rapidly corroded by melted oxide of lead.

Hessian crucibles were formerly employed to a much greater extent in metallurgical operations than they are at present. They are made principally from a clay found at Gross-Almerode, and in their composition resemble very closely the Cornish crucibles. The form is triangular, and they are generally packed in nest's of six; the smaller sizes fitting into the larger. These crucibles are tolerably lasting at moderate temperatures, but are apt to fuse when exposed to very great heat.

Several kinds of French crucibles are manufactured, some of which are of very excellent quality, especially those of Beaufay, called the creusets de Paris, and thosc of Deyeux, termed creusets de Saveignies. Both kinds, however, contain a large percentage of oxide of iron, which renders them objectionable for some purposes.

London crucibles are of a reddish-brown tint, very close grained, and capable of resisting the corrosive action of oxide of lead, but liable to crack when suddenly heated. Of late years white fluxing pots, manufactured by the Plumbago Crucible Company, Battersea, have been very much employed on account of their smooth surface, and their power of resisting the action of fluxes. They are made in various sizes, from $2 \frac{1}{4} \mathrm{in}$. up to $8 \frac{1}{2} \mathrm{in}$. in height.

The large crucibles used in the manufacture of glass, glass-house pots, are made of the best Stourbridge clay, mixed with about $\frac{1}{3}$ its weight of cement of old pots ground to fine powder.

For special metallurgical or chemical purposes, crucibles are sometimes made of platinum, lime, bone dust, magnesia, pure carbon, and other materials.

CRUSHING and AMALGAMATING MACHINE. Fr., Bocard, Machine à broyer; Ger., Pochwerk; Ital., Macchina da stritolare e amalgamare; Span., Bocarte, Triturador.

See Gold. Silver.

## DAMMING.

CULVERT. Fr., Conduit souterrain, Ponceau ; Ger., Abzugscanal; Ital., Condotto; Span., Alcantarilla.

An arched drain for the passage of water under a road or canal is a culvert.
DAM. Fr., Digue; Ger., Damm; Ital., Pescuia, Diga ; Span., Represa.
See Damming. Docks. Embankuents. Gravity, Centre of. Locks. Water-Supply. Waterworks. Weirs.

DAMMING. Fr., Arrêter les eaux par des digues; Ger., Verdämmen; Ital., Sostenere un corso d'acqua conpescaia; Span., El acto de represar.

Pressure of Water.-Let L F B D, Fig. 2208, be a vessel of any form whatever, filled with water, $a b$ any portion of the surface F K CB in contact with the water, G the centre of gravity of $a b$, $G \mathrm{R}$ the perpendicular depth of $G$ below the surface of the water; then if $\mathrm{RG}=10 \mathrm{ft}$., and the area of the surface $a b=3 \mathrm{sq}$. ft., the lbs. pressure on this surface will be $=3 \times 10 \times$ $62 \cdot 5=1875 \mathrm{lbs} .1875 \mathrm{lbs}$. is the weight of a column of water whose base is the area $a b$, and perpendicular height the depth of the centre of gravity of $a b$.

If the area of $c e$ on the bottom of the vessel $=5 \mathrm{sq}$. ft., and $Q \mathrm{G}_{1}=14 \mathrm{ft}$. the perpendicular depth of the centre of gravity $\mathrm{G}_{1}$ below the surface of the water; then the lbs. pressure on the area $c e=5 \times 14 \times 62 \cdot 5=$ 4375 lbs.

A gain, if the area of the surface $h f$, on the slanting face $\mathrm{AL}=4 \mathrm{sq}$. ft., $P \mathrm{G}_{2}=11 \mathrm{ft} ., \mathrm{G}_{2}$
 being the centre of gravity of the area $h f$; then the pressure on this slanting surface will be $4 \times 11 \times 62 \cdot 5=2750 \mathrm{lbs}$.

As in former cases, the weight of a cubic of water is taken $=62 \frac{1}{2} \mathrm{lbs}$. It may be further observed that the positions or inclinations of the surfaces $a b, c e, h f$, are not taken into account, but merely their areas and the perpendicular distances of the centres of gravity from the horizontal surface of the fluid. On this simple principle rests that department of mechanics termed hydrostatics; it is easily demonstrated, as gravity acts on all the particles of the fluid, and each particle presses on that next below it, and, further, because, from the peculiar property of the fluid, this pressure is transmitted in all directions equally.

Ques. Find the lbs. pressure on a floodgate whose breadth is 9 ft . and depth 7 ft .
$7 \times 9=63 \mathrm{sq}$. ft. area, depth of centre of gravity $=\frac{7}{2}=3 \cdot 4 \mathrm{ft} . \therefore 63 \times \frac{7}{2} \times 62 \frac{1}{2}=13781 \cdot 25 \mathrm{lbs}$.
Ques. What is the pressure upon 10 ft . length of an embankment, the depth of the water pressing against it being 11 ft .

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10 \times 11 \times \frac{11}{2} \times 62.5=37812 \cdot 5 \mathrm{lbs}
$$

Ques. Required the relation of the pressure upon the four sides of a cubical vessel filled with water, and the pressure on the bottom which is horizontal.

Put $n=$ the length of the side of the cube in feet, $n^{3} \times 62 \cdot 5=$ pressure on the base; $n \times n \times \frac{n}{2} \times 62.5=$ pressure on one of the sides, $\therefore 2 n^{3} \times 62.5=$ pressure on the four sides. Then $n^{3} \times 62 \cdot 5: 2 n^{3} \times 62 \cdot 5:: 1: 2$; that is, the pressure on the sides is $=$ to twice the pressure on the base. In these calculations the fluid is supposed to be at rest, and acted on only by gravity.

Let ABCD be a vessel filled with water, the pressure $Q$ on any point $n$, in the side AD, Fig. 2209, is due to the perpendicular depth An. If in the base D C produced we take D F = D A, the perpendicular depth of the water, then the pressure upon the point $D$ will be due to the pressure of a column of the fluid, whose height is = D F. Draw F A, and from any point $n$ draw $m n$ perpendicular to $\mathrm{A} D$; hence $m n=\mathrm{A} n$, and the pressure Q on the point $n$ will be due to a column whose height is $m n$; the same reasoning applies to any other point in the side of the vessel.

Let us take an example and compare this method of viewing the subject with the one previously enunciated and illustrated.

Ques. What is the pressure on an embankment whose length is 21 ft . and depth of the water AD $=12 \mathrm{ft}$.

The whole pressure upon the side of the embankment is equivalent to the pressure or weight of a mass of fluid of the form of a wedge, A F D, Fig. 2209.


Area of the triangle A D.F $=12 \times \frac{12}{2}=72$ sq. ft., $\therefore 72 \times 21=1512$ cub. ft. content of the wedge. $\therefore 1512 \times 62.5=94500$ lbs. pressure.

Before it was stated that the pressure in lbs. on the side is equal to a column of water whose base is the area of the surface, and perpendicular height the depth of the centre of gravity.

It is evident, since the side of the embankment is a parallelogram, the depth of its centre of
gravity $=\frac{12}{2}=6 \mathrm{ft}$. , area of the surface $=12 \times 21=252 \mathrm{sq} . \mathrm{ft} . \quad \therefore 252 \times 6 \times 62 \cdot 5=94500 \mathrm{lbs}$, the pressure before found.

It may be easily perceived that there is a certain point in the side $\mathrm{A} D$ of an embankment or vessel filled with water, where a single pressure will counterbalance the pressure of the water against the whole side. This point is called the centre of pressure.

The centre of pressure must evidently lie in the line $\mathbf{P}$ G passing through the centre of gravity, $G$, of the wedge of pressures, of which the plane A FD is a cross-section.

Bisect FD in E, and D A in H, draw A E and FH; these lines cut one another in the centre of gravity G. $\mathrm{DP}=\frac{1}{3} \mathrm{DA}$, that is, the centre of pressure, P , in this case lies at $\frac{1}{3}$ of DA , from the bottom.

Ques. Required the pressure on the staves of a cylindrical vessel filled with water, the diameter of the base being 10 ft . and the perpendicular height 8 ft .
$3 \cdot 1416 \times 10=31 \cdot 416 \mathrm{ft}$. the circumference of the cylinder,

$$
\therefore 31 \cdot 416 \times \frac{8}{2} \times 8 \times 62 \cdot 5=62832 \text { lbs. pressure. }
$$

If the staves of this barrel are to be kept together by a single honp, that hoop should be $\frac{10}{3}=3 \frac{1}{3} \mathrm{ft}$. from the bottom.

Ques. An embankment H D, Fig. 2210, resists a pressure of water whose centre of pressure is at $P$; it is required to determine by construction the conditions of equilibrium, supposing when the pressure is sufficient to overturn the embankment it will turn upon A, as a centre.

Let F O C be the vertical line drawn through $(\underset{\sim}{x}$, the centre of gravity of the embankment. Draw PL perpendicular to $\mathrm{F} \mathbf{C}$, intersecting FC in O . Make $\mathrm{O} n=$ the lbs. pressure in the embankment, and $\mathrm{O} m=$ the pressure of the water, complete the parallelogram $\mathrm{O} m p n$, then if the diagonal Op or Op produced falls as at B inside the base, the embankment will stand, but if the diagonal cuts outside of A, embankment will fall by turning over upon 0 .

Otherwise, since the pressure of the water $P$, in pounds multiplied by the length of A L in feet, gives the moment of the water tending to turn the embankment, HD , on A as a centre; and the product of the weight
 of the embankment, HD , in pounds by the length of A C in feet, gives the momentum of the embankment that acts against the pressure of the water; consequently, when these moments are equal the embankment, A HED, is upon the point of turning over the point $A$; if the moment of the water be the greater of the two, the structure will fall, but if it be the lesser of the two, it will stand.

Ques. Suppose 10 ft . to be the length of an embankment whose height, DE, from the surface of the water at E , is 28 ft ., $\mathrm{AD}=6 \mathrm{ft}$., will the embankment stand or fall when a cubic foot of the material of which it is composed $=160 \mathrm{lbs}$.

Surface upon which the water presses $=28 \times 10=280 \mathrm{sq}$. ft. Pressure of the water $=$ $280 \times \frac{28}{2} \times 62.5=245000 \mathrm{lbs} . \frac{28}{3}=\mathrm{DP}=\mathrm{AL}=$ the distance of the centre of pressure P , from the bottom A D. $\therefore 245000 \times \frac{28}{3}=2286666 \frac{2}{3}$ the moment of the water. Weight of the embankment $=28 \times 10 \times 6 \times 160=268800 \mathrm{lbs}$. Moment of the embankment

$$
=268800 \times \frac{6}{2}=806400
$$

This structure must fall, since the moment of the water is greater than the moment of the embankment.

Ques. What must be the height of the water in the last question, so that the embankment may be upon the point of overturning.

Putting $x$ for the required height, then the moment of the water will be
$x \times 10 \times \frac{x}{2} \times 62 \cdot 5 \times \frac{x}{3}=\frac{x^{3}}{6} \times 625 . \quad \therefore \frac{x^{3}}{6} \times 625=806400 . \quad \therefore x=20.59967 \mathrm{ft}$, height required.
Ques. Required the thickness of a rectangular embankment that supports a pressure of water rising its full height of 16 it . when the structure is upon the point of turning over; the weight of a cubic foot of the material, of which the embankment is composed, 128 lbs.

We may take the length of the embankment $=1 \mathrm{ft}$., for if it stands for 1 ft . of length, it will stand for any other length. $1 \times 16 \times \frac{16}{2} \times 62 \cdot 5=8000 \mathrm{lbs}$., the pressure of the water.

$$
\therefore 8000 \times \frac{16}{3}=\text { the moment of the water. }
$$

If $x$ be put for the thickness of the embankment, its moment will be

$$
16 \times x \times 1 \times 128 \times \frac{x}{2}=x^{2} \times 8 \times 128
$$

Putting $8 \times 128 \times x^{2}=8000 \times \frac{16}{3}$, gives $x^{2}=\frac{125}{3} ; \quad \therefore x=6.45497 \mathrm{ft}$.
Ques. Let the cross-section of the embankment, A B C D, Fig. 2211, have the form of a trapezoid, where $A E=6 \mathrm{ft}$., $\mathrm{EB}=5 \mathrm{ft}$., $\mathrm{BC}=15 \mathrm{ft}$., and the weight of a cubic foot of the material = 120 lbs ; as in former cases, the cubic foot of water is supposed to weigh $62 \frac{1}{2} \mathrm{lbs}$.

Let us consider the circumstances with respect to 8 ft . length of embankment, and suppose the cross-section, ABCD, to be divided into two parts, namely, the rectangular part, BCDE, and the triangular part, AE D. It has been before shown that a vertical line, $g \mathrm{~L}$, passing through the centre of gravity, $g$, of the triangular part, cuts the base, A L, supposed to be horizontal ; so that $\mathrm{AL}=\frac{2}{3}$ of $\dot{\mathrm{A}} \mathrm{E}=4 \mathrm{ft}$. The vertical line, GF, passing through the centre of gravity, G , of the parallelogram, EDCB , cuts the base at F . so that $\mathrm{AF}=\mathrm{AE}+\frac{1}{2} \mathrm{~EB}=8.5 \mathrm{ft}$.

It is supposed that the pressure, P , of the water tends to turn the embankment over a horizontal line passing through $A$, perpendicular to the plane of the paper. $\frac{6 \times 1}{2} \times 8 \times 120=43200 \mathrm{lbs}$. weight of the part of which A D E is a cross-section; the moment of this part will be $=43200 \times$ $4=172800$.

Weight of BCDE $=15 \times 5 \times 8 \times 120=72000 \mathrm{lbs}$. Moment of this part $=72000 \times 8 \cdot 5=612000$. $\therefore 612000+172800=784800$, the moment of 8 ft . length of embankment.

Since the moment of the water will be $15 \times 8 \times \frac{15}{2} \times 62.5 \times \frac{15}{3}=281250$, it follows that the embankment will stand.

Ques. The breadth of a floodgate is 12 ft .; the depth $\mathrm{AB}=8 \mathrm{ft}$.; the centre of the hinge, Q , is 18 in . from the bottom $A$, and the hinge, $R$, is 18 in . from the surface, $B$; the pressure on $Q$, Fig. 2212, is required.


Since one-half the pressure of the water on the gate only acts on the hinges $Q$ and $R$, that pressure in lbs. will be $=8 \times 6 \times \frac{8}{2} \times 62.5=12000 \mathrm{lbs}$.

Let P be the centre of pressure of the water, then $\mathrm{AP}=\frac{8}{3} ; \mathrm{QR}=8-3=5 \mathrm{ft}$;

$$
\mathrm{PR}=\mathrm{PB}-\mathrm{BR}=\frac{8}{3} \times 2-1 \frac{1}{2}=3 \frac{5}{6} \mathrm{ft} .
$$

Because the pressure of the water at $P$ is supported by the hinges at $Q$ and $R$, then, on the principle of the lever, supposing R to be the fulcrum, $\therefore$ putting $x$ for the pressure on Q , $x \times \mathrm{QR}=\mathrm{P} \times \mathrm{PR}$, that is $x \times 5=12000 \times 3 \frac{5}{6} ; \therefore x=9200 \mathrm{lbs}$.

Ques. If one side of an equilateral triangle, immersed in a fluid, be perpendicular to the surface of the fluid, find the relation of the pressures on the three sides.

Let the side, A B, be perpendicular to the surface of the fluid L N, Fig. 2213. From F and G, the points of bisection, and therefore the centres of gravity of A C, C B , draw E F, D C, H G, perpendicular to AB .

It is evident that the perpendicular depths, $M \mathbf{F}=\mathbf{A E}, \mathbf{A D}, \mathbf{M G}=\mathbf{A H}$, of the centres of gravity of the sides, $\mathrm{AC}, \hat{\mathrm{A}} \mathrm{B}, \mathrm{BC}$, are as $1: 2: 3$. Hence the pressure on the side BC is equal to the sum of the pressures on A B, A C.

Geometrical Proposition.-If from any of the angles of a triangle, A B C, Fig. 2214, a line, A $m$, be drawn to $m$, the middle of opposite side, C B, the point $\mathbf{G}$ is the centre of gravity of the triangle if $m \mathbf{G}=\frac{1}{3}$ of $m \mathrm{~A}$.

Draw B $n$, bisecting A C, join $m, n$, then it is evident that all lines, as $p q$, parallel to C B are bisected by $\mathrm{A} m$; hence the centre of gravity of the triangle must lie in A m. In the same manner it may be shown that B $n$ bisects all lines, as $r s$, parallel to CA; therefore the centre of gravity is also in $\mathbf{B} n$; consequently the point $\mathbf{G}$, where $\mathbf{A} m$ and $\mathbf{B} n$ intersect, is the centre of gravity of the triangle A B C.

But $m n=\frac{1}{2} \mathrm{AB}$, and is also parallel to AB: and because the triangles
 $m n \mathbf{G}$ and $\mathbf{G A B}$ are similar, $m \mathbf{G}=\frac{1}{2} \mathbf{G} \mathbf{A}$, whence $m \mathbf{G}=\frac{1}{3} m \mathbf{A}$.

As the knowledge of the position of the centre of gravity of a body is of much importance in

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almost every department of mechanics, and to save the trouble of distinct investigations in cases that often occur, we have thought it proper, in this place, to add the succeeding results.

The centre of gravity, G, of a trapezium, A B CD, Fig. 2215.-Let L be the centre of gravity of the triangle ADC, H of ABC, E of ABD,F of BDC; join HL and EF; these lines cut in G, the centre of gravity of A B CD.

To find the centre of gravity, G, of a quiddrilateral, A B C D, when two sides, A D, B C, are parallel, Fig. 2216. $a=\mathrm{A} \mathrm{L}=\mathrm{L} \mathrm{D}$, and $\mathrm{B} \mathrm{K}=\mathrm{K} \mathrm{C}=b ; \mathbf{K} \mathrm{L}=3 c . \quad \mathrm{K} \mathbf{G}=c \frac{b+2 a}{b+a}$.

2217.


To finl the centre of gravity, G , of any triangular pyramid, ABCD , Fig. 2217.-Put $\mathrm{AB}=a$, $\mathrm{A} \mathrm{C}=b, \mathrm{~A} \mathbf{D}=c$, and $\mathrm{B} \mathbf{C}=d, \mathrm{~B} \mathbf{D}=c, \mathrm{C} \mathbf{D}=f$; then $\mathrm{A} \mathrm{G}=\frac{1}{4} \sqrt{3\left(u^{2}+b^{2}+c^{2}\right)-\left(d^{2}+c^{2}+f^{2}\right)}$. Or, bisect BC in F, draw F D, FA; make EF $=\frac{1}{3}$ of FD, and HF $=\frac{1}{3}$ of AF, and draw $H D, A E$. The triangles $H G E, G A D$ are similar. $\therefore \mathrm{HG}=\frac{1}{3} G D=\frac{1}{4} \mathrm{HD} ; \mathrm{EG}=\frac{1}{3} G A=$ ${ }_{4}^{1} \mathrm{EA}$.

Case 2.- When $\mathrm{B} \mathbf{C}=\mathrm{CD}=\mathrm{D} B$. Then $\mathbf{A} \mathrm{G}^{2}=\frac{3}{16}\left(a^{2}+b^{2}+c^{2}-d^{2}\right)$.
Case 3.-When $\mathrm{BC}=\mathrm{CD}=\mathrm{D} \mathrm{B}$, and also $\mathrm{A} \mathrm{B}=\stackrel{1}{16} \mathrm{C}=\mathrm{A} \mathrm{D}$. Then $\mathrm{A}^{2}=\frac{3}{16}\left(3 a^{2}-d^{2}\right)$.
Case 4.-If all the edges are equal, A B CD becomes a regular tetredron. Then $\mathrm{A} G=\frac{1}{4} a \sqrt{6}$.
To find the centre of gravity of a pyramid whose base is any polygon.-The centre of gravity will be on the line drawn from the vertex to the centre of gravity of the base, and at the distance of $\frac{3}{4}$ of its length from the vertex.

The centres of gravity of the surface of a cylinder, of a cone, and of a conic frustrum, are respectively the centres of gravity of the parallelogram, triangle, and trapezoid, which are vertical sections of the respective solids.

The centre of the surface of a spherical segment is at the middle of its versed sine, or height.
In the cone, as well as in the pyramid, the distance of the centre of gravity from the vertex is $\frac{3}{4}$ of the axis.

In a conical frustrum, the distance of the centre of gravity, measured on the axis from the centre of the less end $=\frac{h}{4} \frac{3 \mathrm{R}^{2}+2 \mathrm{R} r+r^{2}}{\mathrm{R}^{2}+\mathrm{R} r+r^{2}}, h=$ the height; $\mathrm{R}, r$, the radii of the greater and lesser ends.

The last theorem will apply for the frustrum of any regular pyramid, taking R and $r$ for the sides of the two ends.

In a paraboloid, the distance of the centre of gravity from the vertex $=\frac{2}{3}$ of the axis.
In the frustrum of a paraboloid, the distance of the centre of gravity from the centre of the lesser end, along the axis $=\frac{h}{3} \frac{2 \mathrm{R}^{2}+r^{2}}{\mathrm{R}^{2}+r^{2}} ; h$ denotes the height, R and $r$ the radii of the greater and lesser ends.

To find the centre of gravity, G, of a circular arc, M A m, Fig. 2218.-From the middle point of the arc A, draw AO to O, the centre of the circle; put $x=\mathrm{AP}$, $y=\mathrm{MP}=\mathrm{P} m$, radius $\mathrm{AO}=$ $r$; the length of the half-are, or $\mathrm{MA}=z=\mathrm{A} m$. Then OG $=\frac{r y}{z}$.


When the arc is a semicircle, then $y=1$; and $z=\frac{1}{2} \pi=1 \cdot 57079$, or $\frac{y}{z}=\frac{1}{1 \cdot 57079}=\cdot 63662$; and then $\mathrm{OG}=\cdot 63662 r$.

To find the centre of gravity, G , of a circular segment, M A m, Fig. 2219.-O $\mathrm{G}=\frac{\mathrm{MP}^{3}}{3 \text { area of } \mathrm{M} \mathrm{P}}$.

When the circulur segment becomes a semicircle.

$$
\mathrm{OG}=\frac{r^{3}}{(3 \text { quadrants }) \text { radius } r}=\frac{r^{3}}{2 \cdot 356194 r^{2}}=\frac{}{2 \cdot 356194}=\cdot 42441 r
$$

In a circular sector, O B A C, Fig. 2220, the distance from the centre of the circle to the centre of gravity, or $\mathrm{OG},=\frac{2 c r}{3 a}$; in which $r=\mathrm{OA} ; c=\mathrm{BC} ; a=$ the length of the are BAC .


To find the centre of gravity of a common parabola, Fig. 2221. $\mathrm{A} \mathrm{G}=\frac{3}{5} \mathrm{AC}$.

To find the centre of gravity, $g$, of the semi-parabola, CA D, Fig. 2221, take G $g=\frac{3}{8} \mathrm{CD}$.

To find the centre of gravity, G, of the sector of a sphere, ODAB, Fig. 2222.-Let $\mathrm{AC}=x, \mathrm{O} \mathrm{D}=r$, then $\mathrm{A} \mathrm{G}=$ $\frac{1}{8}(2 r+3 x)$. When $x=r$, the sector becomes a hemi-
 sphere, then $\mathrm{A} \mathrm{G}=\frac{5}{8} r$.

To find the centre of gravity, G , of the segment of a spheroid.- Let A be the rertex of the fixed axis $a$, putting $c$ for the length of the revolving axis, then $A G=\frac{4 a-3 x}{6 a-4 x} x$.

When the segment becomes a hemispheroid, then $x=\frac{a}{2}$ and $\frac{4 a-3 x}{6 a-4 x} x=\frac{5}{8} x$, for the distance of the centre of gravity from the vertex, $\therefore \frac{3}{8} x$ is its distance from the centre of the base.

If $c=a$, the spheroid becomes a sphere, and, as the theorem is independent of $c$, it is alike applicable to both solids and their corresponding segments.

To find the centre of gravity of a hyperboloid.-- P utting $g=$ the distance of the centre of gravity G , from the vertex A, and taking $y^{2}=\frac{c^{2}}{a^{2}}\left(a x+x^{2}\right) ; g=\frac{4 a+3 x}{6 a+4 x}$

The position of the centre of gravity, G, of any irregular body, A B C, may be determined when balanced in the manner represented in Fig. 2223, and applying the proportion, $\mathrm{W}: w:: a: z=\mathrm{R} \mathrm{Q}$. $\therefore z=\frac{a w}{\mathrm{~W}}$. RS is put $=a$, and is horizontal. $\mathrm{K} \mathrm{L}, \mathrm{Q} \mathrm{G}, \mathrm{S} \mathrm{T}$ are perpendicular to SR .


Ques. Let the shores, ac, Fig. 2224, support the wall, A C B, that sustains a pressure of water up to the top $A$; the stay delivers its thrust opposite the centre of pressure $P$, of the water; the thrust on the stays is required, when the embankment is upon the point of turning over. Suppose AB = 15 ft .; $C D=3 \mathrm{ft}$.; the weight of a cubic foot of the material of which the wall is composed 120 lbs . $; a d=\mathrm{PB}=5 \mathrm{ft}$.

We will estimate for 8 ft . length of wall, but any other length may be selected. Weight of wall $3 \times 15 \times 120=43200 \mathrm{lbs}$. Moment of the wall $=43200 \times \frac{3}{2}=64800$.

Let $b d$ be perpendicular to $a c ; \therefore b d=\sqrt{\frac{25}{2}}=\frac{5}{2} \sqrt{2}=3 \cdot 5355$.
Put $x$ for the thrust on the stays, $a c$, that prop 8 ft . length of wall; the moment of this thrust will be $x \times 3 \cdot 5355$.

Moment of the pressure of the water will be
$15 \times 8 \times \frac{15}{2} \times 62.5 \times \frac{15}{3 .}=281250 . \quad \therefore x \times 3.5355+64800=281250 \quad \therefore x=61222 \mathrm{lbs}$.
Ques. Required the modulus of stability of the stone structure, A D H O, Fig. 2225. A D=3 ft.; $0 \mathrm{H}=8 \mathrm{ft} . ; \mathrm{BR}$, drawn from the middle of AD to the middle of $\mathrm{OH}=18 \mathrm{ft}$; the height of the water, $\mathrm{HD}=17 \cdot 4 \mathrm{ft}$.; weight of a cubic foot of the material of which the wall is composed $=$ 200 lbs.

It is only necessary to investigate the action of the forces on a length of one foot. A DHO is a cross-sector of the wall.

$$
R G=\frac{1}{3} B R \frac{O R+A D}{O R+A B}=6 \frac{4+3}{4+1 \frac{1}{2}}=\frac{84}{11}, G \text { being the centre of.gravity of the wall. }
$$

$$
\mathrm{RT}=4-1 \frac{1}{2}=\frac{5}{2} \quad \text { R B }: \mathrm{RT}:: \mathrm{RG}: \mathrm{RS}=1 \frac{2}{33} . \quad \therefore \mathrm{SO}=5 \frac{2}{33} .
$$

$B T^{2}=B R^{2}-R^{2} T^{2}$, that is, $B T$ is equal to the square root of $18^{2}-\left(\frac{5}{2}\right)^{2}=\sqrt{1271}=17 \cdot 82554$.
Pressure of the wall, acting in the line GS, through the centre of gravity $\mathbf{G}=$

$$
\frac{8+3}{2} \times 1 \times 17 \cdot 82554 \times 200=19608 \cdot 094 \mathrm{lbs}
$$

The centre of pressure of the water is at P , and $\mathrm{H}^{\urcorner}=\frac{17 \cdot 4}{3}=5 \cdot 8 \mathrm{ft}$. $=\mathrm{CS}$.
Pressure of the water $=$
$17 \cdot 4 \times 1 \times \frac{17 \cdot 4}{2} \times 65 \cdot 2=9211 \cdot 25 \mathrm{lbs} . \quad \therefore 19608 \cdot 094: 5 \cdot 8:: 9211 \cdot 25: 2 \cdot 724652=\mathrm{C} m=\mathrm{S} n$.
In the parallelogram $\mathrm{CS} n m$, if the line $\mathrm{CS}=5 \cdot 8$, represents the pressure of the action in a vertical line passing through its centre of gravity; $\mathrm{C} m=2 \cdot 724652$ represents the whole pressure of the water acting on $\mathbf{P}$, its centre of pressure.

The ratio of SO to $\mathrm{S} n$ is termed the modulus of stability, which, in a good structure, should not be much less than $\frac{1}{2}$. In the present case, $\frac{\mathrm{S} n}{\mathrm{SO}}=\frac{2 \cdot 724652}{5 \frac{2}{33}}={ }^{\circ} 5384$, which is greater than $\cdot 5$. Hence the structure is secure.

2227.

2228.


A square, A B CD, Fig. 2226, is immersed vertically in a fluid, the side A B coinciding with the surface; if the diagonal, B D, be drawn, compare the pressure on the triangles ABD, B DC.

Bisect A B, D C, in E and F ; join DE, BF ; take EG $=\frac{1}{3} \mathrm{ED}$, and $\mathbf{F} g=\frac{1}{3} \mathrm{~B} \mathbf{F}$; G and $g$ are the centres of gravity of the triangles ABD, B DC.

DE is equal and parallel to $\mathrm{B} \mathbf{F}, \mathrm{B} g=2 g \mathrm{~F}=2 \mathrm{EG} ; \therefore$ the perpendicular $m g=$ twice the length of the perpendicular $\mathrm{G} n$.
$\therefore$ The pressure on the triangle BCD is double the pressure on the triangle ABD. The same is true in the case of a rectangle, and the proportions remain the same whatever be the inclinations of the immersed planes, provided only that A B coincides with the surface of the fluid; for the perpendicular depths of the centres of gravity will be altered in the same ratio.

Given a rectangular parallelogram immersed vertically in water, with one side A B, Fig. 2227, coincident with the surface; it is required to draw from one of the angles, $B$, to the base a straight line, B E, so that the pressures on the parts A D E B, E B C, may be in the given ratio of $m$ to $n$.

It is evident that the pressure on the whole parallelogram is to the pressure on the triangle, so is $m+n$ to $n$.

Put $\mathrm{AB}=a ; \mathrm{A}=\mathrm{BC}=b$, and $\mathrm{EC}=x ; \quad \therefore a \times b \times \frac{b}{2}: \frac{1}{2} x \times b \times \frac{2 b}{3}: m+n: n$ $\therefore \frac{a b^{2}}{2}: x \frac{b^{2}}{3}:: m+n: n \quad \therefore x=\frac{3}{2} \frac{n}{m+n} a$.

To compare the pressures on the rectangles A C, C F, Fig. 2228, immersed vertically in water, A B coinciding with the surface of the water.

The pressure on ABCD: that on ABEF : : AD $\times \frac{1}{2} A D: A F \times \frac{1}{2} A E:: A^{2}: A F^{2} ;$ $\therefore$ the pressure ABCD: the pressure on DCEF::AD ${ }^{2}: A F^{2}-A D^{2}$. Put AF $=b$ and $\mathrm{D} \mathrm{A}=x$, then $\mathrm{D} \mathrm{F}=b-x$.

When the pressure on $\mathbf{A C}$ is equal to the pressure on CF , then

$$
x^{2}=b^{2}-x^{2} \quad \therefore 2 x^{2}=b^{2} ; \text { or } x=\frac{b}{\sqrt{2}} \text {. Or } 1: \sqrt{2}:: x: b
$$

If the pressure on ABCD is to be to the pressure on $\mathrm{DCEF}, 5$ to 7 , then

$$
x^{2}: b^{2}-x^{2}:: 5: 7 \quad \therefore 7 x^{2}=5 b^{2}-5 x^{2} \quad \therefore x=\sqrt{\frac{5}{12}} b
$$

Let A C D F, Fig. 2229, be a rectangular parallelogram immersed in water, the side A C coinciding with the surface, $\mathrm{AB}=a ; \mathrm{AF}=b . \quad \mathrm{FBD}$ is the inscribed parabola; find the ratio of the pressure on the parallelogram and the pressure on the parabola.

If $G$ be the centre of gravity of ACDF, then $B G=\frac{b}{2}$ and the pressure

$$
=a b \times \frac{b}{2} \times 62 \frac{1}{2}=\frac{a b}{2} \times 62 \frac{1}{2} .
$$

The area of the parabol: $=\frac{2}{3}$ of the parallelogram; $\mathbf{B} g=\frac{3}{5} b$, if $g$ be the centre of gravity of the parabola; $\therefore \frac{2 a b}{3} \times \frac{3 b}{5} \times 62 \frac{1}{2}=\frac{2 a b^{2}}{5} \times 62 \frac{1}{2}=$ the pressure on the parabola

$$
\therefore \frac{a b}{2} \times 62 \frac{1}{2}: \frac{2 a b^{2}}{5} \times 62 \frac{1}{2}:: \frac{1}{2}: \frac{2}{5}
$$

$\therefore 5: 4$, is the ratio of the pressure on the parallelogram to the pressure on the parabola.
Show that if a hollow sphere be filled with a fluid, whose specific gravity is $s$, that the whole pressure against the internal surface is three times the weight of the contained fluid.

Let $r=$ the radius; then $4 \pi r^{2} \times$ internal surface, $\pi$ as usual $=3 \cdot 14159265 . \quad \therefore$ the pressure $=4 \pi r^{2} \times r \times s=4 \pi \pi^{3.3} s$; the solid content of the sphere $=\frac{4}{3} \pi r^{3}$, and its weight $=\frac{4}{3} \pi r^{3} s$. $4 \pi r^{3} s: \frac{4}{3} \pi r^{3} s \cdots 1: \frac{1}{3}$ or $3: 1$.


Let G be the centre of gravity of a trapezoid ABCO, Fig. 2230, A B = a, and parallel to $\mathrm{OC}=b ; \mathrm{BC}=c$, and perpendicular to both AB and OC . The straight line OY is drawn perpendicular to OCX , it is required to find general expressions for the perpendicular co-ordinates $x=m \mathrm{G}=\mathrm{O} n$, and $\mathrm{Y}=n \mathrm{G}=\mathrm{O} m$.

Let $p$ be the centre of gravity of the rectangle $\mathrm{ABCD}, p \mathrm{Q}=\frac{c}{2}$ and $\mathrm{Q} \mathrm{O}=b-\frac{a}{2}=\frac{2 p-a}{2}$.
If $t$ be the centre of gravity of the triangle OAD , then the co-ordinates of $t$ will be

$$
t \mathrm{R}=\frac{1}{3} \mathrm{AD}=\frac{1}{3} \mathrm{BC}=\frac{c}{3} \cdot \quad \mathrm{OR}=\frac{2}{3} \text { of } \mathrm{OD}=\frac{2(b-a)}{3}
$$

Area of $\mathrm{ABCO} \times \mathrm{G} n=$ area of $\mathrm{ABCD} \times p \mathrm{Q}+$ area of $\mathrm{AOD} \times t \mathrm{R}$; that is,

$$
c \frac{a+b}{2} y=a c \times \frac{c}{2}+\frac{(b-a) c}{2} \times \frac{c}{3} \quad \therefore y=\frac{c}{3}\left(1+\frac{a}{a+b}\right) .
$$

Again the area $\mathrm{OABC} \times \mathrm{G} m=$ area of $\mathrm{ABCD} \times \mathrm{OQ}+$ area of $\mathrm{AOD} \times \mathrm{RO}$; that is,

$$
\begin{gathered}
c \frac{b+a}{2} x=a c \times \frac{2 b-a}{2}+\frac{c(b-a)}{2} \frac{2(b-a)}{3} \therefore \frac{b+a}{2} x=a \frac{2 b-a}{2}+\frac{(b-a)^{2}}{3} \\
\therefore x=\frac{2 b}{3}-\frac{a^{2}}{3(a+b)} .
\end{gathered}
$$

Ques. Given in A BCO, Fig. 2231, which represents the cross-section of an embankment made of brickwork, a cubic foot of which weighs 112 lbs.; A B $=1 \mathrm{ft}$., parallel to $\mathrm{OC}=2 \mathrm{ft}$; find the height, B C (which is perpendicular to both A B and OC ), when the embankment is upon the point of overturning upon the edge at $\mathbf{O}$ by the pressure of the water which stands at the brim, $\mathbf{B}$.

Put $v=B C$, the required height, then the pressure of the water on 1 ft . length of embankment $=v \times 1 \times \frac{v}{2} \times 62.5=\frac{v^{2}}{2} \times 62.5$. Moment of the water $=\frac{v^{2}}{2} \times 62.5 \times \frac{v}{3}=\frac{r^{3}}{6} \times 62.5$.

If $G$ be the centre of gravity of the trapezoid ABCO , then by the last proposition,

$$
\mathrm{OD}=\frac{2}{3} \times 2-\frac{1}{3(1+2)}=\frac{11}{9}
$$

Pressure of $\_$langth of 1 foot of the embankment $=\frac{1+2}{2} v \times 112=168 v \mathrm{lbs} . \cdot$ moment of embankment $=168 v \times \frac{11}{9} . \quad \therefore \frac{v^{3}}{6} \times 62.5=168 v \times \frac{11}{9} ; \therefore v=\sqrt{19 \cdot 712}=4 \cdot 439821$. Weight of embankment $=168 v=745 \cdot 89 \mathrm{lbs}$.

Ques. Let the embankment A B C O, Fig. 2232, be the same as in the previous question; now if the embankment be raised until $\mathrm{HL}=3 \mathrm{ft}$., what must be the perpendicular depth BL, so that when the part ABCO be upon the point of overturning at O , at the same time the whole embankment, A BLH, will be upon the point of turning over the edge through H

$$
\mathrm{EL}=\mathrm{DC}=2-\frac{11}{9}=\frac{7}{9} \quad \therefore \mathrm{HE}=3-\frac{7}{9}=\frac{20}{9}
$$

$\therefore$ the moment of OABC round the point H , will be $745.89 \times$ $\frac{20}{9}$. If $g$ be the centre of gravity of OCLH, then

$$
\mathrm{HF}=\frac{2}{3} \times 3-\frac{4}{3(2+3)}=\frac{26}{15}
$$

putting $z$ for $B L$, the moment of the part OCLH round $H$ as a fulcrum will be $\frac{2+3}{2}(z-v)(112) \times \frac{26}{15}$.


The moment of the water acting in opposition will be $z \times 1 \times \frac{z}{2} \times 62.5 \times \frac{z}{3}=\frac{z^{3}}{6} \times 62.5$ $\therefore \frac{z^{3}}{6}(62 \cdot 5)=\frac{2+3}{2}(z-4 \cdot 43982)(112) \frac{26}{15}+745 \cdot 89+\frac{20}{9}$.

$$
\therefore z^{3}-46 \cdot 592 z=-47 \cdot 73689344 ;
$$

$$
\therefore z=6 \downarrow 0,4,0,5^{\prime} 1^{\prime} 9^{\prime} 9^{\prime} 5=6 \cdot 24037812 .
$$

Sard Dams in Holland.-The following description of the method adopted in preparing the foundation and in building the bridge over the Poldervaart, on $t$.e line of the Amsterdam and Rotterdam Railway, by the Chevalier Conrad, translated and cumpiled by Charles Manby, in the 'Trans. Inst. C. E.,' includes an interesting account of these important dams.

The Poldervaart is a canal for the purpose of conveying away the waters from the Polders, whicb it encompasses, in the commune of Kethel; it is under the government of the Hoogheemraadschap, or Direction of the High Dykes, of Delfland, and runs from the river Sclie to the Five Sluices, on the river Maas. The railway crossing this canal at a considerable angle, rendered a skew bridge necessary, and at length the follnwing provisions were agreed upon:-

The bridge was to have three openings; the centre one, for the navigation, to measure $13 \mathrm{ft}$.1 in ., and the two lateral openings for the drainage 21 ft each, giving a total width of water-way of 54 ft .11 in ., while the level of the timber planking of the land piers was fixed at $9 \mathrm{ft} .8 \frac{3}{4} \mathrm{in}$., below A.P., and that of the top of the landings at $2 \mathrm{ft} .7 \frac{1}{2} \mathrm{in}$. above A.P.

Although the treacherous quality of the soil was generally known, and it was evident that great difficulties must be encountered, yet such harassing casualties as did actually occur were scarcely anticipated, and they proved in reality greater than could have been imagined. The means, therefore, that were adopted for obtaining solid foundations, and for rendering the structure firm under the passage of heavy trains, may, it is hoped, afford data for the pro: ecution of similar works in boggy ground, and aid in avoiding expensive experiments in parallel cases.

The general plan and the details of the work are given in Figs. 2232A to 2232D.
Fig. 2232A shows the gen eral situation of the bridge, with the excavation for the foundations, surrounded by dykes, or dams of sand, and the side-cut, for diverting the water during the progress of the construction. This lateral canal was cut for the purpose of preventing any impediment to the navigation, and for the temporary outlet of the drainage water.

Fig. 2232B gives the general construction of the bridge when finished, and the section of the strata as shown by the borings.

Figs. 2232c, 2232D exhibit the accidental movements of the ground which occurred during the progress of the works; and the various details alluded to in the description, which necessarily assumes almost the character of a journal, or diary of the proceedings.

As in the course of this description the subsidence of the ground will be frequently alluded to, it is necessary to point out the difference existing between the privcipal kinds of subsidence which occur in the execution of works in Holland.

They may be classified under three heads, completely distinct from each other :-

1. Sinkings (Verzinkingen) are those sudden movements whereby, without any warning or perceptible preliminary motion, or any indications such as cracks, bends, or fissures, a considerable area of ground entirely disappears. as if by some unseen operation of natural causes the spot had been undermined; but enough solidity remained to preserve the equilibrium, until it was destroyed by some apparently slight cause.
2. Yieldings (Verzakkingen) are the slow but constant movements which are usually indicated by previous crevices or undulations of the surface, with corresponding gradual elevations around. These are occasioned by treacherous subsoils, dislocations of the strata, the saturation of the ground on account of the absence of proper drainage, or the destruction of the equilibrium by the overloading of a particular spot, calculated to bear only a certain weight.
3. Slips (Verschuivengen) arise from the separation of a mass of fresh earth from old banks
upon the side of which it has been thrown, without, however, being well amalgamated with them. These slips arise either from the action of wet, or from the earth being placed at a greater angle than it can stand at.

These several movements are entirely distinct from the subsidence of new-made ground, and can easily be distinguished from their peculiar features.

After the completion of the side-cut, and determining the position of the inclosure made by dams of sand, and the excavation of the foundation pit, the work was commenced on the 20th of April and without any previous indications of extraordinary difficulties the casualties commenced.


The first process was that of shooting in sand to form the dam A B, Fig. 2232c; this progressed satisfactorily until April 23rd, by which time a length of 71 ft . was formed to above the water level. At about five o'clock in the evening, during a violent thunderstorm, the dam suddenly sunk 29 ft ., and simultaneously an immense mass of bog earth C, Fig. 2232c, of an area of $4489 \mathrm{sq} . \mathrm{ft}$., rose to a considerable height above the water level. From the soundings then taken, and the mure accurate investigation of the locality, it was evident that an extensive subterraneous shifting of the bogearth had occurred; and as it was more than probable there would be a recurrence of it, without the existence of any power of prevention, it became necessary to direct attention to some means of rendering the movement as little injurious as possible. It was therefore determined to discontinue for a time the depositing of sand, and to construct a side-dyke D E F G, Fig. 2232c, in order to pre-
serve the dyke of the North Kethel Polder, the security of that spot being of the utmostimportance; also to place the layers of fascines for the railway embankments, and to enlarge and heighten the earthwork by additions of sand. The passage of the side-cut having become interrupted by the rising of the bottom, it was determined to prolong it to beyond the sand-dams, so as to keep it open for the navigation.

The shooting in of sand was then continued uninterıuptedly until April 30th, whilst the construction of the side dyke DEF G was continued with all speed, as another subsidence at $a, b$, of $67 \mathrm{ft} .9 \frac{3}{4} \mathrm{in}$. long, and from $11 \mathrm{ft} .3 \frac{3}{4} \mathrm{in}$. to $19 \mathrm{ft} .4 \frac{1}{2} \mathrm{in}$. de $\mathbf{p}$ was discovered in front of the old North Kethel Polder, by which the up-raised mass of bog had acquired an additional extension.

In order to prevent the sinking, and to support the base of the North Kethel Polder dyke, which was apparently almost without any foundation, another deposit of sand was made at H I, at a distance of about $32 \mathrm{ft} .3 \frac{1}{2} \mathrm{in}$. from the foot of the dyke, of the entire width of the foundation pit, with a number of transverse dams, e $d, c d$, limiting the extent of any accidental movement which might occur.

On the 5 th of May another sinking, 48 ft . $5 \frac{1}{2} \mathrm{in}$. in length, and from $9 \mathrm{ft} .8 \frac{1}{4} \mathrm{in}$. to $19 \mathrm{ft} .4 \frac{1}{2} \mathrm{in}$. depth, took place in front of the North Kethel Polder dyke, still further increasing the size of the mass of upraised bog-carth C , when the same moans of repair as had been befcre employed were again resorted to.

Considerable agitation of the water in the canal Poldervaart was observed on the 7th of May, and another sinking occurred in front of the same spot, to such an extent tlat the sand, which had been thrown in front of the dyke, subsided over a length of 77 ft . $6 \frac{\mathrm{i}}{5}$. to a depth of from $9 \mathrm{ft} 8 \frac{1}{4} \mathrm{in}$. to 21 ft . The mass of upraised bog-earth C had thus become so enlargel as to chi ke up the mouth of the side-cut, which, laving been already lengthened, was then extended to the Kethelvaart.

The deposit of sand was still continued, and at times masses of bog-enrth rose between the dams, of the areas of 3357 ft . and 1882 sq. ft . respectively, at K K , which at first were connected with the other masses, but were separated by the constaut shooting in of the sand; the sand-dam, which had been raised above the water, still sinking at times to considerable depths.

On the 15th of May, sand was first deposited in front of the second dam L M, and the same symptoms appeared at N , as near the other dam, only to a less extent. An extension of 64 feet 7 in . was therefore given to the side dyke, behind the North Kethel Polder dyke.
May 26 th . -The turf-bog inside the foundation-pit was found to have been raised $1 \mathrm{ft} .7 \frac{1}{4} \mathrm{in}$. above A.P., by the shooting in of the sand for forming the dams A B and LM Fig. 2232c, which by the 28th of May were raised above the water level, and the foundation-pit was complctely surrounded. The draining of the pit was commenced the next day, and the deposit of sand was still continued, in order to increase the strength of the dams. By the 3rd of June the pit was drained, and the excavation of the bog-earth was commenced.

On the 9 th of June a testing pile, $53 \mathrm{ft} .11 \frac{1}{4} \mathrm{in}$. in length, was driven into the foundation, to within 12 in . of its head; another pile of $69 \mathrm{ft} .9 \frac{1}{2} \mathrm{in}$. was then driven to within 6 in . of the head, when it was resolved to drive a large number of thise piles, and to alter the original design of separate foundations for each pier into one general foundation, extending throughout the base of the whole structure. It was then discovered that the bog-earth had again risen considerably, whilst the dams had sunk. In consequence of this, the excavation of the pit was discontinued, and the driving of an entire system of piling was commenced, with the intention of subsequently digging out the foundation to the necessary depth. After, however, driving 65 piles, the pit and the piles together appeared to have moved bo lily nearly $7 \frac{3}{4} \mathrm{in}$. This arrested the process of pile-driving for a time, until other measures of security should have been devised.

On the 24 th of June a row of close-piling O P, Fig. 2232d, was driven, in order to support the bank of the side-cut, where the grif at at anount of movement was perceptible, and for preventing it from slipping into the pit. At the same time it was resolved to form a sunk coffer-dam of sand, loaded with coarse rubble Q R S T', within the foundation-pit. This coffer-dam was sunk to a deptlo of 5 ft . $9 \frac{3}{4} \mathrm{in}$. by a breadth of $9 \mathrm{ft} .8 \frac{1}{4} \mathrm{in}$., to enclose that part of the pit in which the piling was to be driven.

By the 4th of July, the piling for securing the dyke of the side-cut was completed, and that for the land pier on the Kethel side V V was commenced, as the movement of the ground appeared to have ceased at that spot. The sand coffer-dam Q R S T, being completed, the greatest deviation among the piles was found to be about $5 \mathrm{ft} .9 \frac{3}{4} \mathrm{in}$. It, however, became necessary to stop the excavation, as the bottcm continued to rise fast, whilst the dyke of the side-cut began to sink, and numerous fissures were apparent.

On the 3rd of August the excavation was again commenced, with the intention of taking out the whole of the bog-earth within the sand coffer-dam, and filling the space with sand.

On the 5th of August, it was discovcred that the dyke A L, and the row of piles O P were slipping, and had assumed a concave form at O P; the sand-dam A B, on the north side, becoming more extensively cracked and threatening to give way, which would have cuused an irruption of the canal into the foundation-pit.

An examination of the strata was then made, by boring nearly in the centre of the foundationpit, when it was found that, at a depth of 17 metres under A.P. ( 54 ft .11 in .) a bed of coarse river sand would be arrived at. The section of the strata shown by this boring is given in Fig. 2232b.

The saud bedding having been put in, the pit and the piling proceeding regularly, it was determined to form a rectangular foundation at two points laterally with the side-cut. The row of piles O P, Fig. 2232d, had by this time become about $6 \mathrm{ft} .5 \frac{1}{2} \mathrm{in}$. concave.

The fixing of waling-pieces was then commenced; but by the time six of them had been fixed, and the intervening spaces filled with sand and rubbish, the portion near the back dyke had swerved $5 \frac{3}{4} \mathrm{in}$. in a length of 29 ft . $0 \frac{3}{4} \mathrm{in}$. These deviations rendered necessary the driving of more piles outside the foundation, at the foot of the sand-dam, in such a manner that when the planking for the foundation was ready, the diagonal timber could be affixed, so as to prevent any further swerving,


By the 8th of September, the whole of the foundation was completed, and the first stone of the bridge was laid, many parts being then strengthened beyond the original design. The two buttresses, built upon the rectangular projecting foundations, behind the land pier, are portions of these alterations, which were rendered necessary by the nature of the ground.

After this period no perceptible movement of the ground occurred, and the structure was completed without further impediment.

In reviewing these details, the circumstances of the sinking of the first portion of the dam, and the simultaneous rising of the mass of bog-earth during a heavy thunderstorm, deserve particular notice.


The sudden elevation of masses of bog-earth, in the canals and drains of Holland during storms, is not uncommon. It would appear that the adhesion of the masses of bog-turf to the bottom is so slight that the vibration communicated to the water by the thunder suffices to destroy the equilibrium ; and the bog-turf, which from its slight specific gravity will float in water, even when saturated, instantly rises to the surface. When therefore, as in this case, a heavy mass of sand is placed in the vicinity of such bog-earth, the bottom is unable to resist the pressure, and the least vibration causes it to break through the crust and become engulphed amidst the lighter material, which it forces upwards in the direction of the least resistance.

The sudden elevation of the masses of bog earth in this instance may be easily accounted for. The first portion was elevated above the water level, and a cavity was formed of far greater cubic content than the sand-dam which it received. It was evident also that the cavity extended as far

## DAMMING.

as the dyke of the canal Poldervaart, which would probably have been seriously injured if it had bcen recently constructed; but haviug been formed for some years, it had been consecutively weighted with new materials, as it had subsided until the mass had become compact, and had probably penetrated through the upper crust, until it rested upon the more solid strata beneath, and thus the dykes after some time, although their dimensions are apparently inconsiderable, attain great strength, and offer effectual resistance in cases of the sinking of the bottom. Nevertheless, if a subsidence had occurred in the immediate vicinity of the old dyke, a portion might have slid into the chasm, and the whole dyke would have sunk outwards. It was to prevent such a rupture and its injurious consequences that the sand-dam B M, Fig. 2232c, was formed, so as to consolidate the ground immediately beneath the foot of that part of the old dyke, and that another external sanddyke $\mathrm{D} G$ was raised behind it.


The journal of the proceedings shows that but for these precautions and the incessant filling in of sand, the new works would not have acquired solidity enough to support the old dykes, and to have prevented their rupture.

The importance of loading the spot with sand was demonstrated by the fact of the eleration of the masses of bog-earth before the draining was commenced, and even after the pit was pumped out, when the earth was relieved from the pressure of the water.

It might have been expected that in consequence of the piling the ground would have become more compact; but such was the nature of the substratum that, as has been described, the whole of the piles were forced by the general movement out of their direction, and a cessation of piling, for a
time, was deemed desirable. The vibration communicated by the operation of driving the piles probably assisted this movement.

The extent of the movement was also shown by the line of close piling O P, Fig. 2232p, being unable to resist the weight impinging upon it, and its being forced outwards in a concave form.

The internal dam of sand Q R S T, sunk in the pit and loaded with rubbish, proved the most efficacious means for restraining the movement and for enabling the bog-earth to be excavated from within the pit; as unless that had been effectually done, the fouudations could not have been securely laid. Great precautions were necessary in excavating this bog-eartl, and it was only accomplished by cutting out consecutive masses of $3 \mathrm{ft} .2 \frac{3}{4} \mathrm{in}$. sq , and immediately filling the spaces up with sand; thus exchanging the light for a heavy material, proceeding round the limits of the spot, and clos ng inwards, until the whole area was covered with sand, or those portions of bog-earth that remained were, by the process, rendered so compact as to be innocuous.

The operation succeeded in arresting the morement, so as to permit tl. e piles to be driven through the artificial ground, and the foundation was rendered sufficiently solid to bear the bridge.

After what has been described it is perhaps scarcely necessary to make any general remarks. It may, however, be argued, that for obtaining a solid foundation in bog, or other liglit soils of similar character, filling in sand, or other dry material, in large quantities, is the simplest method; but that stability cannot be insured until the whole of the light soil has been removed and the space has been filled by more compact material.

Whenever, then, the new works are isolated, so that the process cannot cause any injury around, the result could be attained by dredging trenches under water, down to the solid strata, to enable the new material for forming the dams to be filled in; or the filling in of the sand could be commenced in the centre of the intended foundation and be carried on to the riglit and the left, so as to press the bog-earth upwards on either side, continuing the process until the new material rested on a solid substratum, and all that portion of bog-earth which was not entirely removed was consolidated by the pressure. The embankment so formed being brought up abore the water level, the foundation-pit might be excavated within the embankment or mound, and the foundations of the structure be laid with perfect security.

Hydraulic System of the Reservoir on the River Furcns.-The town of Saint-Etienne is supplied with water to some extent by means of a subterranean passage or aqueduct which conveys the water directly from the springs at the source of the Furens. In the year 1858, for the purpose of protecting the town from inundations, the French Government undertook to construct an immense reservoir upon this river at a cost of about $1,570,000$ francs, and it was agreed that the excess of the cost above 570,000 francs should be borne by the town on the condition that it should have a right to a portion of the reservoir to store up for its own use the surplus water of the river. Fig. 2233 shows the general arrangements adopted to secure the proper working of the reservoir under the complex conditions imposed on it.


Previous to the construction of these works, the Furens followed the course M APCDGS. A barrage 50 mètres in height bars the valley at the point C. Its section and elevation are shown by Figs. 2234, 2235. At B, Fig. 2233, a by-wash B N D has been constructed, and this now forms the bed of the Furens, the portion A P C D of the former bed being now covered by the reservoir. At $A$ and $B$ are two water-gates or sluices, one of which communicates with the reservoir by the old bed A P , and the other with the by-wash BND which restores the water to the river at D.

Let us now see how the water in the reservoir is utilized, and first we must remark that the level at which the town of Saint-Etienne may retain its water is fixed at $44^{\mathrm{m}} \cdot 50$ above the bottom in front of the great wall, as shown in Figs. 2234, 2235. Above this level there is a height of $5 \mathrm{~m} \cdot 50$, which is always to be kept empty to receive the surplus water in the case of a sudden rise of the river. As soon as the river has returned to its ordinary state, the surplus water is drawn off by a subterranean passage $\mathrm{E}^{\prime}$ N, Fig. 2235, into the by-wash or present bed of the Furens. We shall now see how the reservoir is adapted to supply both the town of Saint-Etienne and the factories in accordance with the conditions stated above.

A subterranean channel is cut in the counter-fort against which the great wall rests, in the direction of the line E F, Fig. 2233, and in this channel, stopped with masonry at the reservoir and, see Fig. 2235 , there are two cast-iron pipes, each of 0 , stopped with masonry at the reservoir masonry, which pipes convey the water to a bag F , by means of cocks $r$ capable of opening to any degree. When the water has been brought into the bay $F$ the double duty of supplying the town and the factories remains to be performed; to accomplish this, an open canal $\mathbf{F} d \mathrm{G}$, Fig. 2233, has been constructed with a regulator-sluice at $d$ to convey the reserved water into the bed of the

Furens at G, and a second covered channel F $l \mathrm{HK}$, provided at $l$ with a regulator-sluice, conveys the water into the Saint-Etienne aqueduct, either directly at H by means of a cock, or through a small reservoir $\mathrm{K} L$ which communicates with the aqueduct by a pipe LV , provided at V with a regulator-cock. Fig. 2235 gives a sectional view of the arrangements of the channel which connects the bay $F$ with the Saint-Etienne aqueduct.

During the summer the watering of the streets and the washing of the sewers is done by means of the resources of the reservoir, the water of the aqueduct being insufficient for this purpose. For this operation communication is established between the aqueduct and the reservoir by the canal FlHK, Figs. 2233 and 2235. If it is required during this season, when the factories on the Furens are obliged to cease work for considerable periods, to increase the discharge of the river, communication is opened with the reservoir by the canal $\mathbf{F} d \mathrm{G}$.

The aqueduct which conveys the water from the source of the Furens is everywhere quite independent of the reservoir, with which it communicates only by the


Section through A C, Fig. 2233.


Section through line TEFNHK, Fig. 2233.
channel $\mathrm{Fl} \boldsymbol{l} \mathrm{HK}$. Fig. 2236 shows the relative positions of the aqueduct and the Furens near the point $A$ at the mouth of the reservoir.

Functions of the Water-gates above the Reservoir.We have now to explain the working of the watergates A and B placed at the head of the feed-canal of the reservoir A P, Fig. 2233, and the by-wash BND . When the discharge of the river reaches the point M, where there is a scale showing the depth of the water, 93 cubic mètres a second, which discharge corresponds to a height of 2 metres on the scale, the town of Saint-Etienne begins to be inundated. Let us suppose the rise to take place when the reservoir is full, that is to the permanent height of $44^{\mathrm{m}} \cdot 50$, the most unfavourable case. The gate A will be left shut and the gate B open so long as the water does not rise above this height of 2 mètres on the scale at the point M, the height corresponding to a discharge of 93 cubic mètres, and below which no injury is to be feared. In this case all the water will flow through the by-wash to join the


Section through M R, Fig. 2233. river again at $D$. But as soon as the water rises on the scale $M$ above 2 mètres, that is as soon as danger becomes imminent, the gate $B$ remaining open, the gate $A$ is opened and by its action the height of the water will be kept at 2 mètres on the scale M. This is easily accomplished, as the gate $\mathbf{A}$ is constructed to discharge the difference, 38 cubic mètres, between the maximum discharge, 131 cubic mètres, a second of the greatest known rise, and 93 cubic mètres. The excess of water will thus be received into the reservoir, and will accumulate in the space of $5{ }^{\mathrm{m}} \cdot 50$ reserved above the line of the permanent level.

Let us now consider the working of these gates A and B in furnishing the permanent reserve, and to this end we will suppose the reservoir empty, which is the case at the close of the summer.

In the first place it is of course necessary to ensure the regular working of the factories under the conditions which existed before the construction of the reservoir, and for this we will suppose a discharge of 350 litres a second requisite. The depth of water corresponding to this discharge is marked on the scale M. So long as the level of the water in the bed of the river remains below his mark, the gate A must be left closed, as all the water will be needed by the factories situated below the reservoir. But as soon as the mark is submerged the gate A may be opened, so as to maintain the proper depth, and then all the execss of the discharge above 350 litres will pass into the reservoir by its feed-canal AP. If the flow of water that produced the rise ceases, the gate $\mathbf{A}$
must be progressively closed, and entirely shut when the discharge of the river has sunk to 350 litres.

It will be seen from this that only that portion of the water which is not required by the factories is taken from the river. When the Furens discharges more than 350 litres a second, the useless surplus is stored up, to be used in the summer when the river, as always happens in dry summers, discharges only from 80 to 100 litres a second. The advantages accruing to the manufacturers from this arrangement will be at once perceived.

Discharge of the Furens.-Capacity of the Reservoir.-According to the daily measurements made during eight years by the engineers who constructed the barrage, in very dry years the quantity of water discharged by the Furens descends as low as 100 and even 80 litres a second; the mean quantity a second throughout the year being about 500 litres. The superficies of that portion of the bed of the river, situated above the reservoir, which furnishes this discharge, is about 2500 hectares, and the mean depth of water falling into it a year is 1 mètre.

The discharge during the greatest rises observed in the Furens between 1858 and 1868 did not exceed 15 cubic mètres a second; but on the 10 th of July, 1849 , a water-spout having burst in the upper part of the valley, an extraordinary rise took place, inundating the town of Saint-Etienne. This was the discharge which it was necessary to determine approximatively in order to fix the capacity of the reservoir; this the engineers to whom the work was entrusted were enabled to do from information obtained on the spot, the value which they found for this unusual rise being 131 cubic mètres. Fig. 2237 is the curve of the discharges of this exceptional rise of 1849. The capacity of the empty portion of the reservoir ought evidently to be equal to the area of the portion of the curve situated above the discharge of 93 cubic mètres, when the inundation of the town begins, which portion is detached in the figure. Now this area is equal to $\left(\frac{131-93}{2}\right) \times 3 \times 3600=205,200$ cubic mètres. The upper portion of the reservoir to be left empty for the purpose of


Axis of the times. receiving the surplus water in case of a rise should, therefore, be capable of containing, in round numbers, 200,000 cubic meetres.

It was seen above that the portion in question was comprised between the horizontal planes situated at 50 mètres and at $44^{\mathrm{m}} \cdot 50$ above the bottom of the reservoir, near the barrage, and, consequently, was $5 \cdot 50$ in depth. From very exact calculations made since the completion of the works, it was found that the contents corresponding to the permanent level of $44 \mathrm{~m} \cdot 50$ were equal to $1,200,000$ cubic metres, and that the contents corresponding to the height of 50 metres were equal to $1,600,000$ cubic mètres. It follows from this that the capacity of the portion intended to receive the surplus water is 400,000 cubic mètres, or the double of that required to contain the destructive portion of the water-spout which burst upon the Furens in 1849. A rise like that of 1849 would give in the reserve portion a depth of only 3 mètres, corresponding to a cube of 200,000 cubic mètres, and to a height of $47^{\mathrm{m}} \cdot 50$ above the bottom of the reservoir, whilst the depth of this portion is $5^{\mathrm{m}} \cdot 50$, corresponding to a cube of 400,000 cubic mètres. Thus it will be seen that arrangements have been made so as to remove all danger from mistakes in calculation.

From measurements and calculations made during a period of eight years, and from the experience of the years 1865 and 1866 had of the reservoir itself, it has been ascertained that the permanent contents of $1,200,000$ cubic mètres are renewed twice a year, in autumn and in spring. The quantity required for the supplementary service of the town of Saint-Etienne can in no case exceed 600,000 cubic mètres a year, so that there remain to be distributed among the factories $2,400,000-600,000$ $=1,800,000$ cubic mètres; or 120 litres a second for six months. Thus the advantages derived from the construction of this reservoir are very great. We have now to explain why a barrage 50 mètres in height, that is, the highest that has ever been constructed, was preferred to two reservoirs with barrages of moderate height.

In the valley in question two reservoirs would have required two barrages each 38 mètres in height, at a cost of $1,810,000$ francs, to obtain the capacity of $1,600,000$ cubic metres, offered by the barrage of 50 mètres for a single reservoir at a cost of $1,600,000$ francs. Thus by constructing only one reservoir a saving was effected of 240,000 francs.

In narrow valleys, the cost of retaining a given quantity of water increases with the number of reservoirs cmployed. This consequence, which we have drawn practically from the numerous comparative studies of reservoirs which we have made, may be arrived at also by theory.

Suppose the reservoir divided into horizontal portions, Fig. 2238, and let A and A' be the lower and the upper sections of one of these portions, the height being K . The expression of the volume of this portion would be $\mathrm{W}=a \mathrm{~K}+\frac{b \mathrm{~K}^{2}}{2}+\frac{c \mathrm{~K}^{3}}{3}$. The values of $a, b, c$, are, if S and $\mathrm{S}^{\prime}, \mathrm{D}$ and $\mathrm{D}^{\prime}$ represent the surfaces and the developments of the perimeters of the sections $A$ and $A^{\prime}$,

$$
\mathrm{A}=\mathrm{S} \quad b=\frac{2 \mathrm{D}\left(\mathrm{~S}^{\prime}-\mathrm{S}\right)}{\mathrm{K}\left(\mathrm{D}+\mathrm{D}^{\prime}\right)} \quad c=\frac{\left(\mathrm{S}^{\prime}-\mathrm{S}\right)\left(\mathrm{D}^{\prime}-\mathrm{D}\right)}{\mathrm{K}^{2}\left(\mathrm{D}+\mathrm{D}^{\prime}\right)} .
$$

When the talus is uniform as in this case, we may admit only one portion, and then K represents
the height of the contents in front of the barrage. Now we see that the expression of W , by substituting the values of $a, b, c$, becomes

$$
W=\frac{D\left(2 S^{\prime}+S\right)+D^{\prime}\left(2 S+S^{\prime}\right)}{3\left(D+D^{\prime}\right)} K
$$

But here $S$ and $D$, that is, the lower section of the reservoir and its perimeter, are very small with respect to $\mathrm{S}^{\prime}$ and $\mathrm{D}^{\prime}$, or the upper section, and we may write, neglecting $S$ and $D, W={ }_{3}^{\prime} K$, which is nothing but the volume of a cone with a base $\mathrm{S}^{\prime}$, as might have been expected.

Hence we see with what rapidity the volume of the contents increases with the height, and what advantages may be derived from constructing very high barrages in narrow valleys.

There is in Spain, near Alicante, a barrage 41 mètres in height, the construction of which dates back as far as the sixteenth century.


Division of a reservoir by sections-Plan. With the excellent hydraulic lime employed by the builders of the barrage of the Furens, there was, therefore, no danger to be apprehended from carrying it up to a height of 50 mètres.

Form and Mode of Construction adapted for Large Dams.-Plan of a Barrage or Dam 50 mètres in height. -We need not in this case consider earthen barrages which are of very doubtful security at a height of 20 mètres; at 50 mètres they would, of course, be quite out of the question. We have, therefore, to discuss only stone barrages, and the first question that presents itself is, Ought a barrage to be curvilineal or straight?

In France, previous to the construction of the Furens, the curved form had never been adopted; in Spain they are nearly all of this form. Theoretically it is admitted that they cannot act as an arch against the pressure of the water when they have the curved form, an opinion that is open to grave doubts if we take sufficiently into consideration the cohesion of good hydraulic mortar. But there is another and we think a sufficient reason for giving the preference to the curvilineal form. This reason is derived from the elasticity of blocks of masonry which in the present day is a proved fact. And if we admit this elasticity, it is obvious that the form which offers the greatest safety is the curved. This form was adopted for the barrage of the Furens, and is shown in Fig. 2239. The versed sine of the arc forming the axis of the crown is 5 metres, and the chord 100 metres.


With respect to the profile to be adopted, M. de Sazilly had already (Annales, 1853) pointed out the only rational arrangements, but his profile was open to the objection of being constructed in a graduated form, which requires a larger cube of cut stone and, consequently, a greater cost. M. Delocre, in his pamphlet, which is far more complete than M. de Sazilly's, has determined the plans of a barrage 50 mètres in height in the two cases of very broad and very narrow valleys, and in the two hypotheses of continuous and graduated facings. Figs. 2240, 2241, represent the former case, and Figs. 2242, 2243, the latter.
M. Delocre's profiles are nearly profiles of equal resistance giving a maximum pressure of 6 kilogrammes to the square centimetre, so that they are available for any height; thus to construct a barrage 26 mètres in height, it would be sufficient to adopt that portion which is situated below the horizontal line A B, Fig. 2240

There are in France barrages in which the pressure greatly exceeds this limit of 6 kilogrammes to the square centimètre. At Almanza, in Spain, there is a barrage of which we shall have
 occasion to speak later, and in which this pressure is as great as 14 kilogrammes. This barrage, which dates from the sixteenth century, is still in a good state of preservation, which is explained by the fact that in calculating the theoretical type abstraction
was made of the cohesion of the mortar, and the weight of the mass only was considered. But the Theil lime, for instance, resists, after six months, a considerably greater pressure than 14 kilogrammes to the square centimètre; it is evident, therefore, that in the case of well-constructed masonry no danger is to be apprehended from subjecting it to this pressure.

We have calculated for a barrage of 50 mètres a profile for a pressure of 14 kilogrammes to the square centimètre, adopting, like M. Delocre, 2000 kilogrammes as the weight of a cubic mètre of the masonry; this profile is shown in Fig. 2244. It gives as the thickness of the base $31^{\mathrm{m} \cdot 02}$, whilst the profile calculated for 6 kilogrammes, Fig. 2240 , gives $49^{\mathrm{m}} \cdot 46$. We see at once what enormous saving, in this case 264 cubic mètres of masonry to the lineal mètre, would be effected if the nature of the materials employed warranted a pressure of 14 kilogrammes to the square centimitre, Vicet cement, for instance. For a pressure between 6 and 14 kilogrammes we shall have a type combining the facings of the two types, Figs. 2240 and 2244 , by making them coincide at their summits, which are both 5 mètres broad. Fig. 2240 gives, in our opinion, the highest and Fig. 2244 the lowest limit of boldness; it will be for the builder to vary between them in accordance with the nature of the materials at his disposal. For the barrage of the Furens, which was to be of greater height than any existing structure, the engineers wisely chose as their model the less bold kind. Emboldened by the success of their undertaking, they have proposed for the barrage which they are about to construct on the Ban for the town of Saint-Chamond an intermediate type, in which the limit of the pressure provided against was 8 kilogrammes to the square centimetre. This type, of which we shall speak later, has been approved by the Administration, as well as the plan of the reservoir for the town of Saint-Chamond.


The type given in Fig. 2240 presents polygonal facings, and if we consider besides that the form of the barrage is circular, it will be seen that these angles must have a bad effect upon the open facings. On this account, curved facings were substituted in the barrage of the Furens, the profile of which is given in Fig. 224.5. Comparing this with Fig. 2240, it will be noticed that the thickness has been considerably increased at the top and slightly diminished at the bottom; this was to provide against the action of ice and floods. This barrage is situated 800 mètres above the level of the sea, and in severe winters the ice is $0^{\mathrm{m} \cdot 50}$ thick. In the event of a rapid thaw, the ice breaks up into large masses, and these masses are liable to be driven against the masonry by the violent gales which are very frequent in that part of the country. It was on this account that, in
 12 mètres and 26 mètres in the theoretical type, Fig. 2240, were increased to $9 \mathrm{~m} \cdot 56$ and $18^{\mathrm{m}} \cdot 01$ respectively. The action of the ice and waves is most to be feared at the permanent level of $44^{\mathrm{m} \cdot 50}$, for above this level the surplus water of a sudden rise is retained only a few hours. By increasing

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the thickness at this level, it became necessary, in order to have continuous and graceful curves on the facings, to increase also the thickness of the summit at the height of 50 mètres from 5 to $5 \mathrm{~m} \cdot 70$. Ahove this level there is a guard-wall to prevent the waves from leaping over, and to serve as a means of communication from one side of the valley to the otler.

In calculating for the type of the barrage of the Furens, the pressures in the case of a valley of indefinite breadth, we see that in no point of the profile does the pressure exceed $6^{k} \cdot 50$ to the square centimètre; but it must be remarked that here we are in a narrow valley, and that the thickness of the barrage begins to be equal to the breadth of the valley at 32 mètres downwards from the general summit. At this level the type of the Furens is $23^{\mathrm{m}} \cdot 49$ in breadth, and the theoretical type, Fig. 2242, $23^{\mathrm{m}} \cdot 40$. The profile to be adopted should, therefore, from this line be in accordance with the profile of Fig. 2242, and thus in the one executed the portion of the masonry $a c b$ and $d e h$, Fig. 2245, is excess. Hence it follows that downwards from the line in question the pressures must be considerably less than those marked on Fig. 2245, which were calculated as for a valley of indefinite breadth. If, therefore, the pressure reaches 6 kilogrammes at the lower portion of the barrage, it will certainly not exceed this limit, and hence we see that this barrage has been constructed with a considerable excess of resistance, which must in part be attributed to the timidity of the engineers, who had to


Scale $0^{\mathrm{m}} \cdot 002$ to the mètre. face for the first time the enormous height of 50 mètres.


We come now to consider the surfaces of the facings ; the following is the way in which they are generated;-Suppose the profile of Fig. 2245 placed in the vertical plane passing through the axis C D, Fig. 2239, in such a way that the middle point of the top, $5^{\mathrm{m} \cdot 70}$ thick, Fig. 2245, shall fall upon the arc of the circle A C B, Fig. 2239, and the whole profile to move successively in
vertical planes cutting each other in the direction of the vertical axis passing through the centre $D$, Fig. 2239, of the circle, the middle point above mentioned remaining during this movement upon the axis ACB of the circle. The stream-ward and opposite lines of the profile will trace the stream-ward and opposite facings of the barrage, which will be surfaces very graceful to the eye. To enable scaffolding to be placed against the facings when the joints in the masonry need repairing, cut stones, jutting out $0^{\mathrm{m}} \cdot 30$, have been placed in quincunx order in the outside facing $4^{\mathrm{m}} \cdot 60$ apart. These projecting stones have also a good effect in breaking the monotony of such an extensive facing. In the facing on the side of the reservoir where these projections were impracticable on account of the action of the waves, rings have been placed in the same order, through which ropes may be passed to fix the scaffolding; these rings serve also to tie the boats to which are used on the reservoir.

Mode of Construction.-The soil upon which the barrage of the Furens is built is mica schist. The barrage is sunk into the rock at its foundations and its sides. For the foundations, all the loose or doubtful blocks were carefully removed until the solid bed of rock was reached which joins the two slopes against which the structure is fixed, and which, as shown in Fig. 2245, is at the base let into this compact bed. For the sides, the earth and rock loosened by contact with the atmosphere where removed until, as before, the solid rock was reached, into which the masonry is inserted in the same way as for the foundations. The structure may be said to be held in a vice, which renders any slipping impossible, and the only movement to which it is liable is the vertical sinking of the masonry.

We think that for works of this nature this founding upon the solid rock both at the base and at the sides is a condition sine quâ non, and if it is not to be obtained, the work ought not to be undertaken. M. Aymard has given, in his interesting work on Irrigation in Spain, the history of the barrage of Puentès; this barrage, 50 mètres in height, gave way in 1802 at the base, the builder having conceived the unfortunate notion of founding it upon piles in an alluvial soil, instead of going down at any cost to the solid rock.

All the masonry of the barrage of the Furens is composed of common stone, the finest being reserved for the facings, the joints in which are irregular, being built without regular courses like the mass of the masonry. In our opinion, in structures which have to bear a great pressure of water, care should be taken not to lay the stones in horizontal courses, and bonders should be placed in all directions. Indeed, the upper surface of the work during construction should look like a field studded with projecting stones; in a word, the masonry should be so executed as to form a monolith. It was to avoid breaking the continuity of the mass that the lateral tunnels were adopted of which we have already spoken. Any opening in a barrage of so great a height as that of the Furens would be a source of danger.

One essential condition in the construction of such works is not to employ materials of too different a character. Masonry consisting of cut stone and stones of regular shape sinks less than the rough mass used for the inner filling. We continually sce in canal locks the cut-stone facings become detached after a certain number of years; and when the water can penetrate between the two kinds of masonry the facings fall with the first frost. In walls not subject to the action of water, the same phenomenon occurs: only it is less marked, and the facing of a wall may stand many years though partly detached from the mass. In a wall of the height of that of the Furens and subject to the action of water, this displacement would have been certain and inevitable. On this account, one of the principal conditions of building this barrage was that "only materials of a similar nature should be employed and the interior should not be filled up with concrete." This latter device would have separated the wall into two parts by the certain disruption of the concrete and the masonry due to the different degree of sinking or settling down, as it is called, and neither of these two parts would have possessed the thickness necessary to resist the pressure.

All the masonry of the barrage, including the foundations and the facings, is thus of rough common stone. It is almost useless to add that this masonry requires the greatest perfection of workmanship, and that this cannot be had except under the immediate and continual superintendence of the engineer. In some parts, where the removal of the rocks caused the surfaces to be too regular, in order to give a firmer hold to the masonry the following method was adopted. On the surface of the rock, previously roughened, a layer of Vassy cement was spread, into which building stones were stuck. In a few hours these stones were very solidly joined to the rock by the hardening of the cement, and by this means an excellent hold was provided for the masonry.

It was at first decided to use Vassy cement for the joints of the facing on the side of the reservoir, to render it more water-tight; but this was discontinued after a height of about 15 mètres had been reached, the successive introductions of water as the work progressed proving that the ordinary mortar held quite as well.

The only portions of the barrage in cut stone are the angle of the upper retreat, the plinths, the parapets, and the corbels upon the outside facing.

The masonry was subjected to the action of the water as the work advanced. The work of one season was left to settle down and harden till the next season, when it was submerged. It was not till the end of the season of 1865 that the work could be tested by a great depth of water. At the beginning of December in that year, the Furens was greatly swollen, and the reservoir was filled up to the height of 46 mètres. In March, 1866, this height was increased to 47 mètres, that is up to the level of the part completed in the season of 1865. These tests produced no movement in the mass nor any escape at the sides. The only phenomenon that occurred was traces of dampness upon the open facing without any apparent local leakage. This fact is explained by the inevitable sinking of such a mass, and by the porous nature of the mortar and the stone itself, which becomes visible only when they are subjected to enormous pressures. A small ditch was dug at the foot of the barrage to show the amount of leakage; but though $46^{\mathrm{m}} \cdot 60$ of water was retained in the reservoir for four months, it remained quite dry.

In short, the results of this great undertaking have been in all respects satisfactory. The success is rendered more conspicuous by the fact that important leakages at the sides, where the masonry is joined to the rock, seemed inevitable; the absence of this leakage must be attributed to the care and foresight with which the work was executed. We have also to call attention to an excellent means of preventing these lateral leakages employed at the reservoir of the Furens. This consists in hermetically closing with cement in the immediate neighbourhood of the barrage all the fissures in the rocks and in cementing as thickly as possible the joint in the angle formed by the facing of the barrage with that of the rocks on each side. The cement is made to cover from 8 to 10 centimètres of the rocks and the masonry. This method was practised by the Romans on the inner angles of their aqueducts, as shown in Fig. 2246.

Comparison of various kinds of Stone Dams. - We have lioman Ciuduits. already stated that a barrage was to be constructed on the Ban, a tributary of the Gier, for the service of the town of SaintChaumond and the factories on the Gier. The profile of this barrage, which has been calculated by M. Mongolfier, is given in Fig. 2247; this barrage will be 42 mètres in height, that is eight less than that of the Furens.

Comparing the portion of the profile above the line RS , situated at 42 mètres below the top, with the corresponding part of the profile of the Furens, Fig. 2245, we see that the thickness has been considerably reduced; and if we increase the profile to
 the height of 50 metres and calculate the pressures, we see by the figures given in lig. 2247 that in no point will the pressure exceed 8 kilogrammes to the square centimètre, a limit that may be adopted with the excellent Theil lime. This profile, which has been approved by the "Administration," is bolder than that of the Furens, and offers a saving of 105 cubic metres of masonry a lineal mètre.

The curves of the pressures, when empty and when full, shown in Figs. 2240, $2244,2245,2247$, enable us to see that these curves leave the middle curve between them, and that they approach it in proportion as the pressure to the square centimètre becomes less. This is shown clearly by the following Table;-

| Profiles. |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

It will be seen that for the same general type the more the limit of pressure is increased the nearer the curves approach the lines of the facings and, consequently, recede from the middle line. The conditions of stability, therefore, necessarily diminish in proportion as the limit of pressure to be adopted is increased. It will be for the builder to consider well the nature of his materials and the circumstances of the place before fixing this limit, and then to choose one of the profiles which we have been examining.

Let us now compare some existing types of barrages with those which we considered in connection with the barrage of the Furens. Figs. 2248 to 2253 represent some Spanish barrages from designs which M. Aymard has given of these structures in his work on Irrigation in Spain.

The first thing that strikes the attention is the colossal proportions of these structures, with the exception of the barrage of Almanza, Fig. 2251, which is the oldest. This barrage is of unusual boldness, at least in its upper portion, which throughout a height of $8^{\mathrm{m} \cdot} \cdot 20$ has a mean thickness of only $3^{\mathrm{m}} \cdot 50$. Five of these barrages, those of Puentès, Val de Infierno, Nijar, Almanza, and Alicante, are of the graduated type represented by Figs. 2241 and 2243 , and one only, that of Elche, Fig. 2252, is of the type with continuous facings, the theoretical form of which is given in Figs. 2240 and 2242.

Calculating in these theoretical types the cubes of masonry corresponding to the heights of the several barrages, and taking into account the pressures at the outer edge when loaded, which pressures we have calculated for each of these barrages, we obtain the following comparative Table;

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| Height of the Barrages. | Name of Barrage. | Maximum Pressure to the square centimètre. | Cubes of Masonry to the lineal mètre of Barrage. |  |  | Differences. | Observations. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | According to the type executed. | According to the theoretical type of Figs. 2241 and 2243. | According to the theoretical type of Figs. 2240 and 2242. |  |  |
| mètres. $50 \cdot 00$ | Barrages of Puentès | kilos. <br> $7 \cdot 90$ | cub. mèt. 1519 | cub. mèt. 1029 | cub. mèt. | cub. mèt. 490 |  |
| $35 \cdot 70$ | Barrages of Val de Infierno | $6 \cdot 50$ | 1084 | 391 | . | 693 |  |
| $27 \cdot 50$ | Barrage of Nijar .. | $7 \cdot 50$ | 499 | 308 |  | 191 |  |
| $20 \cdot 70$ | Barrage of Almanza | $14 \cdot 00$ | 139 | 141 |  | -2 |  |
| $23 \cdot 20$ | Barrage of Elche .. .. | $12 \cdot 70$ | 213 |  | 187 | 56 | Continuous facings. |
| $41 \cdot 00$ | Barrage of Alicante | $11 \cdot 30$ | 1100 | 566 | . | 534 | Graduated facings. |

We see that these great pressures might have been reduced, by adopting the rational type, to the lower limit of 6 kilogrammes to the square centimètre, with a large gain in the masonry in all except the barrage of Almanza; and in this case with the same cube within 2 mètres, we might, by a better arrangement, have reduced the pressure from 14 to 6 kilogrammes.


Barrage of Nijar.


Barrage of Bosméléac.


Barrage of Gros-Bois.


Barrage of Val de Infierno.


Barrage of Almanza.


Barrage of Elche.


Barrage of Alicante.

Figs. 2254, 2255, represent the barrages of Bosméléac and Gros-Bois. The former of these two profiles is similar to the type of Figs. 2240 and 2242, and the latter to that of Figs. 2241 and 2243.

The profile of the barrage of Bosmééac gives a cube of 90 cubic mètres to the lineal mètre, and a pressure of $6 \mathrm{k} \cdot 09$; the profile of Figs. 2240 and 2242 would give for the height of 15 mètres, which is the height of this barrage, a maximum pressure of 6 kilogrammes and a cube of 91 cubic mètres. The barrage of Bosmeléac is thus well designed. As to the barrage of Gros-Bois, its profile is the most irrational of all, and it is the only one in which the two curves of the pressures leave the middle one on the same side, the curve of the pressures when loaded passing within 3 metres of the lower outside edge. Besides this it gives, with a pressure of $10^{\mathrm{k}} \cdot 40$, an excess of masonry equal to $226-156$, or 70 cubic mètres above the type, Figs. 2241 and 2243 , applied to its height of $21^{\mathrm{m}} \cdot 80$.

It is obvious that if the profile of this barrage were turned the other way, with the retreats or gradations on the outside instead of on the water-side, it would have deviated from the theoretical type only by an excess of resistance.

The faulty arrangement of the profile of this barrage must, in our opinion, have tended considerably to cause the giving way of the masonry which has occurred, and which has necessitated the erection of counterforts.

Our readers will perceive, from what we have said on the subject of barrages, the importance of selecting a good profile, and we hope that we have practically demonstrated by the experiment of the barrage of the Furens, the superiority of the form of profile arrived at theoretically by M. Delocre.

The Form of the Profile to be adopted for Large Stone Dams.-We have already had occasion to refer to the treatise of M. Delocre on the form of stone dams; in the following pages we will reproduce the methods of calculation by which he arrived at his decisions.

Type of Rectilineal Barrage or Dam in Valleys of considerable Breadth.-Condition of Stability.-A barrage which does not transmit laterally to the sides of the valley the pressures which it supports, must resist these pressures in all its points by its own weight. We may, therefore, in seeking the conditions of stability of a structure of this nature, consider a single section only equal in length to a lineal unit. If the materials employed were of indefinite resisting power, as well as the soil of the foundation, and if there were between them an unlimited degree of adhesion, the only condition of stability to be fulfilled would be to give to the wall such a profile that the resultant of the thrust of the water and of the weight of the structure should pass within the polygon of the base. But this condition is not sufficient in practice; the materials and the soil of the foundations will, in fact, support only a limited pressure depending upon their nature, and they have not between them an unlimited degree of adhesion. Hence the two following indispensable conditions:-1. In no point of the structure may the materials employed, or the soil of the foundations, be required to bear too great a pressure; 2. The several courses of masonry in the wall must be incapable of slipping one over another, and the wall must be incapable of sliding upon its base.

Hitherto none of the walls that have given way have done so by slipping; these accidents have occurred in all cases from the first condition not having been fulfilled. We shall, therefore, in the study on which we are about to enter, determine first the dimensions of a barrage with respect to this condition, and then ascertain if the second is satisfied.

A B CD, Fig. 2256, being the profile of a barrage, any section of this barrage equal in length to a lineal unit may be considered as subject to the action of two forces; the vertical component $P$

of the resultant of the weight of the structure and of the thrust against the facing D A, and the horizontal component $\mathbf{F}$ of the thrust. These two forces produce a resultant R which cuts the base $\mathrm{A} B$ in the point E . The force R may be considered as applied to the point E and resolved into two at this point, the vertical force being equal to the force P and the horizontal equal to the component F. The horizontal force tends to cause the wall to slide upon its foundations; this effect we will consider later. The vertical force spreads itself over the base from the extremity B, which is nearest the point of application of the resultant, according to a decreasing law.

Denoting the breadth of the base AB by $l$, and the distance $\operatorname{BE}$ by $u$, the pressure $p^{\prime}$ at the point $\mathbf{B}$ will be given by one of the following formulæ;

$$
\begin{equation*}
p^{\prime}=2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{l}, \quad[1] \quad p^{\prime}=\frac{2}{3} \frac{\mathrm{P}}{u} \tag{2}
\end{equation*}
$$

according as $u$ is greater or less than $\frac{1}{3} l$.
These formulæ follow naturally from those given by O. Byrne in his 'Esseutial Elements of Practical Mechanics,' page 236;

$$
p=\frac{\mathrm{N}}{\Omega}(1+3 n) \quad[\alpha] \quad p=\frac{\mathrm{N}}{\Omega} \times \frac{4}{3(1-n)}
$$

and which apply to a homogeneous rectangle pressed by a force acting upon one of the symmetrical axes.

In these formulæ N represents the whole load, $\Omega$ the whole area of the surface, and $n$ a quantity which, with our notation, is equal to $\frac{l-2 u}{l}$. We have represented the load N by P and the surface $\Omega$ is replaced by $l$.

Making in the above formulæ $\mathbf{N}=\mathrm{P}, \Omega=l, n=\frac{l-2 u}{l}$, and substituting $p^{\prime}$ for $p$, they become

$$
\left.p^{\prime}=\frac{\mathrm{P}}{l}\left(1+\frac{3 l-6 u}{l}\right)=2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{l} ; \quad p^{\prime}=\frac{\mathrm{P}}{3\left(1-\frac{4}{l^{\prime}-2 u}\right.} \frac{2}{l}\right) \quad=\frac{2}{3} \frac{\mathrm{P}}{u} .
$$

The formula [a] is applicable when $n<\frac{1}{3}$, and consequently formula [1] is adaptcd to the case in which $\frac{l-2 u}{l}<\frac{1}{3}$, that is, $u>\frac{1}{3} l$.

The formula [ $\beta$ ] is applicable when $n<\frac{1}{3}$, and consequently the formula [2] is adapted to the case in which $\frac{l-2 u}{l}>\frac{1}{3}$, that is, $u<\frac{1}{3} l$.

The stability of the wall requires that this pressure at the point B be equal to or less than the limit of pressure $R^{\prime}$ which each superficial unit may be made to bear. We ought, therefore, to have, according as $u$ is greater or less than $\frac{1}{3} l$,

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{l}=\mathrm{R}^{\prime}, \quad[3] \quad \frac{2}{3} \frac{\mathrm{P}}{u}=\mathrm{R}^{\prime} \tag{4}
\end{equation*}
$$

and this condition should be fulfilled for each horizontal section made in the profile, neglecting the force of cohesion in the mortar, which is unfavourable to resistance.

The expressions [1] and [2] may be put under another form by introducing into the calculation the maximum height $\lambda$ that may be given to a wall with vertical sides, so that the pressure upon the base shall not exceed the limit $\mathrm{R}^{\prime}$. Indeed, representing the density of the masonry or the weight of the cubic mètre by $\delta^{\prime}$, we have $\mathrm{R}^{\prime}=\delta^{\prime} \lambda$.

And the expressions [2] and [3] become

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \quad[5] \quad \frac{2}{3} \frac{\mathrm{P}}{\delta^{\prime} u}=\lambda \tag{5}
\end{equation*}
$$

The preceding conditions would be sufficient if the reservoir were to be always full of water, but the wall must be capable, when the reservoir is empty, of supporting its own weight without being subject in any of its points to a pressure, by unit of surface, exceeding the limit $\delta^{\prime} \lambda$. In this case the resultant of all the forces acting upon the wall is reduced to the weight $\mathrm{P}^{\prime}$; and, denoting the distance $\mathrm{K}^{\prime} \mathrm{A}$ from the vertical passing through the centre of gravity of the figure ABCD, Fig. 2256, to the nearest extremity A of the base by $u$, the pressure at $\mathbf{A}$ will be given according to the case by the formulæ [1] and [2], and the stability of the wall will require that one of the relations [5] and [6] be satisfied when $\mathbf{P}^{\prime}$ is substituted for $\mathbf{P}$.

Form to be given to a Wall having its own weight only to support. - To examine the subject from every point of view, it is important to know what form should be given to a wall having only its own weight to support, so that no point of the masonry may be subject to a pressure above the limit adopted.

It is clear that so long as the height of this wall is less than the limit $\lambda$, it will be sufficient to give it vertical facings, and that the pressure to the unit of surface in the lower part will not exceed $\delta^{\prime} \lambda$. If the wall is to be higher, vertical facings may be adopted throughout a distance from the top equal to $\lambda$, and from this point the thickness of the structure must be increased to prevent the pressure upon any horizontal section from exceeding the limit $\delta^{\prime} \lambda$.

The form to be given to the facings to satisfy this condition is easily determined. We might choose arbitrarily one of the facings of the wall and determine the other; but if we wish to obtain the minimum cube of masonry, we shall give the wall a symmetrical form with respect to its axis. It is clear from the formulæ [1] and [2] that the maximum pressure $p^{\prime}$ cannot acquire a value less than $\frac{\mathrm{P}}{l} ; u$ being by hypothesis greater than $\frac{l}{2}$, or at least equal to $\frac{l}{2}$, and that this minimum value is reached for $u=\frac{l}{2}$.

This being allowed, the curve sought, D N Y, Fig. 2258, must satisfy the condition, that if in any section $\mathrm{M} N$ the pressure to the unit of surface is equal to a given quantity, this pressure will remain the same for a section $\mathrm{M}^{\prime} \mathrm{N}^{\prime}$, infinitely near. This condition will evidently be satisfied if the increase of the surface of the base is proportional to the increase of the pressure, or, as all is symmetrical with respect to the axis OS, if the increase of the half-surface $L N$ is proportional to the increase of the pressure upon this half-surface.

This condition is expressed by the equation $d \mathrm{P}=\mathrm{K} d \mathrm{~B}, \mathrm{P}$ representing the pressure exerted upon the half-section LN by the upper part of the structure, and B the surface of this section. Representing by $b$ the dimension of the wall in the direction perpendicu'ar to the section under
consideration, by $x$ the breadth L N , or the abscissa of the curve sought, D N Y, and by $y$ the distance from the section MN to a horizontal line taken as the axis of $x$, we have

$$
d \mathbf{B}=d b x, \quad d \mathbf{P}=\delta^{\prime} b x d y
$$

$d \mathbf{B}=d b x$,
The differential equation of the curve D N Y is therefore $\delta^{\prime} b x d y=\delta^{\prime} b x d y$. $\quad \mathbf{K} b d x=\frac{\mathbf{K}}{\delta^{\prime}} \frac{d x}{x}$.
The constant K expresses the limit of pressure to the unit of surface, and, consequently, it is equal to $\delta^{\prime} \lambda$; the equation thus becomes $d y=\lambda \frac{d x}{x}$, or, integrating,

$$
\begin{equation*}
y-y_{0}=\lambda \log \cdot\left(\frac{x}{x_{0}}\right) \tag{7}
\end{equation*}
$$

thus the curve D N Y is logarithmic.
If we make $x_{0}=\lambda$, the two equations above give $\frac{d y_{0}}{d x_{0}}=1$, and $y_{0}=0$.
So that the origin of the co-ordinates should be taken at the point where the value of $x$ is equal to $\lambda$, and in this point the tangent to the curve makes an angle of $45^{\circ}$ with the axis of the $x$ 's.

Substituting their values for $x_{0}$ and $y_{0}$, equation [7] becomes

$$
\begin{equation*}
y=\lambda \log \cdot \frac{x}{\lambda} \tag{8}
\end{equation*}
$$

or, passing from hyperbolic to common logarithms, $y=2 \cdot 302658509 \lambda \log . \frac{x}{\lambda}$.
The complete curve, which has as an asymptote the axis of the $y$ 's, would give the form of the facings of a wall indefinite in height, for which the pressure to the unit of surface would be equal to the limit K upon any horizontal section.


Fig. 2259 shows the curve constructed, admitting the limit of pressure adopted for the masonry to be 60,000 kilogrammes to the square mètre, or 6 kilogrammes to the square centimètre; and, supposing it be required to give a breadth of 5 metres to the top part of the wall, we have

$$
\mathbf{K}=60,000 ;
$$

and admitting the density of the masonry to be double that of water, say $\delta^{\prime}=2000$, we have $\lambda=30$; the equation of the curve becomes $y=2 \cdot 3026 \times 30 \log \cdot \frac{x}{30}$.

It must not be forgotten, in making use of this formula, that the direction in which the $y$ 's are usually reckoned has been reversed: in other words, the increment $d y$ has been reckoned with the sign + downwards. The values of $y$ negative must therefore be taken in the direction L O.

It will be seen from Fig. 2258 that if a wall 50 mètres in height and 5 mètres in breadth at the top has a breadth of $9 \mathrm{~m} \cdot 7392$ at the base, in no point will it have to support a pressure above the limit of 6 kilogrammes to the square centimètre.

Substituting right lines for the ares C M H, D N Y, the breadth to be given to the wall at its base would be 10 metres. Fig. 2260 is constructed on this hypothesis.

The equation giving the breadth $x$ of the base is immediately found,

$$
\left[30 \times 5+20\left(\frac{5+x}{2}\right)\right] \frac{2000}{x}=2000 \times 30
$$

The influence of the concave profile which we have given to the wall may be clearly seen by finding the thickness to be given to the base in the case of rectilineal facings inclined from the summit.

This thickness, putting $H$ for the height of the wall, and $a$ for the breadth at the top, will be given by the equation, Fig. 2261, H $\left(\frac{a+x}{2}\right) \frac{\delta^{\prime}}{x}=\delta^{\prime} \lambda$, whence $x=\frac{\mathrm{H} a}{2 \lambda-\mathrm{H}}$

It will be seen that for $\mathrm{H}=2 \lambda, x$ will become infinite, and that for any value of H greater than $2 \lambda, \lambda$ will be negative. Whence we infer that the maximum height that may be given to a wall constructed with rectilineal sides inclined equally from the top, without exceeding the limit of resistance in the masomry, is twice that of a wall with vertical sides.

Admitting 6 kilogrammes to the square centimetre as the limit of pressure, the limit of the height to be given to the wall is 60 mètres; this limit is reduced to 40 mètres, if we take 4 kilogrammes to the square centimètre as the limit of resistance in the materials.

Making $\mathrm{H}=50$ mètres, and $a=5$ mètres, as in Fig. 2260, we find $x=25$ mètres. The breadth of the wall at the base is therefore 25 mètres, instead of $9^{\mathrm{m}} \cdot 739$.

The preceding results show of what importance it may be not to have any point in a structure at which the pressure is much less than that taken as the limit; by uselessly admitting excesses of thickness at certain points, it soon happens that we are unable to maintain the pressures within the required limits at other points. In the following considerations we shall endeavour to find a profile of equal resistance, or at least one that shall deviate but little from such a profile.

Stability of a Wall having to support a Load of Witer.-Theuretical Profile of Equal Resistance.We saw, Fig. 2256, that if ABCD represent a wall having to support a pressure of water, the conditions of stability in this wall are given by one of the relations

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \quad[5] \quad \frac{2}{3} \frac{\mathrm{P}}{\delta^{\prime} u}=\lambda, \tag{6}
\end{equation*}
$$

according as we have $u>\frac{1}{3} l$, or $u<\frac{1}{3} l$.
Keeping to the limiting values which correspond to the sign $=$, we shall have for the equations giving the conditions of stability

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2}{3} \frac{\mathrm{P}}{u \delta^{\prime}}=\lambda . \tag{10}
\end{equation*}
$$

Substituting in the formulæ [9] and [10] for the quantities $u, l$, and $P$, their respective values as a function of the height of the wall, of its reduced thickness, and of the inclination of its sides, we see-

1. That the profile offering the least thickness, while satisfying the conditions of stability, is that, the face of which is vertical on the side of the water and inclined on the opposite side.
2. That when the height is increased, the reduced thickness increases less rapidly, so that a profile constructed with a vertical facing on the side of the water, and with a talus on the opposite side, combined so as to satisfy the limiting condition of stability for its base, will offer an excess of stability over the surplus height. A wall satisfying the limiting conditions throughout its height will therefore present, with a vertical face on the side of the water, a concave curve on the opposite side.


Let us suppose for a moment this curve constructed, and let A C N B, Fig. 2262, be the profile determined by the condition that for each horizontal section $M \mathrm{~N}$, the pressure is equal to the limit $R^{\prime}$ which it is required not to exceed.

This profile would have sufficient resistance if the reservoir were always full, but we have to take care that the pressure arising from the weight of the wall do not exceed the limit $\mathrm{R}^{\prime}$ when the reservoir is empty. It is certain that this pressure will not be exceeded for the face C N B; indeed, the developments into which we have already entered show that the effect of the thrust is to drive towards this side the vertical component equal to the weight of the wall : it remains, therefore, to be seen if the limit $R^{\prime}$ is reached in any point of the face CMA. If we refer to the calculations which we have already made with respect to the stability of walls having only their own weight to
support, we shall see that this limit will be passed at a very small height. It will be necessary, from the point $M$ where the limit $\mathrm{R}^{\prime}$ is reached, to carry out the wall according to a curve MLS: from the section MN the two curves N P B , MLS S, should be determined by the condition that for any horizontal section $L P$, the pressure at the point $P$ when the reservoir is full should be equal to $R^{\prime}$, and, when the reservoir is empty, equal to the same limit at the point L .

To solve the question completely, it only remains now to determine exactly the two curves CNB, MLS.

Let us take as the axis of the $x$ 's, Fig. 2264, the vertical face A B of the wall, and for the axis of the $y$ 's, the perpendicular A $y$ to this face passing through the summit, and let II be any point in the curve.

If $\mathbf{B M}=y$ and $\mathrm{M}=x$, the problem to be solved is to find a relation between $x$ and $y$ which shall be the equation of the curve.

We shall continue to consider a section of the wall of a unit of length in the direction perpendicular to the figure. This wall is acted upon by two forces; its weight $P$ and the thrust $F$. These two forces are represcnted by the following expressions; $\mathrm{P}=\delta^{\prime} \int_{0}^{y} y d x, \mathrm{~F}=\frac{\delta x^{2}}{2}$.

The curve A N L should be determined by the condition that the
 pressure in $M$ to the unit of surface is to be equal to the limit $\lambda \delta^{\prime}$, which must not be exceeded. This condition is expressed according to the cases by one of the equations [ 9$]$ and $[10$.

We have $l=y$, and it remains to determine in functions of $x$ and $y$, the quantity $u=\mathbf{E M}$, Now EM - KM - KE. The two similar triangles OKE, OPR give

$$
\frac{\mathrm{KE}}{\mathrm{P} \mathrm{R}}=\frac{\mathrm{OK}}{\mathrm{OP}}, \text { or } \frac{\mathrm{KE}}{\mathrm{~F}}=\frac{x}{3 \mathrm{P}} .
$$

Whence, substituting for F and P their values, and for brevity making $\frac{\delta}{\delta^{\prime}}=\theta$, we deduce

$$
\mathrm{KE}=\frac{\theta x^{3}}{6 \int_{0}^{y} y d x}
$$

Again we have KMI=y-BK.
$\mathbf{B K}$ is the distance from the centre of gravity of the area $\mathbf{A B M N}$ to the axis $\mathbf{A} x$; this distance is obtained by considering that the moment of the total area with respect to this axis is equal to the sum of the moments of the elementary areas such as $a b c d$.
consequently, $\mathrm{K} \mathbf{M}=y-\mathrm{B} \mathbf{K}=\frac{2 y \int_{0}^{y} y^{2} d x-\int_{0}^{y} y^{2} d x}{2 \int_{0}^{y} y d x}$, and $u=\frac{6 y \int_{0}^{y} y d x-3 \int_{0}^{y} y^{2} d x-\theta x^{3}}{6 \int_{0}^{y} y d x}$.
This value of $u$ substituted in equation [9] gives

$$
\begin{equation*}
3 \int_{0}^{y} y^{2} d x-2 y \int_{0}^{y} y d x-\lambda y^{2}+\theta x^{3}=0 \tag{11}
\end{equation*}
$$

The same value of $u$ substituted in equation [10] would give

$$
\begin{equation*}
4\left(\int_{0}^{y} y x\right)^{2}-6 \lambda y \int_{0}^{y} y d x+3 \lambda \int_{0}^{y} y^{2} d x+\lambda \theta x^{3}=0 \tag{12}
\end{equation*}
$$

We have endeavoured to integrate the formulæ [11] and [12] by an exact method, but without success; we have been able to obtain $y$ developed only as a series in function of $x$; the series belonging to equation [11] is of the form, $y=a x^{\frac{3}{3}}+b x_{\frac{5}{2}}+c x^{\frac{7}{2}}+d x^{\frac{9}{2}}+$ etc.

The series belonging to equation [12] is of the form $y=a x+b x^{2}+c x^{3}+d x^{4}+e x^{5}+$ etc.
We had calculated the coefficients of the first terms admitting for $\theta$ and $\lambda$ the values $\theta=\frac{1}{2}, \lambda=30$; but it seems to us useless to reproduce these calculations here; the above formulæ are, indeed, of no use in determining a practical profile; they are applicable only to the portion CN of the inner curve of Fig. 2263, and the calculations into which we should be led in determining the two curves NPB, MLS, of the lower portion of the profile are quite impracticable even if we have recourse to approximative methods.

Conditions which Profiles of Barrages must satisfy in practice.-We shall avoid the difficulties of integration, and solve, in a manner sufficiently accurate for practice, the problem of finding a profile of equal resistance by replacing the curves C N B, M L S, of Fig. 2263 by the polygonal circumferences $\mathrm{C} b^{\prime} b^{\prime \prime} \mathrm{N} n^{\prime} n^{\prime \prime} \mathrm{B}, \mathrm{M} m^{\prime} m^{\prime \prime} \mathrm{S}$, of Fig. 2265 calculated for the condition that for each horizontal section such as $n^{\prime} m^{\prime}$ passing through the corresponding summits of the polygons forming the profile of the face, the pressures to the unit of surface in $n^{\prime}$ and $m^{\prime}$ according as the reservoir is full or empty, are to be equal to the limit $R^{\prime}$. The smaller thes horizontal sections such as $c^{\prime} b^{\prime}, c^{\prime \prime} b^{\prime \prime}$, the less will the circumferences differ from the curves corrs sponding to an exact solution. This solution offers no difficulty, as we shall see later.

But before proceeding farther we must remark that if the profile of Fig. 2263, supposed to be calculated in an exact manner, were sufficient theoretically to resist the load of water which it would be required to support, a thickness that became nul at the summit could not be admitted in practice. The masonry in the upper portion of the wall must be capable of resisting the action of waves, and if it have sufficient dimensions to serve as a passage for vehicles, or, at least, footpassengers, something will be gained in point of convenience.

These considerations lead us to substitute Fig. 2266 for Fig. 2263, having in its upper portion a part CD B A with vertical sides. In this part the masonry will be subject to pressures less than the limit $\mathrm{R}^{\prime}$. The method of calculating this profile would be analogous to that employed for Fig. 2265 ; but the breadth C D at the top of the barrage being determined, we must begin by calculating the height A D for the condition that the pressure to the unit of surface at the point A when the barrage is full shall be equal to the limit $R^{\prime}$.

The height is readily calculated by the following method ;-Let C D = $=a$, Fig. 2267, the breadth chosen for the top of the barrage, and D A $=x$ the height to be calculated.

When the reservoir is full, the upper portion CDBA of the barrage is subject to two forces, its weight $P$ and the thrust $F$; these two forces produce a resultant $R$ cutting the base $B A$ in the point $\mathbf{E}$; the resultant is decomposed into two at this point, a horizontal force tending to make the part CD B A slide upon the plane B A, and a vertical force equal to $\mathbf{P}$; this latter force spreads itself over the base A B according to a law which we have already alluded to.

By expressing the pressure at the point $A$ as equal to the limit $R^{\prime}=\lambda \delta^{\prime}$ which is not to be exceeded, we shall put the problem into an equation; this will lead us to one of the equations [9] and [10] found by solving a similar question.

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \quad[9] \quad \frac{2}{3} \frac{\mathrm{P}}{u \delta^{\prime}}=\lambda \tag{9}
\end{equation*}
$$

We have now to express in these equations the quantity $u=\mathbf{A}$ E as functions of the lengths $a$ and $x$. Now we have $\mathbf{A E}=\mathrm{KA}-\mathbf{E K}=\frac{a}{2}-\mathbf{E K}$, and again $\mathrm{KE}=\frac{\mathrm{OK} \times \mathrm{PR}}{\mathrm{OP}}=\frac{x \times \mathrm{F}}{3 \mathrm{P}}$.

But $\mathrm{P}=a x \delta^{\prime}$, and $\mathrm{F}=\frac{x^{2} \delta}{2}$. Therefore $\mathrm{K} \mathrm{E}=\frac{x^{2} \delta}{6 a \delta^{\prime}}$. And consequently,

$$
\begin{equation*}
u=\frac{a}{2}-\frac{x^{2} \delta}{6 a \delta^{\prime}}=\frac{1}{6 a}\left(3 a^{2}-\theta x^{2}\right) \tag{13}
\end{equation*}
$$

Substituting this value in the equations [9] and [10], we deduce as the equations connecting the quantities $a$ and $x$, remarking that $l=a$;

$$
\begin{equation*}
\theta x^{3}+a^{2} x-a^{2} \lambda=0 . \quad[14] \quad \theta \lambda x^{2}+4 a^{2} x-3 \lambda a^{2}=0 \tag{15}
\end{equation*}
$$

We must employ equation [14] or equation [15] according as $a$ is greater or less than $\frac{a}{3}$, or, which is the same thing (as may be seen by referring to the value of $u$ given above), according as $x^{2}$ is greater or less than $\frac{a^{2}}{\theta}$.

Having determined the upper portion of the profile in this way, the other two portions ANT, B M S, Fig. 2266, may be calculated by the method indicated for Fig. 2263.


The difficulties of integration will be avoided by substituting polygonal lines for the curves A N T, BMS, as shown in Fig. 2268, and the smaller the horizontal sections into which the barrage is supposed to be divided, such as $m^{\prime} n, m^{\prime \prime} n^{\prime \prime}$, the nearer will the profile approximate to one of equal resistance.

The calculation of this profile offers no difficulty ; it is, however, of great length. We will give later the formulæ arrived at.

To determine a practical Profile.-The important matter in constructing a barrage is not to allow in any point of the structure a pressurc greater than the limit $\mathrm{R}^{\prime}$, but it is obviously not indispensable that this pressure should be reached in every point, and plain that profiles may be admitted which deviate slightly from one of equal resistance if they offer other advantages.

We see at a glance that the execution of facings such as those of Fig. 2268 would offer some difficulty, and that the frequent change of inclination which they present would not produce a very happy effect. A more practical form will be obtained by lessening the number of sections into which the wall is supposed to be divided.

This consideration leads us to calculate a profile of the form of Fig. 2269.
The method of calculation will be the same whatever the height of the horizontal sections may be, and it will apply with slight modifications to the determining of the profile of equal resistance by the approximative method.

The profile of Fig. 2269, like that of Fig. 2265, divides itself naturally into two parts, one CD A B, in which the face on the side of the water is vertical, the other A $B, A^{\prime} \mathrm{B}^{\prime}$, the facings of which are inclined on both sides. One of the horizontal planes dividing the profile into sections must be made to pass through the point B from which the inside face is inclined; the first thing to be calculated is, therefore, the height CB. We will suppose in the first place that the outside facing slopes regularly from D to A, as in Fig. 2270; the problem will be expressed as an equation by making the pressure to the unit of surface at the point A when the reser-
 voir is full, and at the point $B$ when the reservoir is empty, equal to the limit $\mathrm{R}^{\prime}$.

Denoting the weight of the portion CDBA of the wall by P, the thrust by F, and the distance AE by $u$, the equation expressing the above conditions will be one of the following;

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \quad[9] \quad \frac{2}{3} \frac{\mathrm{P}}{u \delta^{\prime}}=\lambda \tag{10}
\end{equation*}
$$

according as $u$ is greater or less than $\frac{l}{3}$.
Let $\mathrm{CD}=a, \mathrm{CB}=z$, and $\mathrm{A} \mathrm{V}=x$. We shall have first,

$$
\mathrm{P}=\left(\frac{2 a+x}{2}\right) \delta^{\prime} z, \quad \mathbf{F}=\frac{z^{2} \delta}{2}, \quad l=a+x, \quad u=\mathbf{A} \mathbf{E}=\mathrm{K} \mathrm{~A}-\mathrm{K} \mathrm{E}
$$

Now we have

$$
\mathrm{KE}=\mathrm{OK} \times \frac{\mathrm{F}}{\mathrm{P}}=\frac{z^{2} \delta}{3(2 a+x) \delta^{\prime}}=\frac{z^{2} \theta}{3(2 a+x)}
$$

We shall obtain KA by considering that the moment of the whole weight of the figure CBAD, with respect to the point $A$, is equal to the sum of the moments of the two parts $\mathrm{BCDV}, \mathrm{DVA}$, of which it is composed.

$$
\mathrm{K} \mathrm{~A} \times \frac{(2 a+x) z \delta^{\prime}}{2}=\frac{(2 x+a) a z \delta^{\prime}}{2}+\frac{z x^{2} \delta^{\prime}}{3}
$$

Whence we deduce, $\mathrm{K} \mathrm{A}=\frac{2 x(x+3 a)+3 a^{2}}{3(2 a+x)}$, and, consequently, $u=\frac{2 x(x+3 a)-\theta z^{2}+3 a^{2}}{3(2 a+x)}$.
This value of $u$ substituted successively in the equations [9] and [10] gives us the two following;

$$
\begin{gathered}
\theta z^{3}-\lambda x^{2}-2 a \lambda x+a^{2} z-\lambda a^{2}=0 . \\
x^{2} z-2 \lambda x^{2}+4 a x z+\theta \lambda z^{2}-6 a \lambda x+4 a^{2} z-3 a^{2} \lambda=0 .
\end{gathered}
$$

Equations [16] or [17] will be employed according as $u$ is greater or less than $\frac{b}{3}$.
Hitherto we have only one equation between the unknowns $x$ and $z$, but we may obtain a second by expressing the pressure to the unit of surface at the point $\mathbf{B}$ when the reservoir is empty as equal to the limit R'. This condition will always be given by the relations [9] and [10], but the quantity $u$ will not remain the same; the thrust of the water no longer existing, the vertical force P acts in the direction O P , and we have $u=\mathrm{B}$ K.

This quantity is immediately deduced from the preceding calculations; we have

$$
u=\mathrm{BK}=\mathrm{AB}-\mathrm{A} \mathrm{~K}=\frac{x(x+3 a)+3 a^{2}}{3(2 a+x)}
$$

Substituting the preceding value in the expressions [9] and [10], we obtain the two equations

$$
\begin{gather*}
x^{2} z-\lambda x^{2}+3 a x z-2 a \lambda x+a^{2} z-\lambda a^{2}=0  \tag{18}\\
x^{2} z-\lambda x^{2}+4 a x z-3 a \lambda x+4 a^{2} z-3 a^{2} \lambda=0 . \tag{19}
\end{gather*}
$$

The values of $x$ and $z$ may be determined in each particular case by combining one of the equations [16] and [17] with one of the equations [18] and [19]. This operation must be effected by tentative experiments, for the value of $u$ being a function of the unknowns, we cannot ascertain in a precise manner a priori which equation is required. It will be necessary, in this case, to have recourse to an hypothesis on the relation of the value of $u$ to that of $\frac{r}{3}$. Having solved the two equations chosen in accordance with this hypothesis, we must verify the supposition, and if it be untrue we shall be driven to take those of the equations which suit the values found for $u$ and $z$, which, though inexact, will be near enough to enable us to choose the equations that will determine the true values.

The equations [16], [17], [18], and [19] being of the third degree, by combining two of them, we shall be led to the solution of an equation of the sixth degree. This solution will offer no difficulty in practice, the equations to be dealt with being numerical. Supposing, for example, the two values of $u, \mathrm{~A} \mathrm{E}$ and BK to be less than $\frac{\mathrm{BA}}{3}=\frac{l}{3}$, the equations [17] and [19] must be combined. Equation [19] being of the first degree in $z$, we may deduce from it the value of this unknown as a function of $x$; and substituting it in equation [17] we shall obtain the following final equation

$$
\begin{gather*}
x^{6}+11 a x^{3}+\left(48 a^{2}-\lambda^{2} \theta\right) x^{4}+2 a\left(52 a^{2}-3 \theta \lambda^{2}\right) x^{3}+a^{2}\left(112 a^{2}-15 \theta \lambda^{2}\right) x^{2} \\
+6 a^{3}\left(8 a^{2}-30 \lambda^{2}\right) x-9 a^{4} \lambda^{2} \theta=0 . \tag{20}
\end{gather*}
$$

The values of $x$ and $z$ being found, the portion CDBA of the barrage, constructed by means of these values, possesses this property, namely, the pressures to the unit of surface in the points A and B according as the reservoir is full or empty are equal to the limit $\mathrm{R}^{\prime}$, and we are at the same time certain, as was shown in our considerations on the profile of equal resistance, that the pressure in any point $a$ of the face $\mathbf{D} A$ is less than the limit $\mathrm{R}^{\prime}$.

It will be very easy to ascertain what pressure is borne by any point of the face $\mathbf{D A}$; if it happen that in certain points the difference between this pressure and the limit $\mathrm{R}^{\prime}$ is considerable, a marked advantage will be gained by subdividing the height CD into two and calculating a profile of the form of Fig. 2271. This calculation may be effected in the manner we have already pointed out, and will offer no difficulty whatever.

In this case, for the upper portion CD M N of the profile, the height CM is given; the only unknown is $\mathrm{V}^{\prime} \mathrm{N}=x$, and as the pressure in $M$ when the reservoir is empty is necessarily less than $R^{\prime}$, we have only one equation for determining $x$; this is evidently one of the equations [16] and [17], in which the value $h$ chosen as the height C M
 will be substituted for the unknown $z$.

The value of $x$ will therefore be given by one of the following equations;

$$
\begin{gather*}
\lambda x^{2}+2 a \lambda x+\lambda a^{2}-\theta h^{3}-a^{2} h=0  \tag{21}\\
(2 \lambda-h) x^{3}+2 a(3 \lambda-2 h) x+3 a^{2} \lambda-4 a^{2} h-\theta \lambda h^{2}=0 . \tag{22}
\end{gather*}
$$

The portion MNBA of the profile may be determined by employing a method analogous to that followed in calculating the profile of Fig. 2270, taking as unknown the height MB=zand $\mathrm{N} \mathrm{A}=x$.

The equations of the problem will always be the relations [9] and [10], in which the quantities $\mathrm{P}, l$, and $u$, will be expressed as functions of the data of the question.

Representing by $s$ the surface of the upper portion MCDN of the profile, by $\beta$ the distance $\mathrm{N} G$ from the vertical passing through the centre of gravity of this portion to the point N , and by $b$ the length MN, we shall have
$\mathbf{P}=s \delta^{\prime}+b z \delta^{\prime}+\frac{z x \delta^{\prime}}{2}=\left(\frac{2 s+2 b z+z x}{2}\right) \delta^{\prime} \quad \mathrm{F}=\frac{(h+z)^{2} \delta}{2}, \quad l=b+x, \quad u=\mathrm{A} \mathbf{E}=x+\mathrm{V} \mathbf{K}-\mathrm{K} \mathbf{E}$.
K E will be given immediately by the relation

$$
\mathbf{K} \mathbf{E}=0 \mathrm{~K} \times \frac{\mathbf{F}}{\mathbf{P}}=\frac{(h+z)^{3} \delta}{3(2 s+2 b z+z x) \delta^{\prime}}=\frac{(h+z)^{3} \theta}{3(2 s+2 b z+z x)}
$$

V K may be found by considering the moment of the resultant P of the vertical forces with respect to the point N as equal to the sum of the moments of the components with respect to the same point.

$$
\mathrm{P} \times \mathrm{VK}=\beta s \delta^{\prime}+b z \delta^{\prime} \times \frac{b}{2}-\frac{z x \delta^{\prime}}{2} \times \frac{x}{3}
$$

whence we deduce, substituting the value of $\mathrm{P}, \mathrm{VK}=\frac{6}{3} \frac{\beta s+3 b^{2} z-z x^{2}}{(2 s+2 b z+z x)}$. And, consequently,
the equation $u=x+\mathrm{VK}-\mathrm{K} \mathrm{E}$ gives $u=\frac{3 x(2 s+2 b z+z x)+6 \beta s+3 b^{2} z-z x^{2}-(h+z)^{3} \theta}{3(2 s+2 b z+z x)}$.
Substituting the values of $\mathrm{P}, l$, and $u$, in the relations [9] and [10], we obtain the following equations;

$$
\begin{gather*}
\theta z^{3}-\lambda x^{2}+3 h \theta z^{2}-2(s+\lambda b) x+\left(b^{2}+3 h^{2} \theta\right) z+4 b s-6 \beta s+\theta h^{3}-\lambda b^{2}=0  \tag{23}\\
z^{2} x^{2}-2 \lambda z x^{2}+4 b z^{2} x+\theta \lambda z^{3}+2(2 s-3 \lambda b) z x+\left(4 b^{2}+3 h \theta \lambda\right) z^{2}-6 \lambda s x+ \\
\left(8 b s+3 h^{2} \theta \lambda-3 \lambda b^{2}\right) z+4 s^{2}-6 \lambda \beta s+\theta \lambda h^{3}=0 . \tag{24}
\end{gather*}
$$

The first or the second of these equations will be employed according as $u$ is greater or less than $\frac{b+x}{3}$ 。

A second equation between $x$ and $z$ may be obtained by expressing the pressure to the unit of surface at the point $B$ as equal to the limit $R^{\prime}$ when the reservoir is empty. The relations [9] and $[10]$ will still be the equations of the problem.

The value of $u$ will thus be equal to $\mathbf{B K}$, and this quantity may be obtained as a function of

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the data of the question by expressing the moment of the whole weight $P$ with respect to the point B as equal to the sum of the moments of the components of this weight.

$$
\mathrm{BK} \times \mathrm{P}=\delta^{\prime} s(b-\beta)+b z \delta^{\prime} \times \frac{b}{2}+\frac{x z \delta^{\prime}}{2}\left(b+\frac{x}{3}\right)
$$

Whence we deduce, substituting the value of P and, for the sake of brevity, putting $b-\beta=\alpha$,

$$
u=\frac{6 a s+3 b^{2} z+3 b z x+z x^{2}}{3(2 s+2 b z+z x)} .
$$

Substituting this expression, as well as the values of P and $l$ in the relations [9] and [10], we obtain the equations

$$
\begin{equation*}
z x^{2}-\lambda x^{2}+3 b z x+2(2 s-\lambda b) x+b^{2} z+4 s b-6 \alpha s-\lambda b^{2}=0 \tag{25}
\end{equation*}
$$

$$
z^{2} x^{2}-\lambda z x^{2}+4 b z^{2} x+(4 s-3 \lambda b) z x+4 b^{2} z^{2}+\left(8 b s-3 \lambda b^{2}\right) z+4 s^{2}-6 \lambda \alpha s=0 \text {. [26] }
$$

Recourse will be had to equation [25] or equation [26], according as $x$ is greater or less than $\frac{b+x}{3}$.

The values of $x$ and $z$ may be obtained by combining one of the equations [23] and [24] with one of the equations [25] and [26].

The calculations to be worked out are long, but they offer no serious difficulty. Elimination is accomplished in a very simple way, for in whatever manner the equations are combined, there will be always one at least of the second degree with respect to one of the unknowns.

The upper portion CD B A, in which the inner face is vertical, being determined by the preceding considerations, the dimensions to be adopted for the lower portion A B, A' $\mathbf{B}^{\prime}$, Fig. 2269, in which the wall has a talus upon both sides, remain to be ascertained.

We will suppose this portion of the wall divided into a number of sections, and make A B, $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$, Fig. 2272, the section, the dimensions of which are to be determined.

The height of this section is chosen beforehand; we will represent it by $h$, and take as unknowns the distances $\mathrm{A}^{\prime} \mathrm{L}=x$, $\mathbf{B}^{\prime} \mathbf{H}=y$. The unknowns may be determined, as in the questions
 we have been considering, by expressing the pressures in $\mathrm{A}^{\prime}$ and $B^{\prime}$, according as the reservoir is full or empty, as equal to the limit $R^{\prime}$. The equations of the problem will still be the relations [9] and [10].

$$
\begin{equation*}
2\left(2-\frac{3 u}{l}\right) \frac{\mathrm{P}}{\delta^{\prime} l}=\lambda, \quad[9] \quad \frac{2}{3} \frac{\mathrm{P}}{u \delta^{\prime}}=\lambda, \tag{9}
\end{equation*}
$$

in which the weight P and the quantities $l=\mathrm{A}^{\prime} \mathrm{B}^{\prime}, u=\mathrm{AE}$, have to be replaced by their values as functions of the data of the question.

The calculations will be effected in the way already pointed out. We have first

$$
\begin{gathered}
\mathbf{P}=s \delta^{\prime}+b h^{\prime} \delta+\frac{h^{\prime} x \delta^{\prime}}{2}+\frac{h^{\prime} y \delta^{\prime}}{2}+\left(\frac{2 h+h^{\prime}}{2}\right) y \delta=\frac{\left(s+b h^{\prime}\right) \delta^{\prime}+h^{\prime}(x+y) \delta^{\prime}+\left(2 h+h^{\prime}\right) y \delta}{2}, \\
\mathbf{F}=\left(\frac{h+h^{\prime}}{2}\right)^{2} \delta, \\
l=b+x+y, \\
u=\mathbf{A}^{\prime} \mathrm{E}=x+\mathbf{L} \mathbf{K}-\mathbf{K ~ E}, \\
\mathbf{K} \mathbf{E}=0 \mathbf{K} \times \frac{\mathbf{F}}{\mathbf{P}}=\frac{\left(h+h^{\prime}\right)^{3} \delta}{3\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y) \delta^{\prime}+\left(2 h+h^{\prime}\right) y \delta\right]} \\
=\frac{\left(h+h^{\prime}\right)^{3} \theta}{3\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y)+\left(2 h+h^{\prime}\right) y \theta\right]} .
\end{gathered}
$$

The value of LK may be obtained by expressing the moment, with respect to the point A, of the whole weight P , which includes that of the water weighing upon the inclined portion $\mathrm{BB}^{\prime}$ of the inner side, as equal to the sum of the moments of the components of this force.

$$
\begin{aligned}
\mathrm{KL} & =\frac{12 s \beta \delta^{\prime}+6 b^{2} h^{\prime} \delta^{\prime}+2 h^{\prime} y \delta^{\prime}(y+3 b)+3\left(2 h+h^{\prime}\right)(y+2 b) y \delta-2 h^{\prime} \delta^{\prime} x^{2}}{6\left[2\left(s+b h^{\prime}\right) \delta^{\prime}+h^{\prime}(x+y) \delta^{\prime}+\left(2 h+h^{\prime}\right) y \delta\right]} \\
& =\frac{12 s \beta+6 b^{2} h^{\prime}+2 h^{\prime} y(y+3 b)+3\left(2 h+h^{\prime}\right)(y+2 b) y \theta-2 h^{\prime} x^{2}}{6\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y)+\left(2 h+h^{\prime}\right) y \theta\right]} .
\end{aligned}
$$

We deduce immediately from these expressions the value of $u$.
$u=\frac{6 x\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y)+\left(2 h+h^{\prime}\right) y \theta\right]+12 s \beta+6 b^{2} h^{\prime}+2 h^{\prime} y(y+3 b)+3\left(2 h+h^{\prime}\right)(y+2 b) y \theta-2 h^{\prime} x^{2}-2 \theta\left(h+h^{\prime}\right)^{3}}{6\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y)+\left(2 h+h^{\prime}\right) y \theta\right]}$. $6\left[2\left(s+b h^{\prime}\right)+h^{\prime}(x+y)+\left(2 h+h^{\prime}\right) y \theta\right]$
This value must be substituted in equation [9] or equation [10], according as $u$ is greater or less than $\frac{l}{3}$.

We thus obtain one of the two following equations [28] and [29], writing for the sake of brevity;

$$
\left\{\begin{align*}
s+b h & =\sigma  \tag{27}\\
s \beta+\frac{b^{2} h^{\prime}}{2} & =\mu \\
h+h^{\prime} & =\mathrm{H} \\
2 h+h^{\prime} & =\mathrm{H}^{\prime}
\end{align*}\right.
$$

$$
\left.\begin{array}{l|l|l|l|l|l}
2 \lambda & y^{2}+4 \lambda & x y+y+2 \lambda & x^{2}+4 b \lambda & y+4 b \lambda & x+2 b^{2} \lambda \\
-H^{\prime} \theta & +2 H^{\prime} \theta & & 2 b H^{\prime} \theta & -4 b h^{\prime} & -8 b \sigma  \tag{29}\\
-2 h^{\prime} & -2 h^{\prime} & & -2 b h^{\prime} & +4 \sigma & -2 H^{3} \theta \\
& -8 \sigma & +12 \mu
\end{array}\right\}=0 .
$$

We have now to express the pressure to the unit of surface at the point $\mathrm{B}^{\prime}$ as equal to the limit $R^{\prime}$, when the reservoir is empty.

The value of $u$ is thus equal to $\mathrm{B}^{\prime} \mathrm{K}^{\prime}=y+\mathrm{HK}^{\prime}$, and the value of HK is found by stating the moment of the whole weight with respect to the point $B$ as equal to the sum of the moments of the components of this weight with respect to the same point.

The pressure of the water no longer existing, the value of P is

$$
\mathbf{P}=\frac{2\left(s+b h^{\prime}\right) \delta^{\prime}+h(x+y) \delta^{\prime}}{2}=\frac{\left(2 \sigma+h^{\prime} x+h^{\prime} y\right) \delta^{\prime}}{2}
$$

And we find $u=\frac{2 h^{\prime} y^{2}+3 h^{\prime} x y+h^{\prime} x^{2}+6 \sigma y+3 b h^{\prime} x+6 \mu^{\prime}}{3\left(h^{\prime} y+h^{\prime} x+2 \sigma\right)}$, making $s a+\frac{b^{2} h^{\prime}}{2}=\mu^{\prime}$.
This value of $u$, substituted in the relations [9] and [10], gives the two following equations [30] and [31];
$\left.\begin{array}{c|l|l|l|l}\lambda y^{2}+2 \lambda \lambda \\ -h^{\prime} & -h^{\prime} & \begin{array}{ll}x^{2}+2 b \lambda & y+2 b \lambda \\ -2 b h^{\prime} & +b h^{\prime} \\ +2 \sigma & -4 \sigma\end{array} & x+\lambda b^{2} \\ +6 \mu^{\prime} \\ -4 \sigma b\end{array}\right\}=0$
$\left.\begin{array}{l|l|l|l}2 h^{\prime} \lambda \\ -h^{\prime 2} & y^{2}+3 h^{\prime} \lambda & x y+h^{\prime} \lambda & x^{2}+6 \lambda \sigma \\ -2 h^{\prime 2} & -h^{\prime 2} & y+3 b h^{\prime} \lambda & x+6 \mu^{\prime} \lambda \\ -4 h^{\prime} \sigma & 4 h^{\prime} \sigma\end{array}\right\}=0$
Equation [30] or equation [31] will be required, according as $u$ is greater or less than $\frac{l}{3}$.
The values of $x$ and $y$ will be obtained by combining one of the equations [28] and [29] with one of the equations [30] and [31]. These equations being of the second degree in $x$ and $y$, there will be no difficulty in the matter of elimination, the rather complicated coefficients of the different terms being, in each particular case, replaced by numbers.

The choice of the equation, adapted to the question, may give occasion for tentative experiments as we have already explained, the value of the relation of $u$ to $\frac{l}{3}$ which serves to determine this choice being a function of the unknowns. It will be necessary to make some hypotheses on the value of this relation, and having solved the equations chosen, to verify the suppositions; if this verification be unsatisfactory, the equations must be chosen by the aid of the values of the relation of $u$ to $\frac{l}{3}$, calculated with the values found for $x$ and $y$, which will be sufficiently near to enable us to follow out the indications given by this relation.

The first section A B $n^{\prime} n^{\prime}$ of the lower portion A B, $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$, of the profile, Fig. 2269, being determined, the dimensions of the second section $m^{\prime} n^{\prime}$ A $\mathrm{B}^{\prime}$ may be calculated in the same manner with the equations just found by modifying them as follows; -
$s \delta^{\prime}$ representing in the calculations the total weight of the portion of the barrage situate above the section considered, we must take into account the weight of the column of water which presses upon the face $\mathbf{B}^{\prime} m^{\prime}$, a weight equal to $s^{\prime \prime} \delta$ if we represent by $s^{\prime \prime}$ the vertical section $c^{\prime} c \mathbf{B} m^{\prime}$ of this column ; in the case under consideration we have $s^{\prime \prime}=\left(h+\frac{h^{\prime}}{2}\right) y_{1}$, denoting by $y_{1}$ the value of $m^{\prime} m^{\prime}$ from $y$ which has been determined.

The weight of that portion of the barrage situate above the section considered will thus be $s^{\prime} \delta^{\prime}+s^{\prime \prime} \delta, s^{\prime}$ representing the surface $c \mathrm{~B} m^{\prime} n^{\prime} \mathrm{AD}$.

This quantity must be substituted for $s \delta^{\prime}$ in the equations [28], [29], [30], and [31], by making $s \delta^{\prime}=s^{\prime} \delta^{\prime}+s^{\prime \prime} \delta$, whence $s=s^{\prime}+s^{\prime \prime} \theta$.

But the modifications to be introduced into the formulæ will not end here; the moment of the welght $s \delta^{\prime}$ with respect to the point $n^{\prime}$ will be changed: we must take into account the moment of the weight $s^{\prime \prime} \delta$ of the water. Denoting by $\beta^{\prime}$ the distance from the centre of gravity of the surface $s^{\prime \prime}$ to the point $n^{\prime}$, we may state $s \delta^{\prime} \beta=s^{\prime} \delta^{\prime} \beta+s^{\prime \prime} \delta \beta$, or $s \beta=s^{\prime} \beta+s^{\prime \prime} \theta \beta$.

Substituting in the formulæ [28], [29], [30], and [31], for $s$ and $s \beta$ the values so determined,
these formulw become perfectly applicable to the determination of the dimensions of the section $\mathrm{A} m^{\prime} n^{\prime} \mathrm{B}$, and in general to the calculation of any section, knowing the preceding ones.

Determination by an Approximative Method of the Profile of Equal Resistance. -The profile to be adopted in practice consists, as we have seen, of three portions. The first CD B A, Fig. 2268, has a vertical face on each side; the second ABMN offers a vertical face on the side of the water and an inclined face on the outside, and the third M N S T has an inclined face on each side.

We have seen how the portion CDBA is determined. Below the horizontal plane BA we will suppose the barrage divided into sections of equal height, and the question is to calculate for each section the projections, such as $b^{\prime} b^{\prime \prime}, \mathbf{N} n, n^{\prime} n^{\prime \prime}$, of the elements of the face upon the lower horizontal plane which serves as its base.

Let us see, in the first place, how any section AB, $c^{\prime} b^{\prime}$, of the portion BAMN may be determined. The height of this section is known, we will represent it by $h^{\prime}$.

The problem will be solved at once by referring to the determination of the part MNBA of Fig. 2269 developed above.

We have merely to substitute in the equations [23] and [24] $h^{\prime}$ for the unknown height $x$, and we thus obtain the following equations:-

$$
\begin{gather*}
\lambda x^{2}+2(\lambda b+s) x+\lambda b^{2}+6 \mu-4 b \dot{\sigma}-\theta \mathrm{H}^{3}=0  \tag{32}\\
\left(2 \lambda h^{\prime}-h^{\prime 2}\right) x^{2}+2 \sigma\left(3 \lambda-2 h^{\prime}\right) x+6 \mu \lambda-4 s^{2}-\theta \lambda \mathrm{H}^{3}=0 . \tag{33}
\end{gather*}
$$

Making, as before [347.

$$
\left\{\begin{aligned}
s+b h^{\prime} & =\sigma \\
s \beta+\frac{b^{2} h^{\prime}}{2} & =\mu \\
h+h^{\prime} & =\mathrm{H}
\end{aligned}\right.
$$

These equations might be obtained directly by the method given for equations [23] and [24]. The first or the second will be employed according as $u$ is greater or less than $\frac{b+x}{3}$.

The preceding formulæ will serve to calculate successively all the sections of the portion B A M N of the profile, Fig. 2268. Having determined each of them, it will be necessary to ascertain if the pressure in the points $c^{\prime}, M$, of the inner facing is below the limit $R^{\prime}$; when this limit is exceeded, a talus must be given to both facings, and we pass on to determine the last portion M N S T of the profile.

This problem has been solved above in determining the lower part of the profile of Fig. 2272. The formulæ to be employed are the equations [28], [29], [30], and [31].
M. de Sazilly, in his considerations, published in 1853 in the 'Annales des Ponts et Chaussées,' on the walls of reservoirs, adopted a profile differing slightly from that of Fig. 2268, for the calculation of a wall the form of which should deviate but little from that of equal resistance. This engineer, instead of supposing the faces of the wall formed of successive inclined surfaces, assumed that they ought to be composed of a system of vertical facings of inconsiderable height; separated by gradations or retreats, as shown in Fig. 2273.

The profile is calculated for the condition that the pressure
 exerted upon each of the inner angles of the outer facing, when the reservoir is full, shall be equal to the limit $\mathrm{R}^{\prime}$, and that this limit shall be reached upon the inner angles of the inside facing when the reservoir is empty.

The upper portion CD B A of the profile is determined in exactly the same way as in Fig. 2268. The breadth of the summit $a$ and the height A $\mathrm{D}=x$, are connected according to the case by one of the equations [14] and [15].

$$
\begin{equation*}
\theta x^{3}+a^{2} x-a^{2} \lambda=0 \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\theta \lambda x^{2}+4 a^{2} x-3 \lambda a^{2}=0 \tag{15}
\end{equation*}
$$

To put these equations in the form of those given by M. Sazilly, they must be solved with respect to $a$; we thus obtain the formulæ [35] and [36], which, with the difference of notation, are precisely those given in p. 244 of Graeff's paper, 'Annales des Ponts et Chaussées,' 1866.

$$
\begin{equation*}
a=x \sqrt{\frac{\theta x}{\lambda-x}} \tag{35}
\end{equation*}
$$

$$
\begin{equation*}
a=x \sqrt{\frac{\theta \lambda}{3 \lambda-4 x}} . \tag{36}
\end{equation*}
$$

The graduations of the second portion A BMN of the barrage may be calculated in the manner described for the corresponding part ABMN of Fig. 2268. We thus get the following equations;-

$$
\begin{gathered}
\left(\lambda-h^{\prime}\right)(b+x)^{2}+2\left(\sigma-b h^{\prime}\right)(b+x)-\theta \mathrm{H}^{3}+6 \mu-6 b \sigma+3 b^{2} h^{\prime}=0 \\
h^{\prime}\left(3 \lambda-4 h^{\prime}\right) x^{2}+2 \sigma\left(3 \lambda-4 h^{\prime}\right) x-4 \sigma^{2}+6 \mu \lambda-\theta \mathrm{H}^{3} \lambda=0
\end{gathered}
$$

Solving these equations, we arrive at the following formulæ ;-

$$
\begin{gather*}
x+b=-\frac{\sigma-b h^{\prime}}{\lambda-h^{\prime}}+\sqrt{\left(\frac{\sigma-b h^{\prime}}{\lambda-h}\right)^{2}+\frac{\theta \mathrm{H}^{3}-6\left(\mu-\sigma b+\frac{1}{2} b^{2} h^{\prime}\right)}{\lambda-h^{\prime}}}  \tag{37}\\
x=-\frac{\sigma}{h^{\prime}}+\sqrt{\left(\frac{\sigma}{h^{\prime}}\right)^{2}+\frac{4 \sigma^{2}-\lambda\left(6 \mu-\theta \mathrm{H}^{3}\right)}{h^{\prime}\left(3 \lambda-4 h^{\prime}\right)}} \tag{38}
\end{gather*}
$$

## DAMMING.

The first or second of these will be employed according as

$$
u=\frac{\mu+\sigma x+\frac{h^{\prime} x^{2}}{2}-\frac{\theta \mathrm{H}^{3}}{6}}{\sigma+h^{\prime} x}<\text { or }>\frac{1}{3}(b+x)
$$

The third portion MNST of the profile may be determined by the methods employed above for Fig. 2268. The following equations must be substituted for those [28, 29, 30, 31] relative to this latter profile.

| $\begin{align*} & \lambda  \tag{39}\\ - & h \theta \\ - & h^{\prime} \end{align*}$ | $\begin{aligned} & y^{2}+2 \lambda \\ & +2 h \theta \\ & -2 h^{\prime} \end{aligned}$ | $\begin{aligned} & x y+\lambda \\ & -h^{\prime} \end{aligned}$ | $\begin{aligned} & x^{2}+2 b \lambda \\ & +2 b h \theta \\ & +2 b h^{\prime} \\ & -4 \sigma \end{aligned}$ | $\begin{aligned} & y+2 b \lambda \\ & -4 b h^{\prime} \\ & +2 \sigma \end{aligned}$ | $\begin{aligned} & x+b^{\prime} \lambda \\ & -4 b \sigma \\ & -H^{3} \theta \\ & +6 \mu \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{align*} & 3 h^{\prime} \lambda  \tag{40}\\ + & 3 h \lambda \theta \\ - & 4 h^{2} \theta^{2} \\ - & 8 h h^{\prime} \theta \\ - & 4 h^{\prime 2} \end{align*}$ | $\begin{aligned} & y^{2}+6 h^{\prime} \lambda \\ & +6 h \lambda \theta \\ & -8 h h^{\prime} \theta \\ & -4 h^{\prime 2} \end{aligned}$ | $\begin{aligned} & x y+3 h^{\prime} \lambda \\ & -4 h^{\prime} 2 \end{aligned}$ | $\begin{aligned} & x^{2}+6 b h^{\prime} \lambda \\ & +6 b h \lambda \theta \\ & -8 h \theta \sigma \\ & -8 h^{\prime} \sigma \end{aligned}$ | $\begin{aligned} & y+6 \lambda \sigma \\ & -8 h^{\prime} \sigma \end{aligned}$ | $x+6 \lambda \mu$ $-H^{3} \lambda \theta \theta$ $-4 \sigma^{2}$ |
| $-\begin{aligned} & \lambda \\ & h^{\prime} \end{aligned}$ | $y^{2}+2 \lambda$ $-2 h^{\prime}$ | $x y+\lambda$ $-h^{\prime}$ | $\begin{align*} & x^{2}+2 \sigma \\ & +2 b \lambda  \tag{41}\\ & -4 b h^{\prime} \end{align*}$ | $y+2 b \lambda$ $-4 \sigma$ $+2 b h^{\prime}$ | $x+\lambda b^{2}$ $+6 \mu^{\prime}$ $-4 b \sigma$ |
| $\begin{array}{r} 3 h^{\prime} \lambda  \tag{42}\\ -4 h^{\prime 2} \end{array}$ | $\begin{aligned} & y^{2}+6 h^{\prime} \lambda \\ & -8 h^{\prime 2} \end{aligned}$ | $x y+3 h^{\prime} \lambda$ | $\begin{aligned} & x^{2}+6 \lambda \sigma \\ & -8 h^{\prime} \sigma \end{aligned}$ | $\begin{aligned} & y+6 b h^{\prime} \lambda \\ & -8 h^{\prime} \sigma \end{aligned}$ | $x+6 \lambda \mu$ $-4 \sigma^{2}$ |

These equations must be employed in the same way as those relative to Fig. 2268.
Conditions of Stability with respect to Slipping.-Having calculated the profile of a barrage in accordance with the preceding considerations, it will be necessary to ascertain if its dimensions are such as to hinder the wall from slipping horizontally upon one of its courses or upon its foundation.

Denoting by $H$ the distance of a course below the top, the force which tends to make this course slide upon its bed is equal to the horizontal component of the thrust of the water against that portion of the inner facing which is situate above this course, and it is given by the equation

$$
\mathrm{F}=\frac{\delta \mathrm{H}^{2}}{2}
$$

The resistances to the action of this force are friction and the cohesion of the masonry. The friction is proportional to the weight of the upper portion of the structure, and the force of cohesion to the thickness of the wall.

Representing the coefficient of friction by $f$, the force of cobesion to the unit of surface by $\gamma$, the surface of that portion of the profile situate above the course considered by $s$, and the thickness of the wall at this point by $b$, the resistance R to slipping will be $\mathrm{R}=s \delta^{\prime} f+\gamma b$, and we must have

$$
\begin{equation*}
s \delta^{\prime} f+\gamma^{b}>\frac{\delta \mathrm{H}^{2}}{2}, \text { or } 2 \frac{\left(s \delta f+\gamma^{b}\right)}{\delta \mathrm{H}^{2}}>1 . \tag{43}
\end{equation*}
$$

This inequality must be verified for all the horizontal sections of the profile. It must also be verified for the base of the foundations; $f$ and $\gamma$ then representing the coefficients relative to the soil upon which the structure stands.

It will be prudent in practice, for greater safety, not only to make the quantity $\frac{2(s \delta f+\gamma b)}{\delta \mathrm{H}^{2}}$ greater than unity, but equal to the value found for existing reservoirs which have not yielded in any degree. We will consider later the application of formula [43].

Having determined by the aid of the preceding formulæ the dimensions of the profile and proved the conditions of stability with respect to slipping between the courses to be satisfactory, we have now to ascertain if the soil of the foundation is capable of supporting the limit of pressure adopted for the masonry, and if the wall is not liable to slip upon the soil which supports it. The precautions to be taken in these cases are to render the soil of the foundations more solid by processes usually employed in important works and to diminish the pressure upon the soil by widening the base of the profile.

Dams or Barrages suitable to Narrow Valleys.-Barrages constructed in the form of an Arch.Hitherto we have not considered the length of the barrages whose dimensions we have determined; the profiles calculated are such that any length of the structure resists by its own weight only the action of the forces to which it is subject. In the case in which the valleys to be barred are narrow and formed of a resisting soil, it is possible, by giving the barrages the form of an arch, to transmit the thrust horizontally to the sides of the valley, and we have to see if, in certain cases, this arrangement will enable us to reduce the dimensions of the profile.

It is easy to ascertain the influence of the arrangement by which the thrust is transmitted laterally to the sides of the valley. Take Fig. 2256 as an example. This profile is subject to the action of two forces, the weight P and the thrust F . In the case in which the wall is rectilineal or indefinite, these two forces combine to produce a resultant $R$, and to ensure the stability of the structure the point E , where this resultant meets the foundation, must be within the base AB,

.
$\qquad$
(-


[^0]:    36. Bearing-rollers.-Cast-iron plates are interposed between the girders and the masonry, where the former rest upon the abutments. For spans of more than 20 mètres, friction-rollers, or, as they are usually called, bearing-rollers, are placed between these plates and the girders to provide for the movement caused by the expansion of the metal. We may calculate in the following manner the maximum load which these rollers are liable to bear.

    The girder is fixed in the direction of CD, Fig. 1631, to a plate, the thickness of which $=\epsilon$, resting upon rollers having a diameter of $2 r$, which, in their turn, rest upon a lower plate, cemented into the masonry. The diametrical plane A B undergoes no distortion, it only sinks in consequence of the depression at the bottom; if we admit that the face CD also remains plane after compression, the planes CD and AB will have approached each other by the quantity $\lambda$, on account of the pressure $\mathbf{Q}$ exerted by the girder, and supposed collected upon 1 lineal mètre of roller. The lower face of the plate offers a depression into which the roller penetrates, and which is itself flattened. The value of the ratio $\frac{\mathrm{GH}}{\mathrm{HI}}$ of the compression of the roller to that of the plate, for an ele-
    

[^1]:    (1)

[^2]:    The slating of a roof begins from the eaves. The slates are shaped and trimmed on the ground, and two holes are punched through them near the upper end for the nails to fasten them to the boards or battens. Slates may be laid either on boards or battens; these are generally at right angles to the last course of bearers, and therefore generally horizontal; there is less cutting to waste of them when they are laid horizontally; the boards or battens are seldom less than 1 ia. thick, on account of the slating nails which have to be driven into them; battens should be at the utmost at the interval apart of two consecutive courses of slates. Boards are generally used for important buildings, they are stiffer, and keep out the wet and the heat better than battens. The first course of slates at the eaves is little more than half the width of the other courses; it is called the doubling eaves' course, and the next course covers it completely, a special batten being laid under it to give it the same slope as the others: the second course is called the covering eaves' course. At the ridge another half-course is laid over the last course. Slating should have a bond or overlap of at least $2 \frac{1}{2} \mathrm{in}$., that is, the third course should overlap the first course by $2 \frac{1}{2} \mathrm{in}$., and the fourth should overlap the second by the same. Hence, if the overlap be deducted from the length of the slate, half the remainder will give what is called the margin or gauge of the slating. At the hips and valleys the slates are cut to fit the required angles.

    For the nails for fastening slates copper is the best material, and is generally used in important buildings. Zinc and iron nails are sometimes used, but the former are not strong enough, and the

